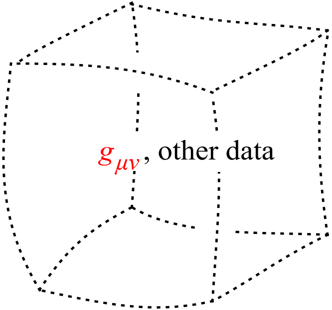
Open String Background Independence

Theodore Erler

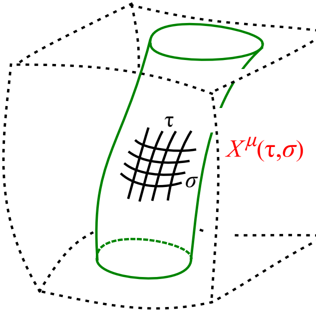
Institute of Physics, Prague

Ingredients of String Theory

1) A background:



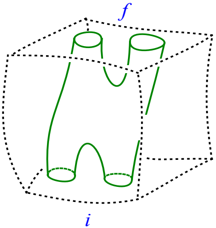
2) String moving in background:

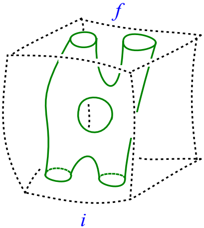


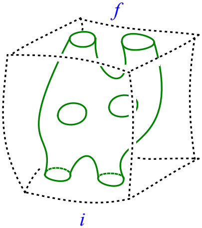
$$\text{Action} = S[X^\mu(\tau,\sigma)] \sim \text{Area}$$

Note that the area depends on the metric of the background.

3) Path integral over string surfaces, weighted by action

$\mathcal{A}_{i \to f} = \int$ 

$+ \int$ 

$+ \int$ 

$+ \ldots$

Result: Scattering amplitudes between strings moving in a given background

But why do we care?

We are interested in the quantum dynamics of spacetime, not of strings.

Miracle of string theory:

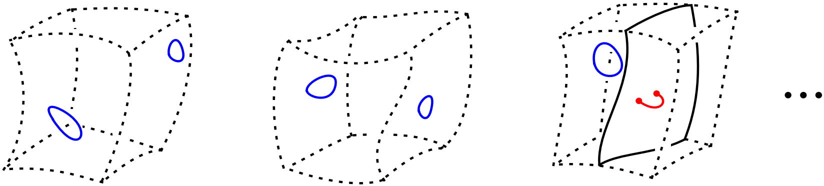
The initial and final scattering states of the string precisely represent quantum fluctuations of the choice of background.

For example, one kind of scattering state of the string is the graviton, which represents a quantum fluctuation of spacetime.

Therefore the quantum dynamics of strings is *the same* as the quantum dynamics of spacetime. (Or, at least, small fluctuations of spacetime).

Background Independence

Problem: We have a virtually unlimited set of quantum theories of small fluctuations of different backgrounds.



How do we see that these theories are related by large fluctuations?

This is the problem of background independence in string theory.

Background independence is one of the oldest and most difficult problems in string theory.

There is overwhelming evidence that string theory is background independent.

But overwhelming evidence is not the same as seeing that it is true.

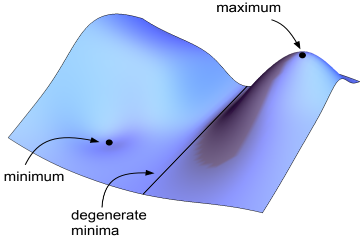
For example, it would be strange to say that we have "overwhelming evidence" that the magnetic field in a solenoid is a configuration in the same theory as the electric field in a capacitor.

While having a consistent theory of graviton scattering is important as a matter of principle, what we really want to know is why the standard model of particle physics takes the form that it does, and how the universe came about through the big bang.

Addressing these questions requires a unified description of string backgrounds which can usefully describe how they are dynamically related.

This requires addressing the problem of background independence in string theory.

One naturally thinks of string backgrounds as unified as critical points of some gigantic potential.



