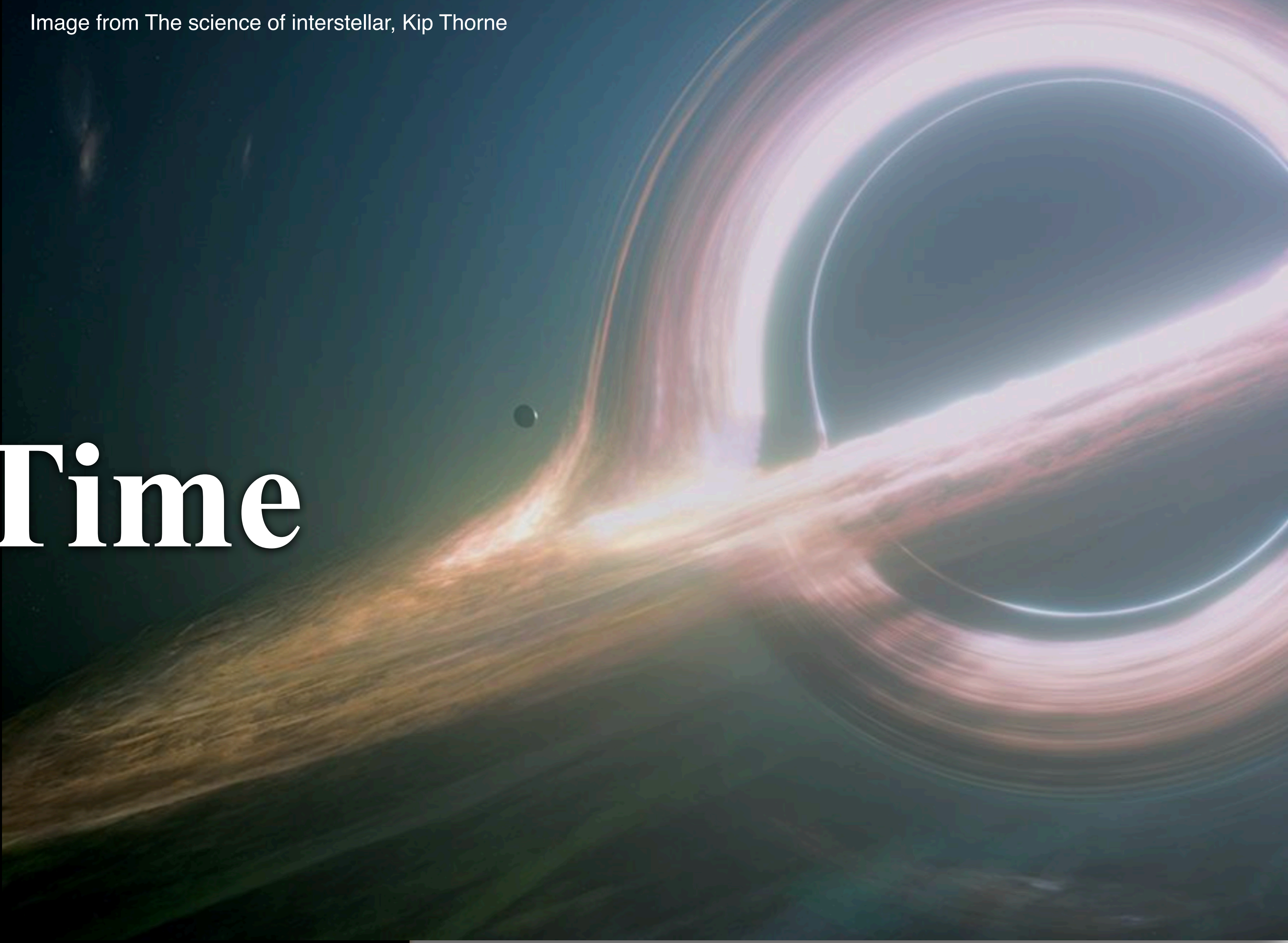
Image from The science of interstellar, Kip Thorne



End of Time

and

Physics Nobel Prize 2020

Alexander Vikman

19.11.2020

FZU Institute of Physics of the Czech Academy of Sciences

CEICO

EUROPEAN UNION
European Structural and Investment Funds
Operational Programme Research, Development and Education

MINISTRY OF EDUCATION, YOUTH AND SPORTS

Black Holes

This year's Nobel Prize in Physics focuses on black holes, which are among the most enigmatic objects in the Universe. The Prize is awarded for establishing that black holes can form within the theory of general relativity, as well as the discovery of a supermassive compact object, **compatible** with a black hole, at the centre of our galaxy

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Black Holes Early History

Dark Star

John Michell (1783)

If there should really exist in nature any bodies, whose density is not less than that of the sun, and whose diameters are more than 500 times the diameter of the sun, since their light could not arrive at us; or if there should exist any other bodies of a somewhat smaller size, which are not naturally luminous; of the existence of bodies under either of these circumstances, we could have no information from sight; yet, if any other luminous bodies should happen to revolve about them we might still perhaps from the motions of these revolving bodies infer the existence of the central ones with some degree of probability, as this might afford a clue to some of the apparent irregularities of the revolving bodies, which would not be easily explicable on any other hypothesis; but as the consequences of such a supposition are very obvious, and the consideration of them somewhat beside my present purpose, I shall not prosecute them any further.

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light cannot escape from

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For our galactic centre Sgr A* observed by **Genzel and Ghez**

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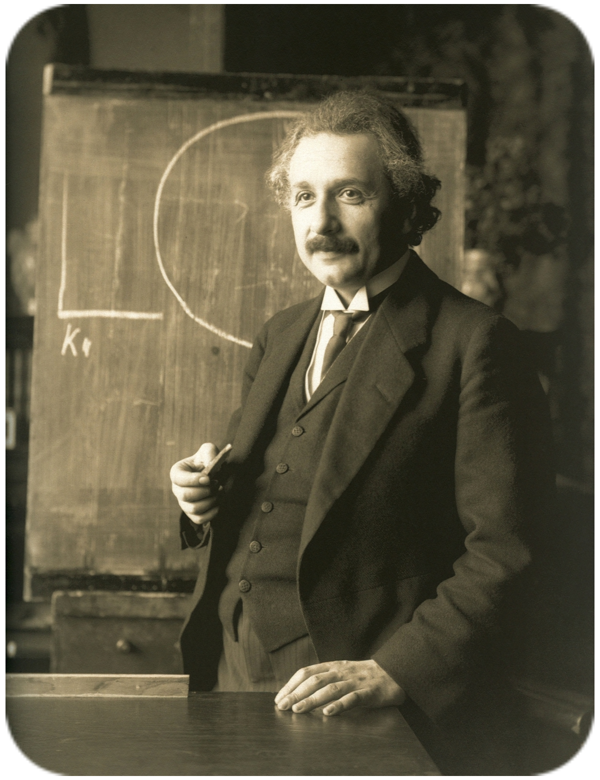
Distance to the Sun $15 \times 10^7 \, \text{km}$

Birth of General Relativity (1915)

From Light Front

$$c^2dt^2 - dx^2 - dy^2 - dz^2 = 0$$

$$ds^2 = g_{\mu\nu}(x)\,dx^\mu dx^\nu$$



To General interval / curved spacetime metric

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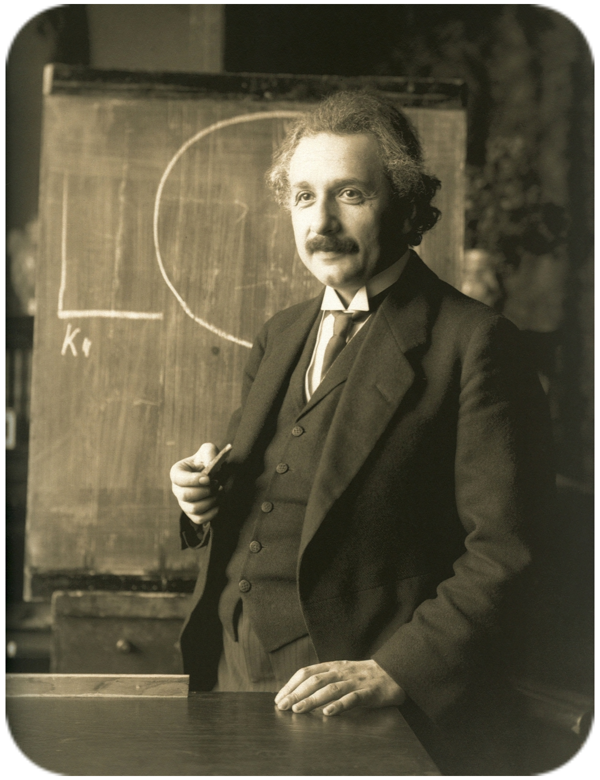

