Institute of Physics
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Models of elastic pp scattering at high energies - possibilities, limitations, assumptions and open questions

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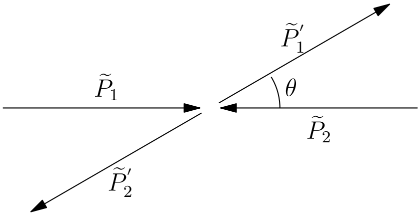
Contents

Elastic proton-proton scattering (ES) - kinematics



- two-body collision process: p+p $\rightarrow$ p + p
- colliding protons change direction of motion, no new particles produced
- center-of-mass system typically used
- kinematical variables
  - four-momentum ... $\widetilde{P} = (E, \vec{p})$
  - proton mass at rest ... m
  - scattering angle ... θ
  - magnitude of momentum of one of the colliding proton (center-of-mass frame) ... $p = |\vec{p}|$
  - two Mandelstam variables t and s typically used (if spins or other quantum numbers are not considered)
    - four-momentum transfer

$$t = (\widetilde{P}_1 - \widetilde{P}_1')^2 = -2p^2(1 - \cos\,\theta) = -4p^2 \sin^2 \frac{\theta}{2} \tag{1}$$

    - $\sqrt{s}$ - collision energy

$$s = (\widetilde{P}_1 + \widetilde{P}_2)^2 = 4(p^2 + m^2) \tag{2}$$

TOTEM experiment - physics programme

TOTal Elastic and diffractive cross section Measurement
one of the LHC experiments [1] at CERN, $\approx$ 80 people

Main aims to measure and study diffractive pp collisions at the LHC energies:

- p+p $\rightarrow$ p+p ... elastic scattering (ES)
- p+p $\rightarrow$ p+X ... single diffraction (SD)
- p+p $\rightarrow$ p+X+p ... central production (CP)
- p+p $\rightarrow$ X+Y ... double diffraction - (DD)
- determine integrated total and inelastic cross sections
- ...

CMS-TOTEM common measurement and analysis to further extend and exploit physics potential
$\Rightarrow$ Precision Proton Spectrometer (PPS) project [2]

Schematic layout of the LHC experiments at CERN

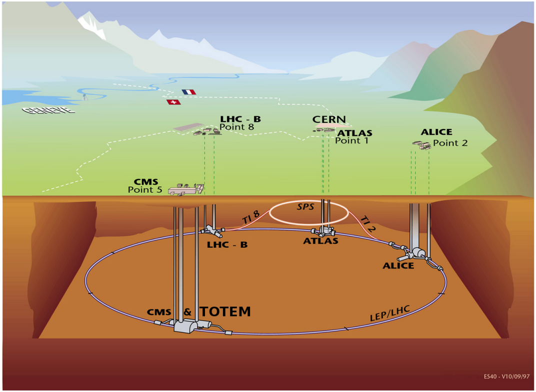
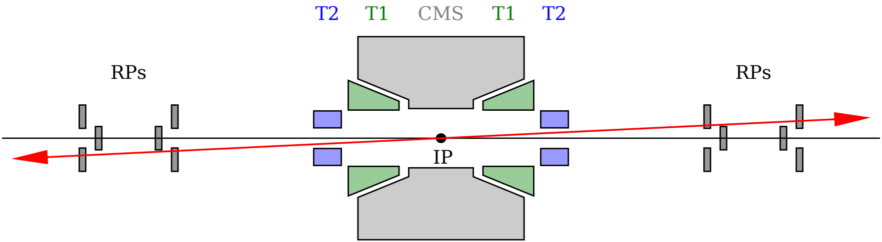


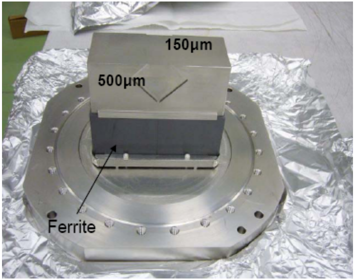
Figure: The LHC accelerator is underground approximately 100 m below the surface (Image: CERN).