Implicit in this picture is the notion of a field theory action, generalizing the Einstein-Hilbert action, whose field equations select string backgrounds.

We need string field theory.

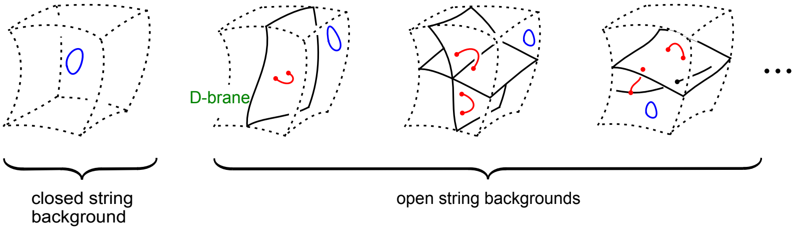
Open string background independence

The general problem of background independence in string theory is too difficult.

The reason is that closed string field theory is complicated.

We therefore focus on the more limited problem of open string background independence.

In a given background where it is possible to quantize the closed string, it is possible to place various configurations of D-branes:



D-branes are solitonic objects in string theory where open strings can attach at their endpoints.

A closed string background is a background where closed strings can propagate.

An open string background is a closed string background together with a set of boundary conditions (D-branes) at endpoints which allow open strings to propagate as well.

Open string background independence is the statement that all D-brane systems in a fixed closed string backgrounds are connected as part of a single (classical) theory of D-branes.

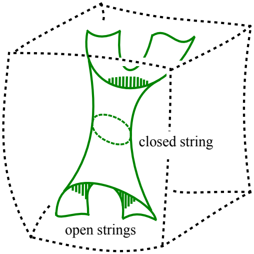
Therefore:

We want to show that all D-brane systems are realized as classical solutions of open string field theory in a fixed closed string background.

The problem of open string background independence is already very difficult.

Still it falls short of the complete result would like to have. However,

- Open string field theory has many features in common with closed string field theory.
- Loop diagrams of open strings include closed strings. If a proof of open string background independence can be extended to the quantum level, we have a general proof of background independence.



Open string field

We need an appropriate field variable for describing the dynamics of D-branes.

Quantum theory of open string: The open string can be in a quantum state Ψ subject to an analogue of the Schrödinger equation:

$$Q\Psi = 0$$

Q ↪ BRST operator

The BRST operator squares to zero

$$Q^2 = 0$$

This leads to a physical equivalence between quantum states:

$$\Psi' = \Psi + Q\Lambda$$

Golden rule of second quantization: The wavefunction in the first quantized theory becomes the classical field in the second quantized theory.

$$\Psi = \text{String field}$$

This represents the classical fluctuation field of the D-brane to which the first quantized open string is attached.

Linearized EOM: $Q\Psi = 0$
Linearized gauge symmetry: $\Psi' = \Psi + Q\Lambda$

We consider D-branes in bosonic string theory. There are no fermions, and the string field always includes a tachyon which indicates that the D-brane is unstable.

Important conceptual point:

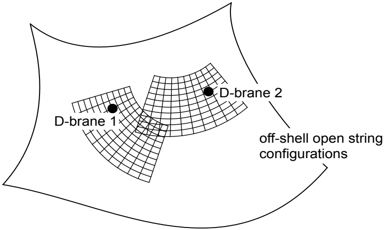
Since Ψ is the fluctuation field of a D-brane, its definition makes special reference to an open string background—namely, the D-brane. In particular, the field configuration $\Psi = 0$ represents the D-brane itself, with no fluctuations.

Contrast with general relativity. The definition of the metric $g_{\mu\nu}$ makes no reference to any specific spacetime. There is no spacetime with $g_{\mu\nu} = 0$.

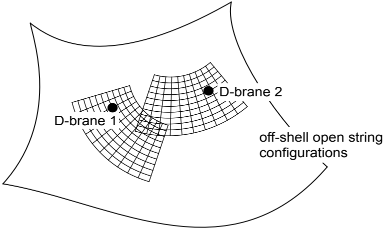
This is a consequence of the fact that the string field was obtained by reinterpreting a quantum state via second quantization. The string field, like the quantum state, lives in a vector space.