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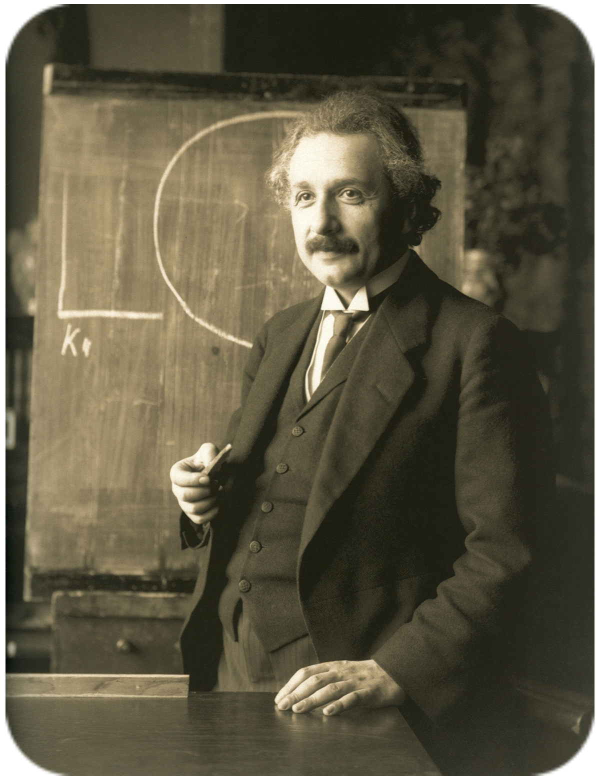

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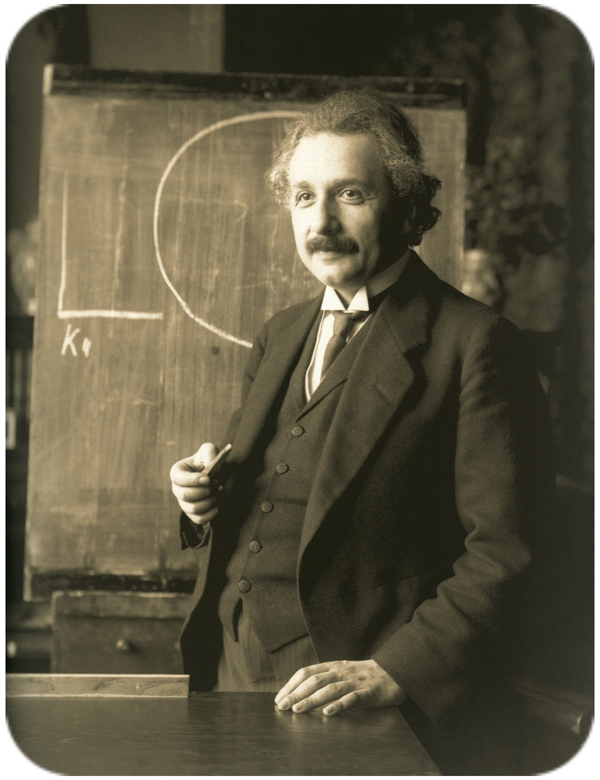

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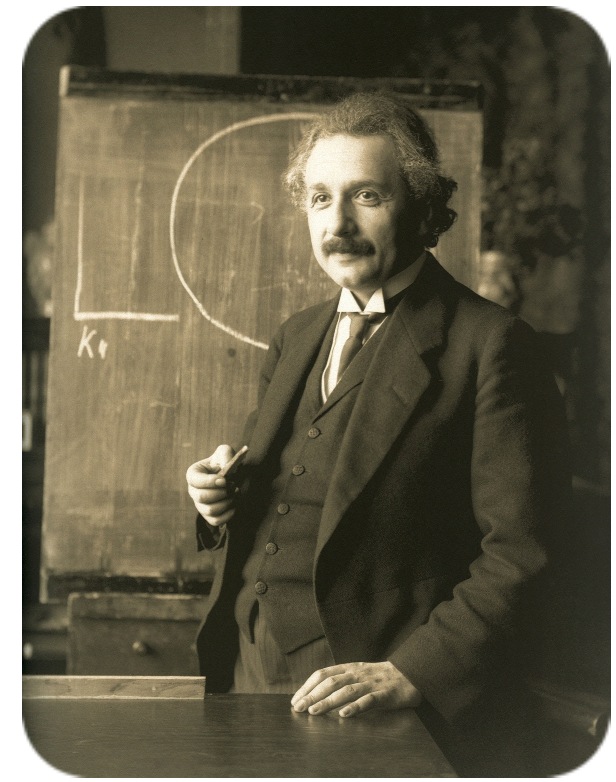

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Energy-Momentum (Stress)
Tensor for matter fields

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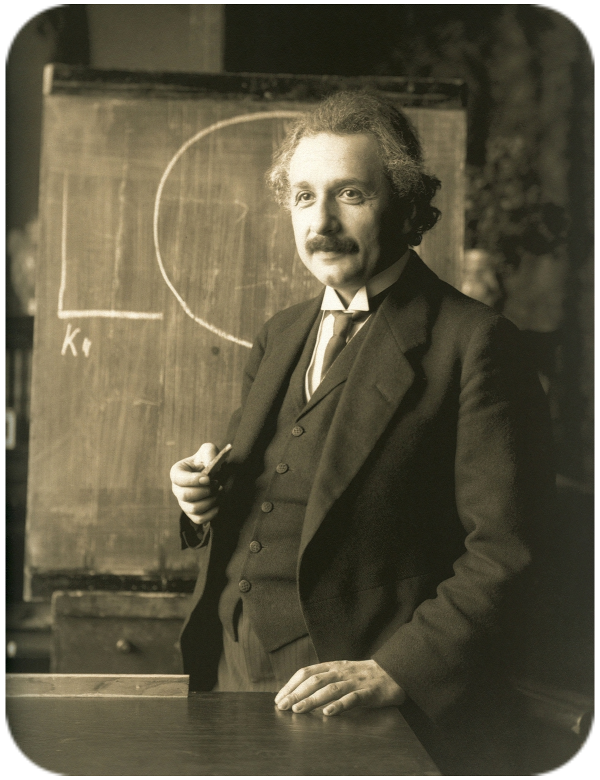

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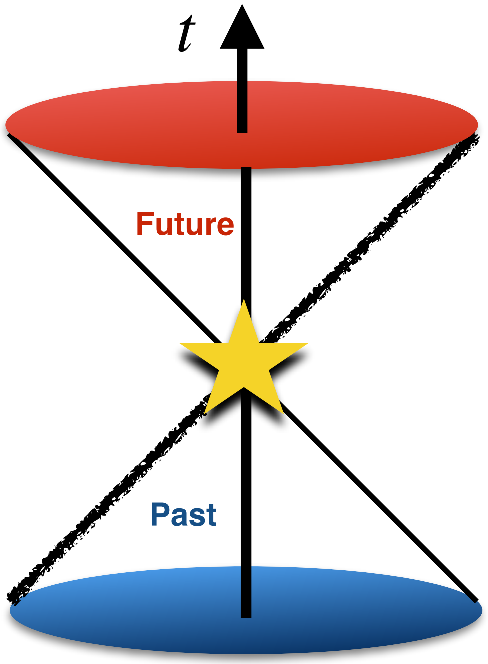

Schwarzschild Solution

$$ds^2 = \left(1 - \frac{r_g}{r}\right) c^2 dt^2 - \left(1 - \frac{r_g}{r}\right)^{-1} dr^2 - r^2 \left(d\theta^2 + \sin^2\theta \, d\phi^2\right)$$

Karl Schwarzschild

(1916)

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t time of the observer at **infinity**

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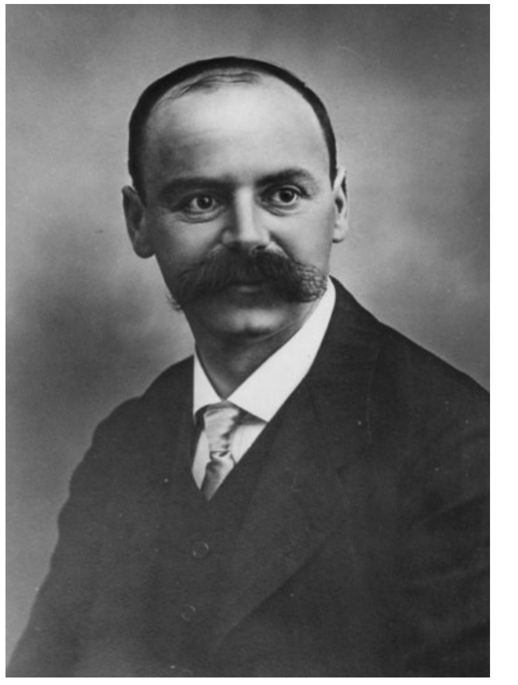