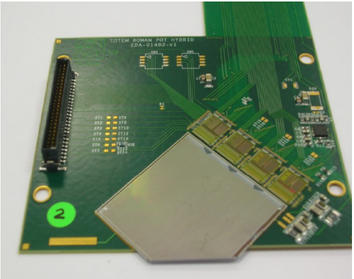
Schema of detection of elastically scattered protons

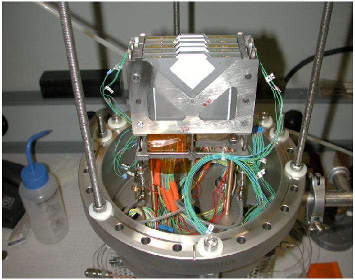


- protons may be scattered at interaction point (IP) at very low scattering angles θ and move along the beam
- $\Rightarrow$ two TOTEM stations of Roman Pots (RPs) at distance about 220 m far from the IP on each side are used to detect the protons; the RPs (moveable devices hosting detectors) may be inserted very close to the beam during dedicated stable beam conditions (staying retracted otherwise)
- motion of a scattered proton and its acceptance in a RP is strongly influenced by magnet settings (optics) between the IP and the RP $\Rightarrow$ dedicated optics necessary to measure very low scattering angles

TOTEM Roman Pot and strip silicon detectors

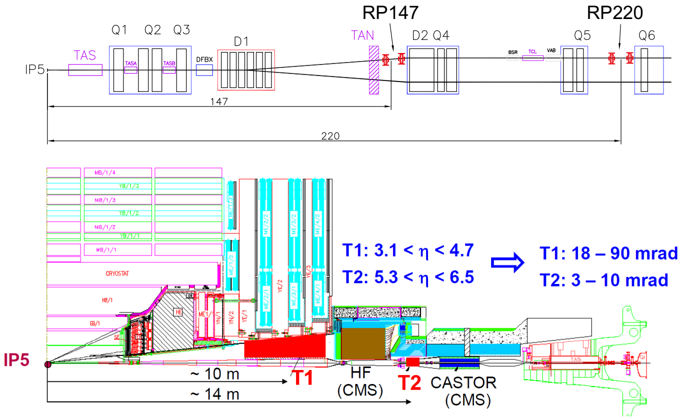






- Roman Pots (RPs) - moveable devices (made by Vakuum Praha)
- each RP may be equipped with planar "edgeless" silicon strip detectors
  - active edge $\approx 50\mu$m from the physical edge
  - 5+5 planes (u and v projection), in each RP
  - one plane: 512 strips, pitch of 66 μm
- different detector technology possible (and already in use)

TOTEM detector apparatus (LHC Run 1)



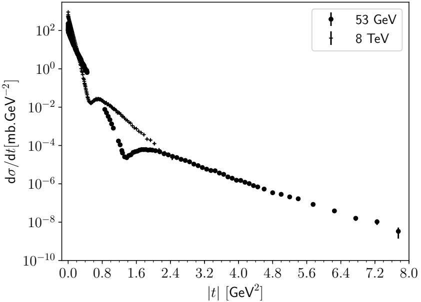
- Roman Pots: measure elastic and diffractive protons close to outgoing beam
- Telescopes T1 and T2: tracking of charged particles from inelastic collisions (in forward direction)
- All TOTEM detectors on both sides of IP5 (symmetrical experiment)
- All TOTEM detectors trigger capable trackers

Elastic differential cross section: difficult precise measurement

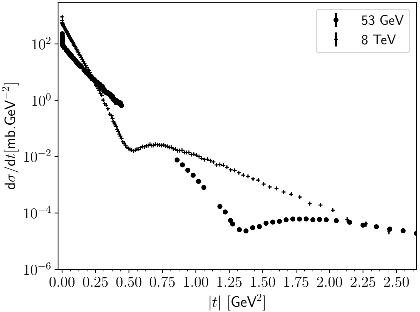
1. Reconstruction of proton kinematics
   - proton tracks in RPs → proton kinematics at IP (alignment + optics)
2. Elastic tagging
   - elastic events: 2 anti-collinear protons from the same vertex (⇒ compare left and right reconstructed protons)
   - no forward momentum loss ⇒ correlation hit position vs. track angle at RPs (due to optics); remove protons shifted due to beam dispersion
3. Acceptance corrections
   - finite size of RP detectors, LHC apertures
   - azimuthal symmetry of el. scattering ⇒ geometrical corrections
   - beam divergence ⇒ correction for missing protons at RP edges
4. Unfolding of resolution effects
   - angular resolution from data (compare left and right protons)
   - Monte Carlo ⇒ impact on t-distribution
5. Inefficiency corrections
   - DAQ inefficiency, trigger inefficiency, uncorrelated one-RP inefficiencies, near-far correlated RP inefficiencies, ...
   - "pile-up" related inefficiencies: elastic event + another track in a RP
6. Luminosity
   - Van der Meer scans
   - measurement based on elastic scattering and optical theorem
   - ...

explained in details in TOTEM papers, see, e.g., [3–5] (and short review of TOTEM results in Chapter 1 and 2 in [6])