General relativity does not come from second quantization, and the metric does not live in a vector space.

Given this, what does it mean to establish open string background independence?



- Open string backgrounds are critical points on a manifold of (off-shell) D-brane configurations.
- Each critical point comes with a coordinate system defined in its vicinity. This is the string field Ψ of the corresponding D-brane configuration.



Open sting background independence means:

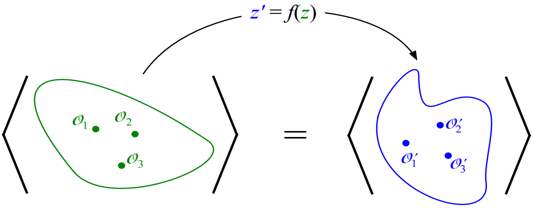
- The coordinate system of each critical point extends to cover all others. In particular, the fluctuation field of a given D-brane system can describe any other D-brane system as solutions to the classical field equations.
- There is a transformation between the coordinate systems of different critical points. In particular, the string fields of two D-brane configurations can be related by field redefinition.

To describe open string field theory, we will need to be slightly less schematic about the nature of the string field.

The quantum theory of an open string is a Euclidean QFT on a two dimensional surface with boundary. The QFT knows about angles on the two dimensional surface, but does not know about distances.

Technically, this is referred to as a boundary conformal field theory (BCFT). A BCFT is defined on a two dimensional Riemann surface with boundary, and it is natural to talk about complex coordinates on the surface $z = \tau + i\sigma$.

BCFT correlation functions on two very differently shaped surfaces are the same provided the surfaces are related by conformal transformation:



The nature of the BCFT path integral on the interior of the surface is what characterizes the closed string background.

The nature of the BCFT path integral on the boundary of the surface is what characterizes the D-brane configuration within the closed string background.

The quantum fields of the BCFT are

$$X^{\mu}(z) = \text{string embedding coordinates}$$

$$b(z), c(z) = \text{Fadeev-Popov ghosts}$$

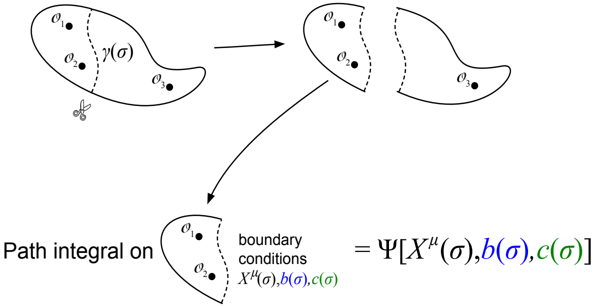
Fadeev-Popov ghosts arise from a determinant in fixing gauge symmetries of the action of the open string.

Incidentally, this is also the origin of the BRST operator Q.

Ghost number $= (\#c\text{'s}) - (\#b\text{'s})$

The BRST operator carries ghost number 1.

An open string quantum state is constructed by considering a correlation function on a surface and "cutting open" the surface along a curve $\gamma(\sigma), \sigma \in [0, \pi]$:



The result is a wavefunctional. By the golden rule of second quantization, it can also be regarded as an open string field, i.e. the fluctuation of the D-brane configuration defined by the BCFT.

The ghost number of a string field is given by the ghost number of all operator insertions in the severed surface.

Open string field theory

Start with a D-brane configuration in a fixed closed string background. This defines the reference boundary conformal field theory BCFT_1.

Dynamical string field: $\Psi =$ ghost number 1 state in BCFT_1

Action: Chen-Simons form:

$$S = \text{Tr}\left(\frac{1}{2}\Psi Q \Psi + \frac{1}{3}\Psi^3\right)$$

Equations of motion:

$$Q\Psi + \Psi^2 = 0$$

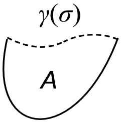
Gauge Invariance:

$$\Psi' = \Psi + Q\Lambda + [\Psi, \Lambda]$$

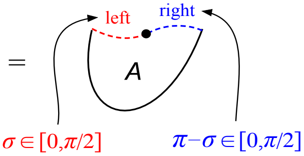
The definition of the theory requires the definition of a product between string fields and a trace operation.