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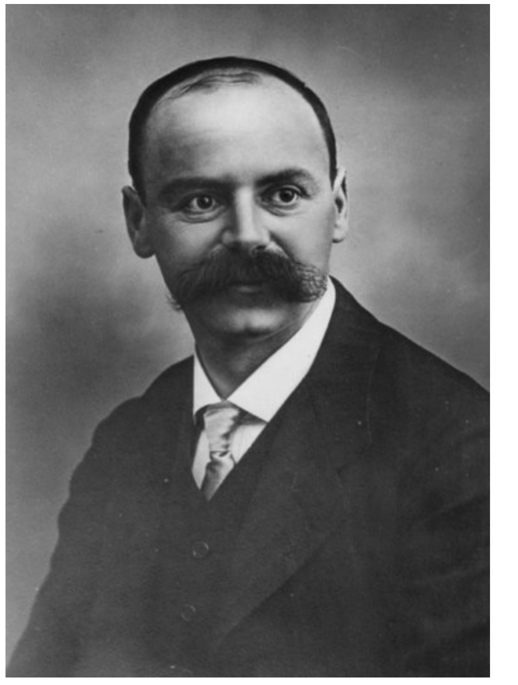

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“Static solution” $g_{\mu\nu}(t, x^i)$

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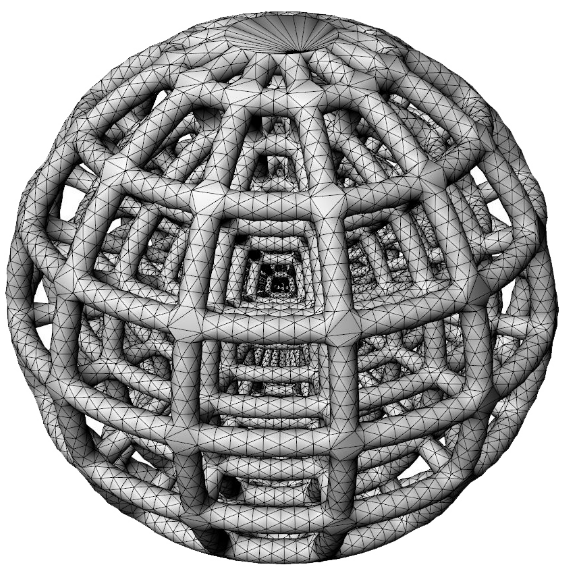

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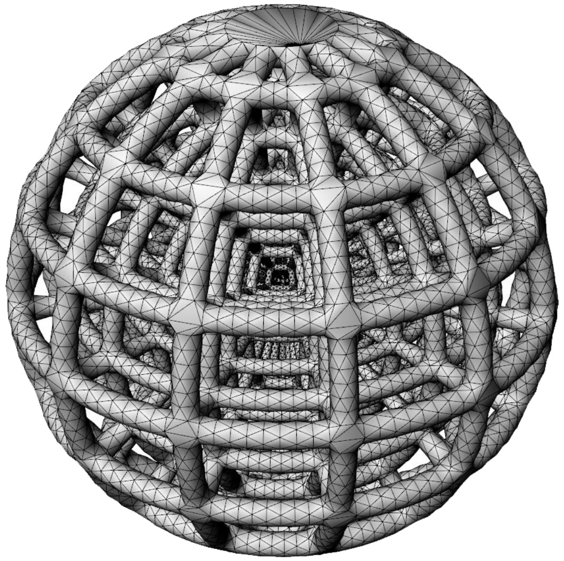

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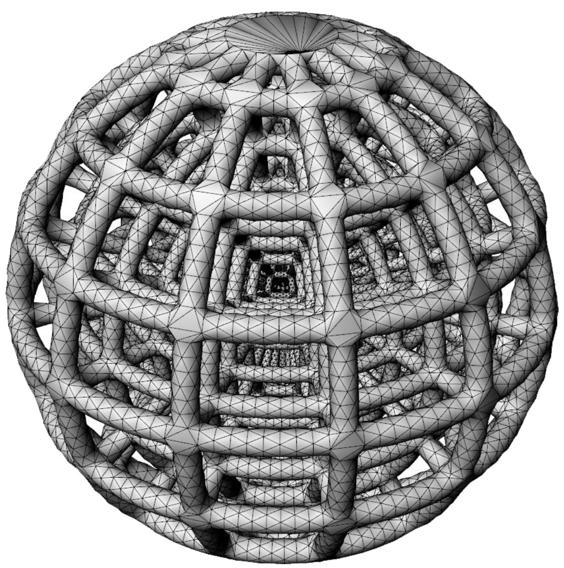

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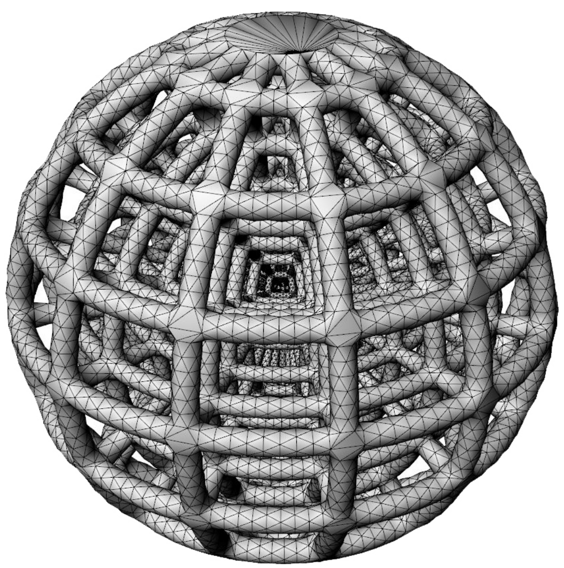

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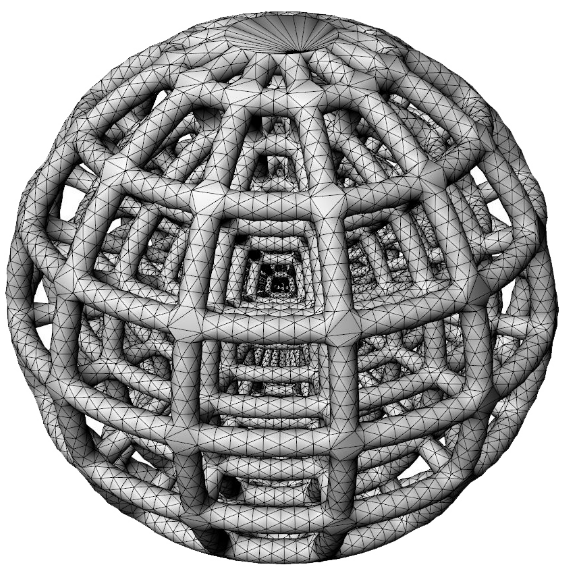

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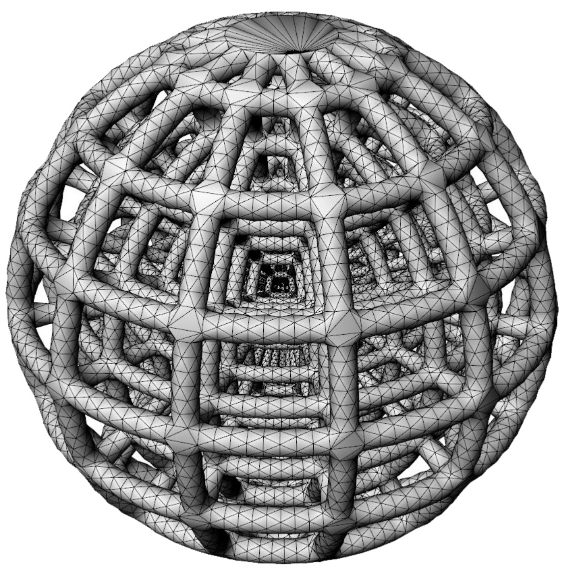

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$c = 1$

Flip the sign

$$ds^2 = \left(1 - \frac{r_g}{r}\right) dt^2 - \left(1 - \frac{r_g}{r}\right)^{-1} dr^2 - r^2 \left(d\theta^2 + \sin^2\theta \, d\phi^2\right)$$

Radial coordinate r is not spacelike inside the Schwarzschild radius!

Time coordinate t is not spacelike inside the Schwarzschild radius!

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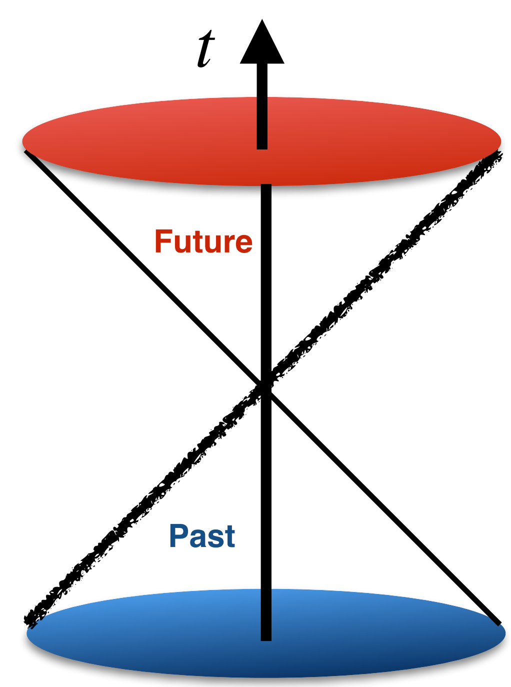

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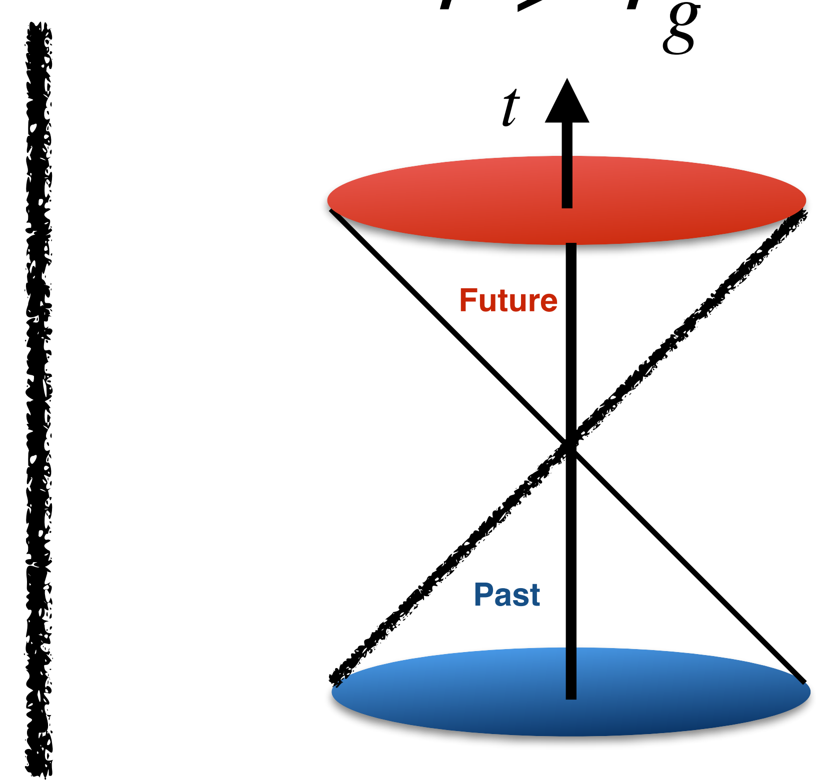
Schwarzschild radius

$$r_g = 2GM$$

$$r > r_g$$



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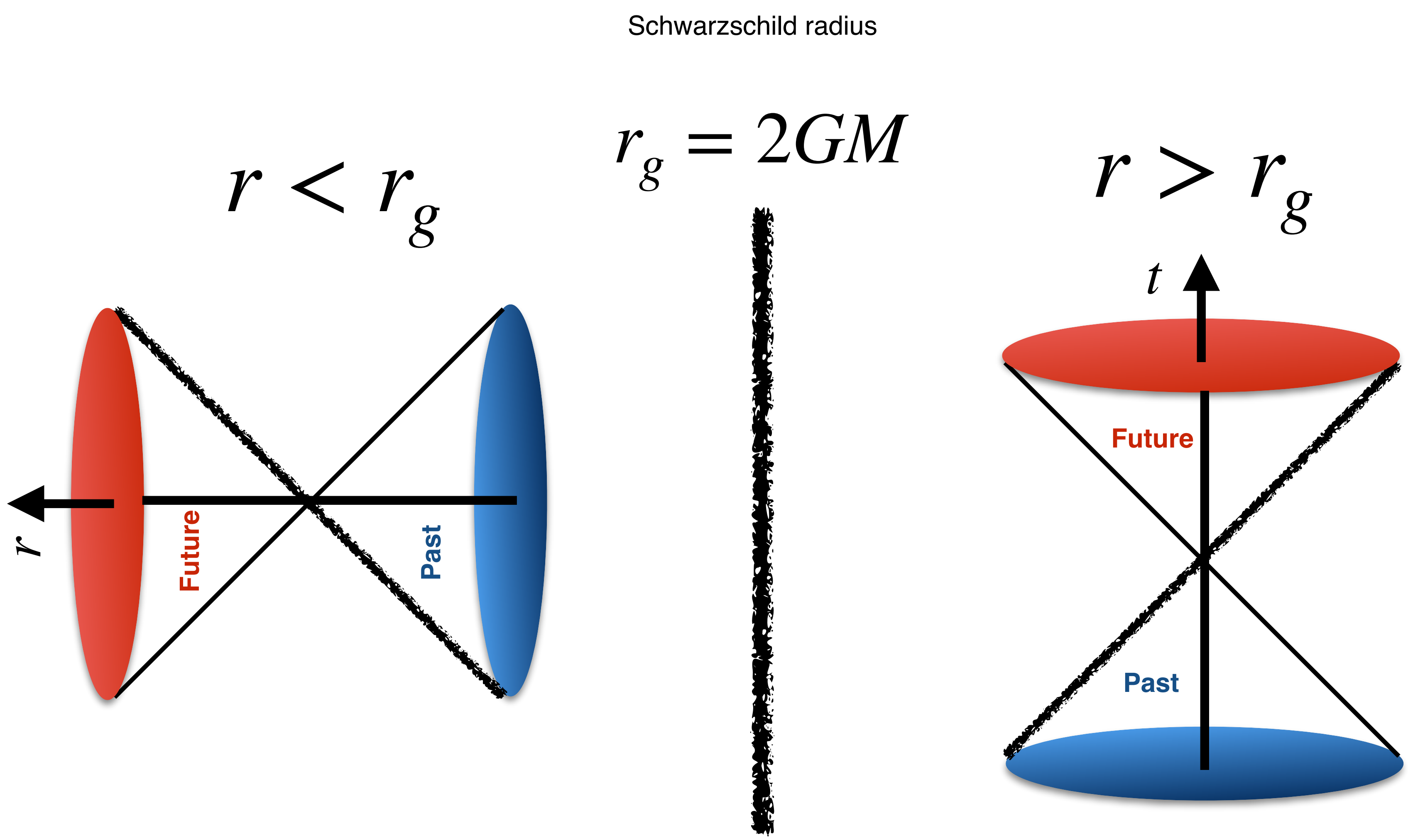
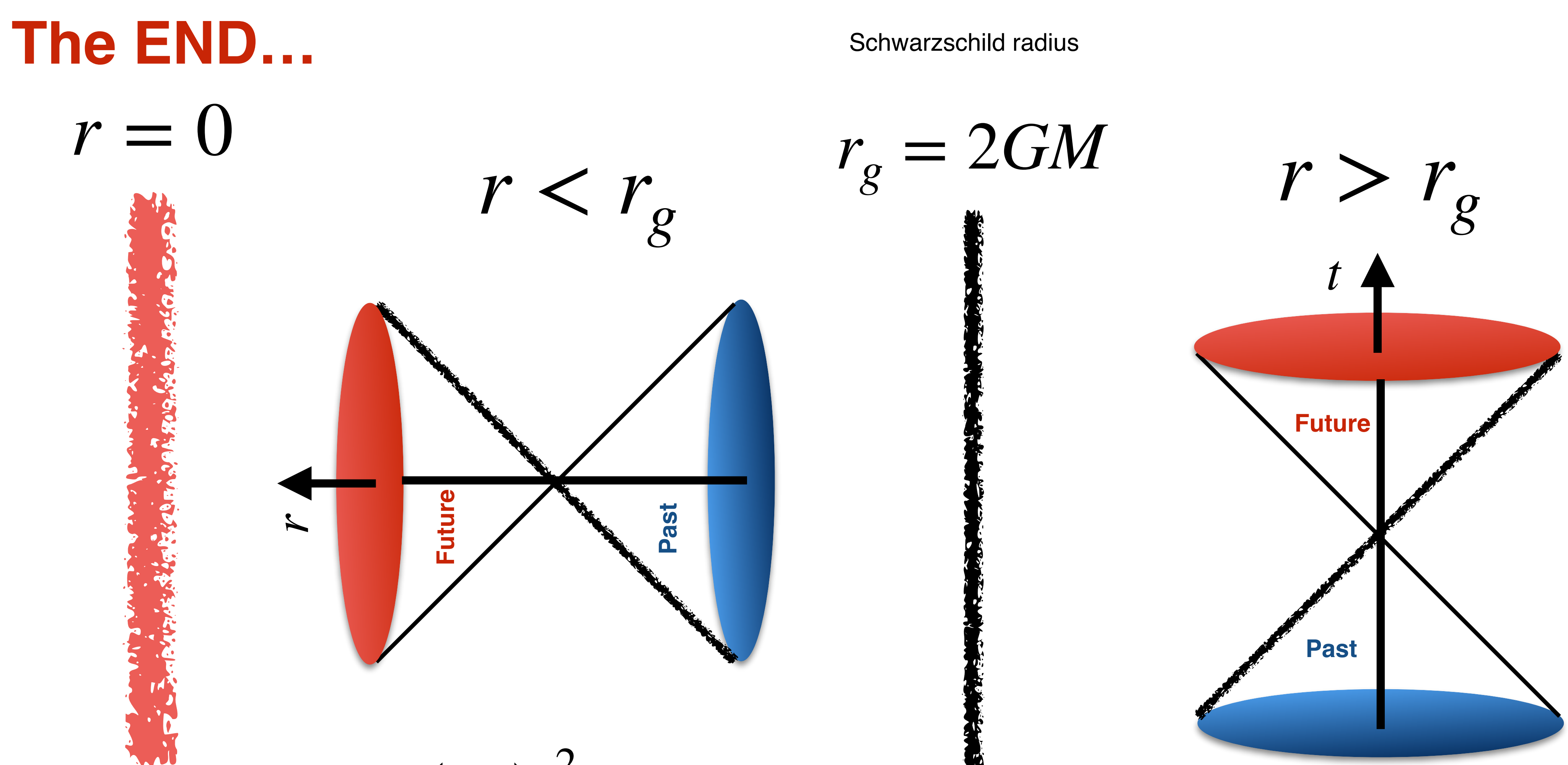