For this it is useful to view the curve $\gamma(\sigma)$ on the severed surface as partitioned into left and right halves.

$A[X^\mu(\sigma), b(\sigma), c(\sigma)]$ =

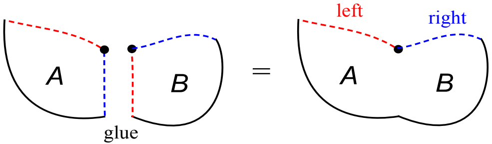


=



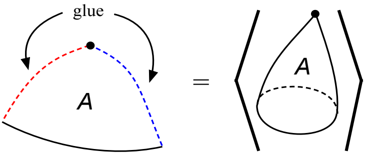
Product:

$AB =$



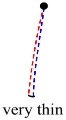
Trace:

$\mathrm{Tr}[A] =$



Identity:

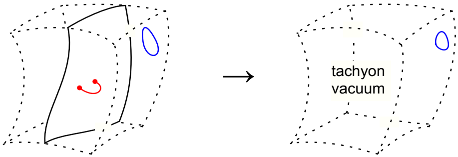
$1 =$



Background independence in open string field theory

The proof of open string background independence starts with the observation that D-branes in bosonic string theory have tachyons, and are unstable.

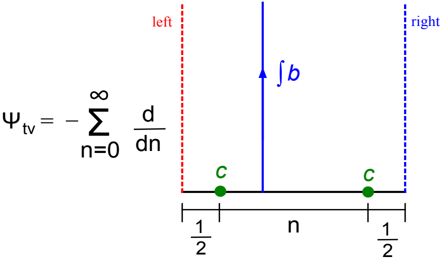
The endpoint of the instability is known to represent a state where the D-brane has completely disappeared, leaving empty space without open strings.



The final state can be represented as a classical solution of open bosonic string field theory on the initial D-brane, and is called the tachyon vacuum.

An exact solution for the tachyon vacuum was found by Martin Schnabl.

The solution is universal, in the sense that its formulation does not require specific data about the BCFT of the initial D-brane.



Satisfies equations of motion $Q\Psi_{\text{tv}} + \Psi_{\text{tv}}^2 = 0$.

Every linearized fluctuation of the solution satisfying $Q_{\Psi_{\text{tv}}}\psi = 0$ must be of the form $\psi = Q_{\Psi_{\text{tv}}}\Lambda$

Therefore, every linearized fluctuation is pure gauge $\rightarrow$ no open strings, no D-branes.

The trivial solution $\Psi_{\text{trivial}} = 0$ represents the D-brane we start with.

Suppose we represent the trivial solution in terms of Schnabl's solution:

$$\Psi_{\text{trivial}} = \underbrace{\Psi_{\text{tv}}}_{(1)} \underbrace{-\Psi_{\text{tv}}}_{(2)}$$

(1) Represents annihilation of the initial D-brane through tachyon condensation.

(2) Is a solution to the equations of motion expanded around the tachyon vacuum:

$$Q_{\Psi_{\text{tv}}}(-\Psi_{\text{tv}}) + (-\Psi_{\text{tv}})^2 = 0$$

It represents recreation of the initial D-brane out of the tachyon vacuum.

This suggests a strategy for constructing other D-brane configurations ass classical solutions.

Let $[\Psi_{\rm tv}]_{\rm BCFT_1}$ represent the tachyon vacuum of the initial D-brane. It is a state of $\rm BCFT_1$.

Let $[\Psi_{\rm tv}]_{\rm BCFT_2}$ represent the tachyon vacuum of the D-brane configuration we wish to describe. It is a state in $\rm BCFT_2$.