Flip the sign

$$ds^2 = \left(1 - \frac{r_g}{r}\right) dt^2 - \left(1 - \frac{r_g}{r}\right)^{-1} dr^2 - r^2 \left(d\theta^2 + \sin^2\theta \, d\phi^2\right)$$

Radial coordinate r is not spacelike inside the Schwarzschild radius!

Time coordinate t is not spacelike inside the Schwarzschild radius!



$$R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} = \frac{3}{r^4}\left(\frac{r_g}{r}\right)^2$$

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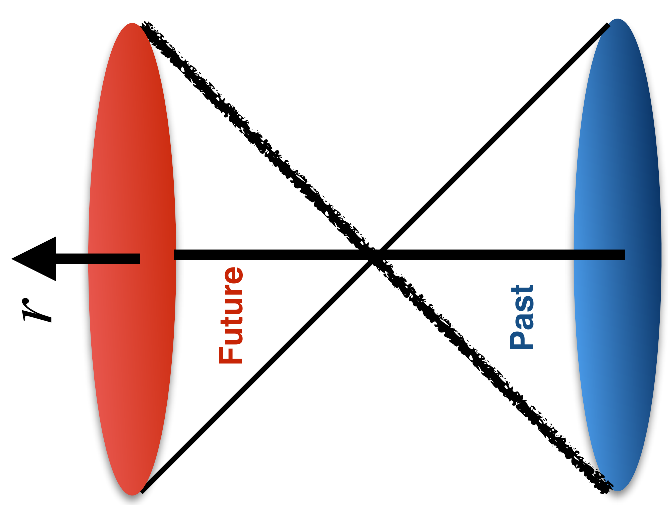
End of Time!

The END…

$r = 0$

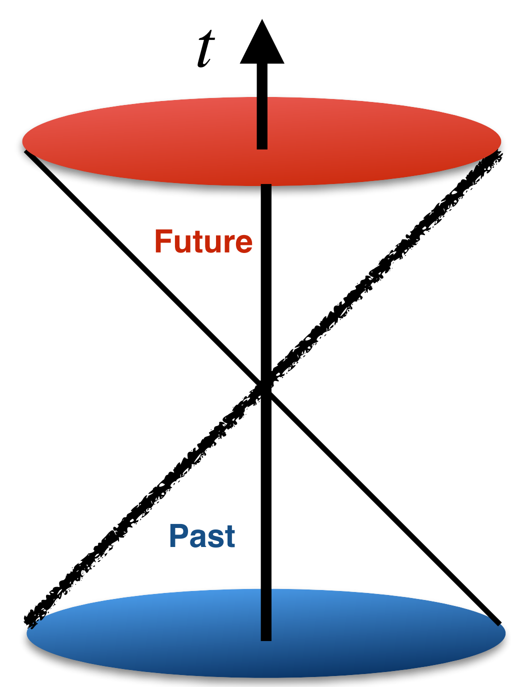
$r < r_g$



Schwarzschild radius

$r_g = 2GM$

$r > r_g$



$$R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} = \frac{3}{r^4}\left(\frac{r_g}{r}\right)^2$$

A whole (anisotropic) Universe Inside!

For $r < r_g$

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Homogeneous, anisotropic time-dependent metric

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$$ds^2 = \left(\frac{r_g}{|\tilde{t}|} - 1\right)^{-1} d\tilde{t}^2 - \left(\frac{r_g}{|\tilde{t}|} - 1\right) d\tilde{r}^2 - \tilde{t}^2 \left(d\theta^2 + \sin^2\theta \, d\phi^2\right)$$

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Stretching

Contraction

A spatially infinite Universe Inside!

$$ds^2 = \left(\frac{r_g}{|\tilde{t}|} - 1\right)^{-1} d\tilde{t}^2 - \left(\frac{r_g}{|\tilde{t}|} - 1\right) d\tilde{r}^2 - \tilde{t}^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)$$

$$-r_g < \tilde{t} < 0 \qquad -\infty < \tilde{r} < +\infty$$

A spatially infinite Universe Inside!

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Infinite spatial volume at $\tilde{t} = const$:

$$V(\tilde{t}) = \int dx\, dy\, dz = \int_{-\infty}^{+\infty} \sqrt{\frac{r_g}{|\tilde{t}|} - 1}\, d\tilde{r} \int \left(|\tilde{t}| \, d\theta\right) \left(|\tilde{t}| \sin\theta \, d\phi\right)$$

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Finite life time

$$T_{end} = \int_{-r_g}^{0} d\tilde{t} \sqrt{g_{00}} = \int_{-r_g}^{0} d\tilde{t} \sqrt{\left(\frac{r_g}{\left|\tilde{t}\right|} - 1\right)^{-1}} = \frac{\pi}{2} r_g$$

Smooth transition through r_g

$$ds^2_{Minkowski} = du\,dv \qquad u = t - r\,, \quad v = t + r$$

Outgoing Ingoing

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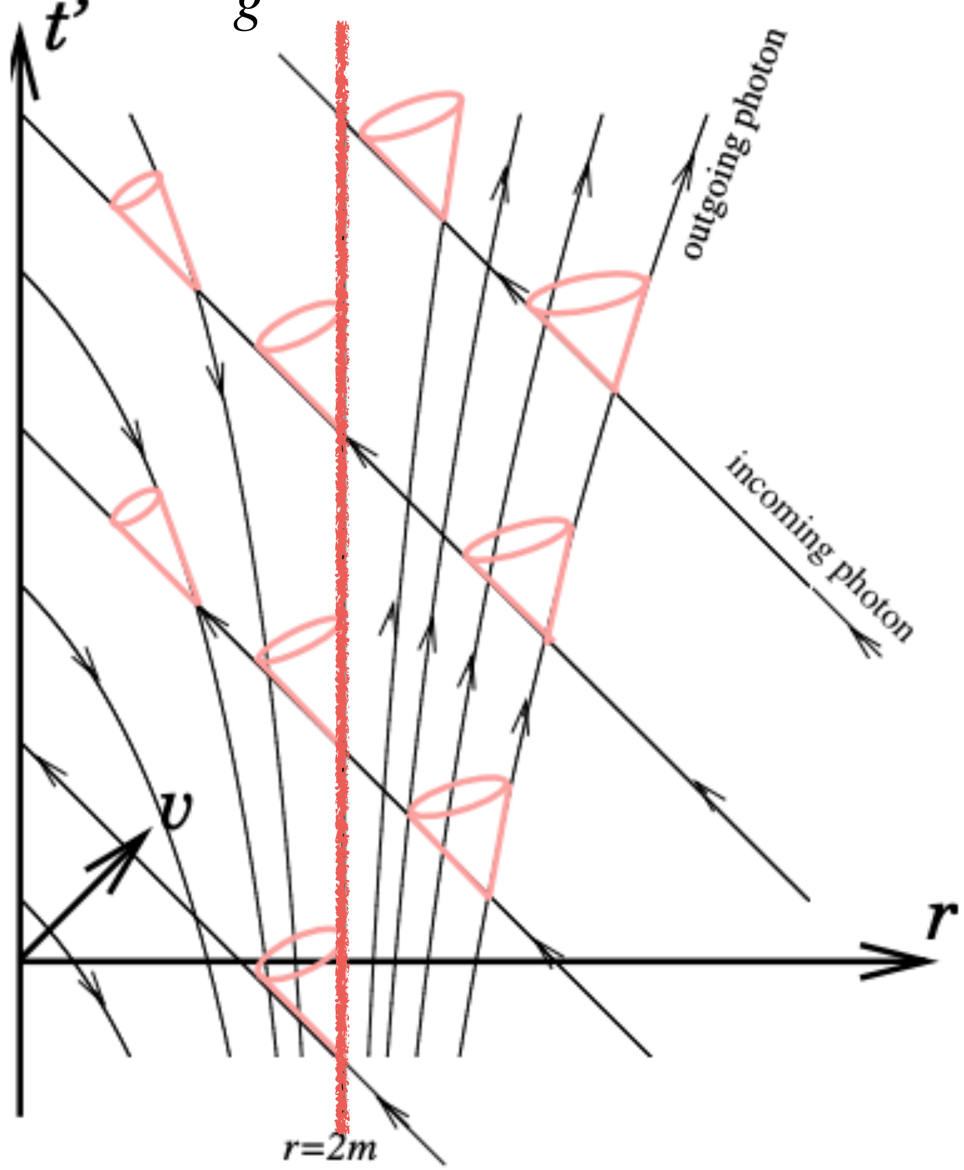
For Schwarzschild