Then we can propose the solution

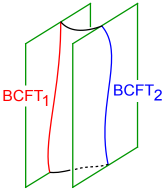
$$\Psi_{12} = [\Psi_{\rm tv}]_{\rm BCFT_1} - [\Psi_{\rm tv}]_{\rm BCFT_2}$$

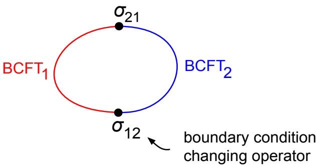
The first term destroys the initial D-brane of $\rm BCFT_1$. The second term creates a new D-brane $\rm BCFT_2$ out of the tachyon vacuum.

Problem: $[\Psi_{\rm tv}]_{\rm BCFT_1}$ and $-[\Psi_{\rm tv}]_{\rm BCFT_2}$ live in different vector spaces, and cannot be added.

We need a dictionary which relates a state of $BCFT_2$ to a state of $BCFT_1$.

For any two D-brane configurations, there are always stretched strings which connect them.



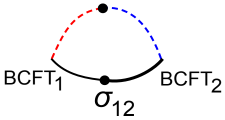


Through conformal transformation, stretched strings define pseudo-local operators in the worldsheet theory called boundary condition changing operators. Boundary condition changing operators appear at the locus of a change of boundary condition in the worldsheet path integral between two BCFTs.

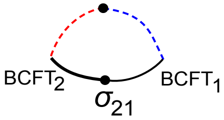
This indicates that the proposed solution must be further articulated by inserting boundary condition changing operators relating the initial and final D-brane configurations.

To do this, we introduce so-called intertwining fields as severed surfaces:

$\Sigma_{12} \sim$



$\Sigma_{21} \sim$



The proposed solution may then be written

$$\Psi_{12} = [\Psi_{\rm tv}]_{\rm BCFT_1} - \Sigma_{12}[\Psi_{\rm tv}]_{\rm BCFT_2}\Sigma_{21}$$

Multiplication by intertwining fields produces the desired map between states of BCFT_2 and BCFT_1.

Note that this works for any two open string backgrounds. For any two D-brane configurations, there are always stretched strings that connect them.

The equations of motion require certain conditions on the intertwining fields:

(1) $Q_{\Psi_{\mathrm{tv}}}\Sigma_{12} = Q_{\Psi_{\mathrm{tv}}}\Sigma_{12} = 0$

(2) $\Sigma_{21}\Sigma_{12} = 1$

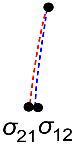
Condition (1) is easy, since it merely implies that the intertwining fields are $Q_{\Psi_{\mathrm{tv}}}$ of something else.

Condition (2) is more challenging.

The problem is that the identity string field is a severed surface of vanishing area. This suggests that Σ_{21} and Σ_{12} must have vanishing area as well.

But this implies

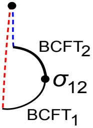
$\Sigma_{21}\,\Sigma_{12} =$

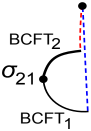


$= \infty$

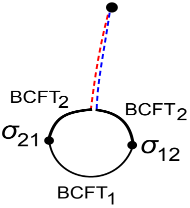
Correlation functions with coincident operator insertions are typically divergent.

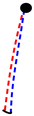
Resolution: Define Σ_{21} and Σ_{12} as "flag shaped severed surfaces:

$\Sigma_{12} \sim$ 

$\Sigma_{21} \sim$ 

Then there is no collision of boundary condition changing operators in the product:

$\Sigma_{21}\Sigma_{12} =$ 

$=$  $= 1$

In this way, we show that open string field theory of an initial D-brane system has classical solutions representing all D-brane systems which share the same closed string background.

We may also relate the fluctuation fields of the initial and final D-brane system:

$$[\Psi]_{\mathrm{BCFT}_1} = \Psi_{21} + \Sigma_{12}[\Psi]_{\mathrm{BCFT}_2}\Sigma_{21}$$

This establishes open string background independence.

Thank you!