$$r_g = 2GM$$



$$ds^2 = \left(1 - \frac{r_g}{r}\right) dv^2 - 2drdv - r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)$$

$$v = t + r + r_g \log\left|\frac{r}{r_g} - 1\right|$$

Finkelstein (1959), "Eddington-Finkelstein" coordinates

Penrose singularity theorem

GRAVITATIONAL COLLAPSE AND SPACE-TIME SINGULARITIES

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The discovery of the quasistellar radio sources has stimulated renewed interest in the question of gravitational collapse. It has been suggested by some authors[1] that the enormous amounts of energy that these objects apparently emit may result from the collapse of a mass of the order of $(10^6\text{-}10^8)M_\odot$ to the neighborhood of its Schwarzschild radius, accompanied by a violent release of energy, possibly in the form of gravitational radiation. The detailed mathematical discussion of such situations is difficult since the full complexity of general relativity is required. Consequently, most exact calculations concerned with the implications of gravitational collapse have employed the simplifying assumption of spherical symmetry. Unfortunately, this precludes any detailed discussion of gravitational radiation—which requires at least a quadripole structure.

The general situation with regard to a spherically symmetrical body is well known.[2] For a sufficiently great mass, there is no final equilibrium state. When sufficient thermal energy has been radiated away, the body contracts and continues to contract until a physical singularity is encountered at $r=0$. As measured by local comoving observers, the body passes within its Schwarzschild radius $r=2m$. (The densities at which this happens need not be enormously high if the total mass is large enough.) To an outside observer the contraction to $r=2m$ appears to take an infinite time. Nevertheless, the existence of a singularity presents a serious problem for any complete discussion of the physics of the interior region.

The question has been raised as to whether this singularity is, in fact, simply a property of the high symmetry assumed. The matter collapses radially inwards to the single point at the center, so that a resulting space-time catastrophe there is perhaps not surprising. Could not the presence of perturbations which destroy the spherical symmetry alter the situation drastically? The recent rotating solution of Kerr[3] also possesses a physical singularity, but since a high degree of symmetry is still present (and the solution is algebraically special), it might again be argued that this is not representative of the general situation.[4] Collapse without assumptions of symmetry[5] will be discussed here.

Consider the time development of a Cauchy hypersurface C^3 representing an initial matter distribution. We may assume Einstein's field equations and suitable equations of state governing the matter. In fact, the only assumption made here about these equations of state will be the non-negative definiteness of Einstein's energy expression (with or without cosmological term). Suppose this matter distribution undergoes gravitational collapse in a way which, at first, qualitatively resembles the spherically symmetrical case. It will be shown that, after a certain critical condition has been fulfilled, deviations from spherical symmetry cannot prevent space-time singularities from arising. If, as seems justifiable, actual physical singularities in space-time are not to be permitted to occur, the conclusion would appear inescapable that inside such a collapsing object at least one of the following holds: (a) Negative local energy occurs.[6] (b) Einstein's equations are violated. (c) The space-time manifold is incomplete.[7] (d) The concept of space-time loses its meaning at very high curvatures—possibly because of quantum phenomena.[2] In fact (a), (b), (c), (d) are somewhat interrelated, the distinction being partly one of attitude of mind.

Before examining the asymmetrical case, consider a spherically symmetrical matter distribution of finite radius in C^3 which collapses symmetrically. The empty region surrounding the matter will, in this case, be a Schwarzschild field, and we can conveniently use the metric $ds^2=-2drdv+dv^2(1-2m/r)-r^2(d\theta^2+\sin^2\theta d\varphi^2)$, with an advanced time parameter v to describe it.[8] The situation is depicted in Fig. 1. Note that an exterior observer will always see matter outside $r=2m$, the collapse through $r=2m$ to the singularity at $r=0$ being invisible to him.

After the matter has contracted within $r=2m$, a spacelike sphere S^2 ($t=\text{const}$, $2m>r=\text{const}$) can be found in the empty region surrounding the matter. This sphere is an example of what will be called here a trapped surface—defined generally as a closed, spacelike, two-surface T^2 with the property that the two systems of null geodesics which meet T^2 orthogonally converge locally in future directions at T^2. Clearly trapped surfaces will still exist if the matter region has no sharp boundary or if spherical symmetry is dropped, provided that the deviations from the above situation are not too great. Indeed, the Kerr solutions with $m>a$ (angular momentum ma) all possess trapped surfaces, whereas those for which $m\leq a$ do not.[9] The argument will be to show that the existence of a trapped surface implies—irrespective of symmetry—that singularities necessarily develop.

The existence of a singularity can never be inferred, however, without an assumption such as completeness for the manifold under consideration. It will be necessary, here, to suppose that the manifold $M_+{}^4$, which is the future time development of an initial Cauchy hypersurface C^3 (past boundary of the $M_+{}^4$ region), is in fact null complete into the future. The various assumptions are, more precisely, as follows: (i) $M_+{}^4$ is a nonsingular $(+---)$ Riemannian manifold for which the null half-cones form two separate systems ("past" and "future"). (ii) Every null geodesic in $M_+{}^4$ can be extended into the future to arbitrarily large affine parameter values (null completeness). (iii) Every timelike or null geodesic in $M_+{}^4$ can be extended

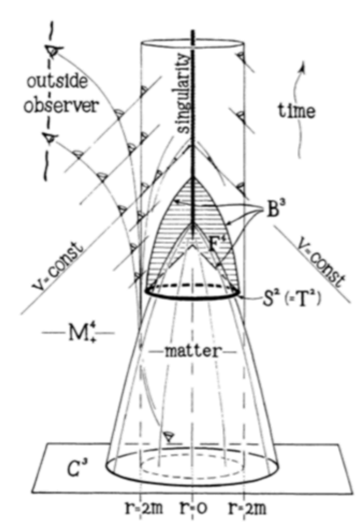


FIG. 1. Spherically symmetrical collapse (one space dimension surpressed). The diagram essentially also serves for the discussion of the asymmetrical case.

into the past until it meets C_3 (Cauchy hypersurface condition). (iv) At every point of $M_+{}^4$, all timelike vectors t^μ satisfy $(-R_{\mu\nu}+\frac{1}{2}Rg_{\mu\nu}-\lambda g_{\mu\nu})t^\mu t^\nu\geq 0$ (non-negativeness of local energy). (v) There exists a trapped surface T^2 in $M_+{}^4$. It will be shown here, in outline, that (i), $\cdots$, (v) are together inconsistent.

Let F^4 be the set of points in $M_+{}^4$ which can be connected to T^2 by a smooth timelike curve leading into the future from T^2. Let B^3 be the boundary of F^4. Local considerations show that B^3 is null where it is nonsingular, being generated by the null geodesic segments which meet T^2 orthogonally at a past endpoint and have a future endpoint if this is a singularity (on a caustic or crossing region) of B^3. Let l^μ (subject to $l^\mu{}_{;\nu}l^\nu=0$), ρ $(=-\frac{1}{2}l^\mu{}_{;\mu})$, and $|\sigma|$ $\{=[\frac{1}{2}l_{(\mu;\nu)}l^{\mu;\nu}-\frac{1}{4}(l^\mu{}_{;\mu})^2]^{1/2}\}$ be, respectively, a future-pointing tangent vector, the convergence, and the shear for these null geodesics,[10] and let A be a corresponding infinitesimal area of cross section of B^3. Then $[(A^{1/2})_{;\mu}l^\mu]_{;\nu}l^\nu=-(A^{1/2}\rho)_{;\mu}l^\mu=-A^{1/2}(|\sigma|^2+\Phi)\leq 0$ where $\Phi=-\frac{1}{2}R_{\mu\nu}l^\mu l^\nu$ [≥ 0 by (iv)]. Since T^2 is trapped, $\rho>0$ at T^2, whence A becomes zero at a finite affine distance to the future of T^2 on each null geodesic. Each geodesic thus encounters a caustic. Hence B^3 is compact (closed), being generated by a compact system of finite segments. We may approximate B^3 arbitrarily closely by a smooth, closed, spacelike hypersurface B^{3*}. Let K^4 denote the set of pairs (P,s) with $P\in B^{3*}$ and $0\leq s\leq 1$. Define a continuous map μ: $K^4\to M_+{}^4$ where, for fixed P, $\mu\{(P,s)\}$ is the past geodesic segment normal to B^{3*} at $P=\mu\{(P,1)\}$ and meeting C^3 [as it must, by (iii)] in the point $\mu\{(P,0)\}$. At each point Q of $\mu\{K^4\}$, we can define the degree $d(Q)$ of μ to be the number of points of K^4 which map to Q (correctly counted). Over any region not containing the image of a boundary point of K^4, $d(Q)$ will be constant. Near B^{3*}, μ is 1-1, so $d(Q)=1$. It follows that $d(Q)=1$ near C^3 also, whence the degree of the map $B^{3*}\to C^3$ induced by μ when $s=0$ must also be unity. The impossibility of this follows from the noncompactness of C^3.

Full details of this and other related results will be given elsewhere.

[1] F. Hoyle and W. A. Fowler, Monthly Notices Roy. Astron. Soc. 125, 169 (1963); F. Hoyle, W. A. Fowler, G. R. Burbidge, and E. M. Burbidge, Astrophys. J. 139, 909 (1964); W. A. Fowler, Rev. Mod. Phys. 36, 545 (1964); Ya. B. Zel'dovich and I. D. Novikov, Dokl. Akad. Nauk SSSR 155, 1033 (1964) [translation: Soviet Phys.—Doklady 9, 246 (1964)]; I. S. Shklovskiĭ and N. S. Kardashev, Dokl. Akad. Nauk SSSR 155, 1039 (1964) [translation: Soviet Phys.—Doklady 9, 252 (1964)]; Ya. B. Zel'dovich and M. A. Podurets, Dokl. Akad. Nauk SSSR 156, 57 (1964) [translation: Soviet Phys.—Doklady 9, 373 (1964)]. Also various articles in the Proceedings of the 1963 Dallas Conference on Gravitational Collapse (University of Chicago Press, Chicago, Illinois, 1964).

[2] J. R. Oppenheimer and H. Snyder, Phys. Rev. 56, 455 (1939). See also J. A. Wheeler, in Relativity, Groups and Topology, edited by C. deWitt and B. deWitt (Gordon and Breach Publishers, Inc., New York, 1964); and reference 1.

[3] R. P. Kerr, Phys. Rev. Letters 11, 237 (1963).

[4] See also E. M. Lifshitz and I. M. Khalatnikov, Advan. Phys. 12, 185 (1963).

[5] See also P. G. Bergmann, Phys. Rev. Letters 12, 139 (1964).

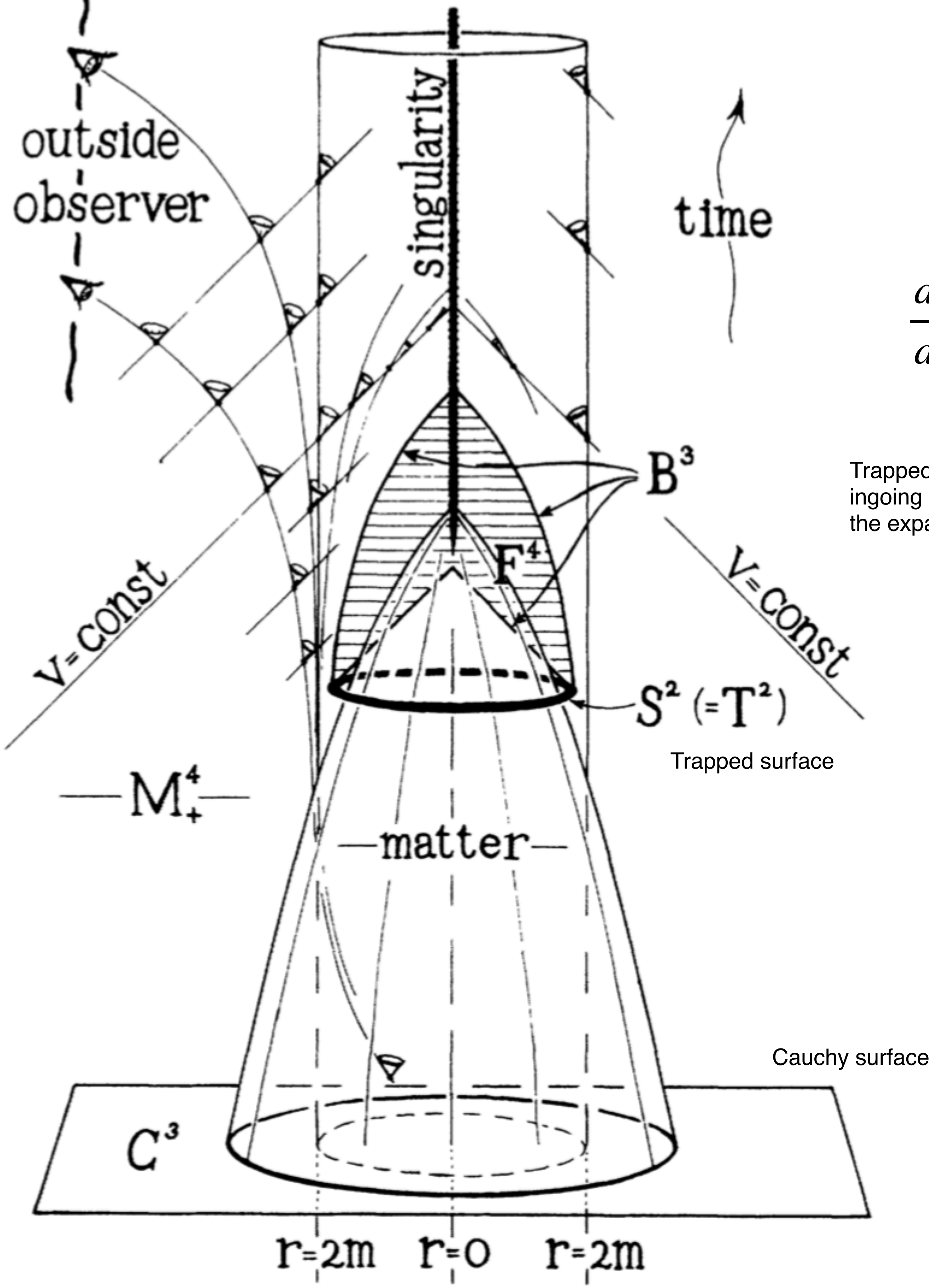
[6] The negative energy of a "C field" may be invoked to avoid singularities: F. Hoyle and J. V. Narlikar, Proc. Roy. Soc. (London) A278, 465 (1964). However, it is difficult to see how even the presence of negative energy could lead to an effective "bounce" if local causality is to be maintained.

[7] The "I'm all right, Jack" philosophy with regard to the singularities would be included under this heading!

[8] D. Finkelstein, Phys. Rev. 110, 965 (1959).

[9] The case $m<a$ is interesting in that here a singularity is "visible" to an outside observer. Whether or not "visible" singularities inevitably arise under appropriate circumstances is an intriguing question not covered by the present discussion.

[10] For the notation, etc., see E. Newman and R. Penrose, J. Math. Phys. 3, 566 (1962).



θ Expansion

$$\frac{d\theta}{d\lambda} = -\frac{1}{2}\theta^2 - \sigma^2 - R_{\mu\nu}k^\mu k^\nu$$

Trapped surface: closed two-surface S with the property that for both ingoing and outgoing congruences of null geodesics orthogonal to S, the expansion is negative everywhere on S.

G=1

Penrose singularity theorem

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- iv) At every point of M_+^4 all time like vectors satisfy $\left(R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - \lambda g_{\mu\nu}\right) t^\mu t^\nu = 8\pi G\, T_{\mu\nu} t^\mu t^\nu \geq 0$ - non-negativeness of local energy
- v) There exists a trapped surface T^2

In will be shown here, in outline that i)-v) are inconsistent.

Penrose singularity theorem

- i) M_+^4 is a nonsingular (+,-,-,-) Riemannian manifold for which the null half-cones form too separate systems ("past" and "future")

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Penrose singularity theorem

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- ii) Every null geodesic in M^4_+ can be extended into the future to arbitrary large affine parameter values (null completeness)

- iii) Every timelike of null geodesic in M^4_+ can be extended into the past until it meets C_3 (Cauchy hypersurface condition)

- iv) At every point of M^4_+ all time like vectors satisfy $\left(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R - \lambda g_{\mu\nu}\right)t^\mu t^\nu = 8\pi G\, T_{\mu\nu}t^\mu t^\nu \geq 0$ - non-negativeness of local energy

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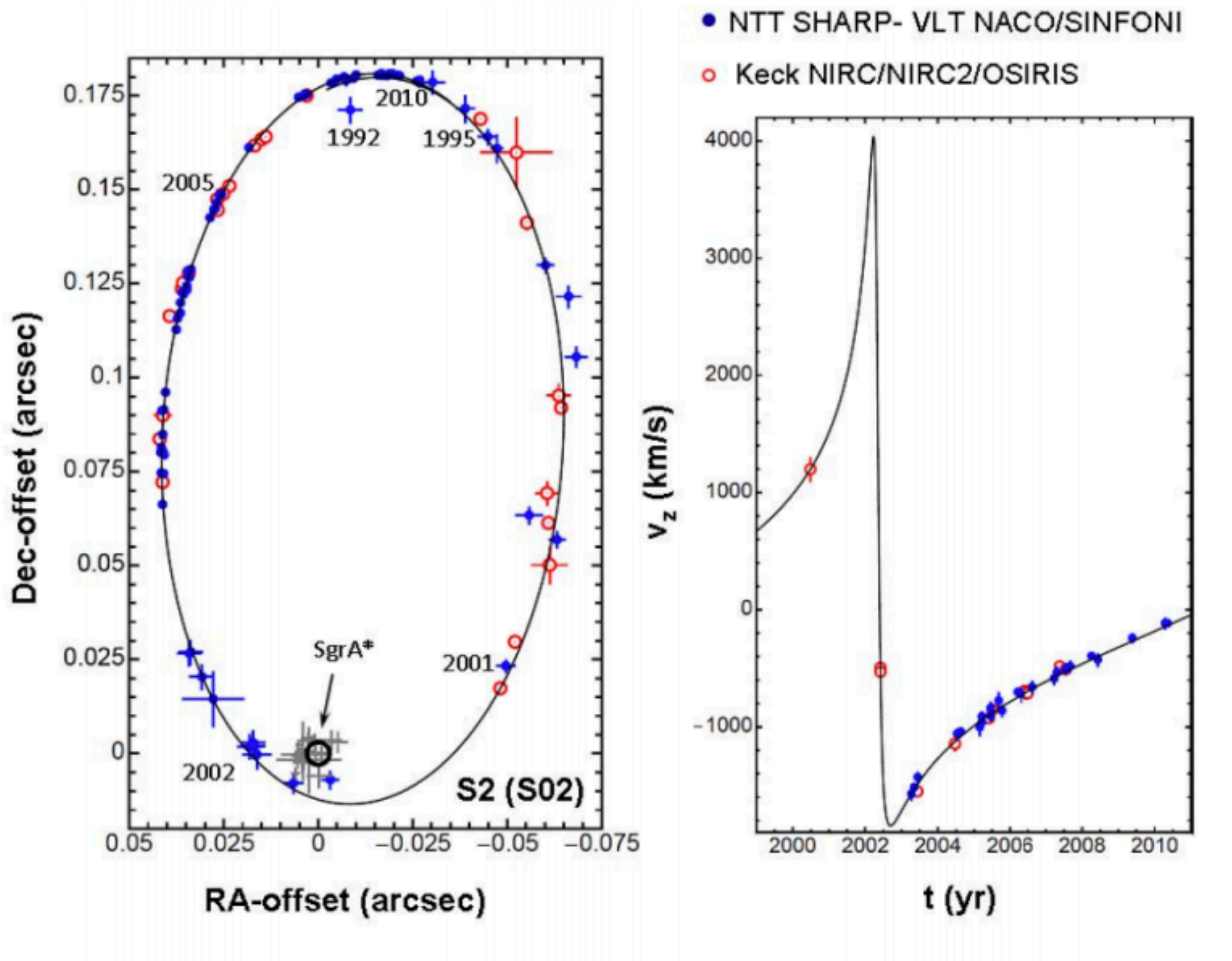

Figure 8. *Orbit of the star S2 (S02) on the sky (left panel) and in radial velocity (right panel). Data from NTT/VLT and Keck are shown. Blue, filled circles, denote the NTT/VLT points and open and filled red circles are the Keck data. The positions are relative to the radio position of Sgr A* (black circle). The grey crosses are the positions of various Sgr A* infrared flares. From Genzel, Eisenhauer & Gillessen (2010).*

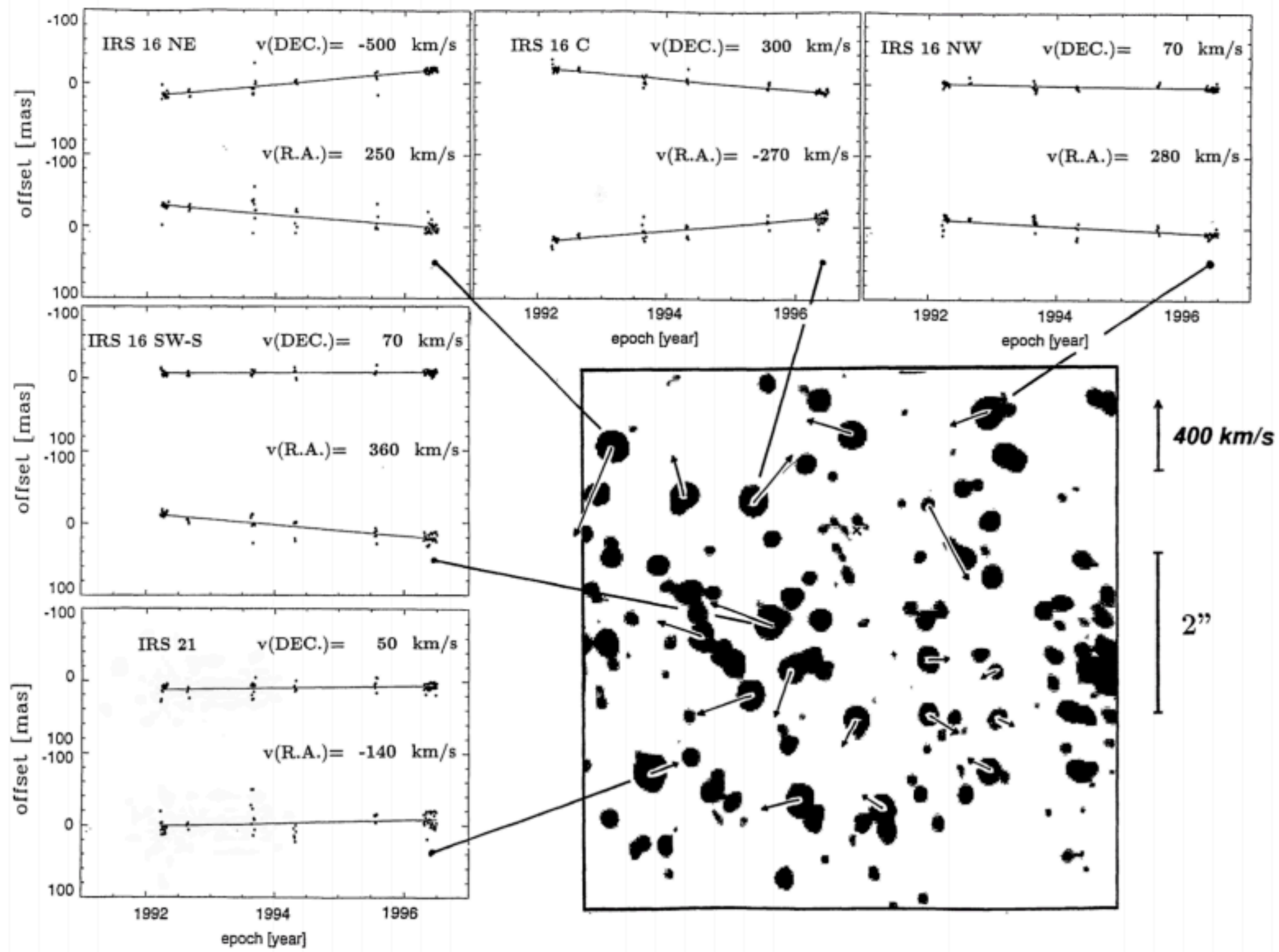

Figure 5. *Proper motions and vectors of selected stars surrounding the central compact radio source Sgr A*, marked with a cross. The measurements were carried out over four years and the offsets were determined with respect to the base epoch in 1994. From Eckart & Genzel (1997).*