Measured elastic pp differential cross section at 52.8 GeV and 8 TeV



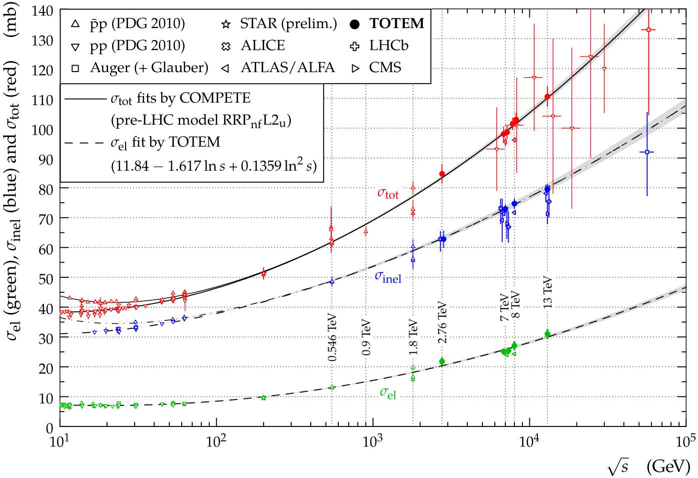
(a) $|t|$ values up to 8 GeV^2



(b) $|t|$ values up to $\approx$ 2.5 GeV^2

- 52.8 GeV: ISR data
- 8 TeV: LHC data (TOTEM measurement)
- $d\sigma/dt$ at both the energies very different, but similar shape:
  - peak at the lowest measured values of $|t|$ (atributed to Coulomb-hadronic interference)
  - followed by nearly exponential t-dependence
  - and dip-bump structure at even higher values of $|t|$
- measured elastic pp $d\sigma/dt$ available at various energies - what can we learn from it?

Total, elastic and inelastic pp and $\bar{p}p$ cross sections



- significant increase with increasing collision energy $\sqrt{s}$, see TOTEM overview [7]
- cross sections determined from different methods (elastic pp and $\bar{p}p$ scattering, p-air scattering (cosmic ray + showers), ...) ⇒ based on assumptions which should be carefully studied

Contemporary situation concerning description of el. scattering

- description and understanding of el. pp scattering is still not fully satisfactory
- contemporary situation summarized recently, e.g., in a strategical document A. Andreazza et al., "What Next: White Paper of the INFN-CSN1", Frascati Phys. Ser. **60**, 1–302 (2015); Section 7.5 - Total, elastic and diffractive cross sections:

  "Several theoretical models have been developed during the last decades to interpret the experimental results. Unfortunately, the perturbative QCD approach cannot be used in this context since most of the processes contributing to the total cross section are characterised by low momentum transfer. Some of the models are still based on Regge theory, while others prefer using optical or eikonal approaches. Moreover, so-called QCD-inspired models are trying to connect the concepts of Pomeron trajectories and proton opacity to the QCD description of elementary interactions between quarks and gluons. At the moment, no model manages to describe qualitatively and quantitatively the large amount of data available; they all have merits and shortcomings. Typically, they successfully describe the experimental results in a certain kinematic range but completely fail in other ones."
- at the moment only the eikonal model approach allows to take into account and study both Coulomb-hadronic interference and dependence of (elastic) particle collisions on impact parameter - more fundamental description than other approaches discussed in the literature

Descriptions of elastic collisions of charged hadrons

- measured elastic $d\sigma/dt$ of two charged hadrons given by

$$\frac{d\sigma(s,t)}{dt} = \frac{\pi}{sp^2}\left|F^{C+N}(s,t)\right|^2 \tag{3}$$

  - $F^{C+N}(s,t)$ - complete elastic scattering amplitude of Coulomb-hadronic interaction depending on both Coulomb $F^{C}(s,t)$ and hadronic $F^{N}(s,t)$ amplitudes
  - Coulomb amplitude is usually assumed to be well known from QED (except from electromagnetic form factors) but elastic hadronic amplitude still not fully known
  - eq. (3) allows "separation" of Coulomb interaction from data and to study less known elastic hadron (nuclear) scattering
  - Coulomb-hadronic interference used to constrain t-dependence of the phase of $F^{N}(s,t)$

- two approaches for description of elastic collisions of charged hadrons (amplitude $F^{C+N}(s,t)$)
  - West and Yennie (Feynman diagram technique)
  - eikonal model

Coulomb-hadronic interference in the West and Yennie approach

- H. A. Bethe, "Scattering and polarization of protons by nuclei", Ann. Phys. **3**, 190–240 (1958)

$$F^{\mathrm{C+N}}(s,t) = F^{\mathrm{C}}(s,t)\, \mathrm{e}^{\mathrm{i}\alpha\phi(s,t)} + F^{\mathrm{N}}(s,t) \tag{4}$$

- G. B. West and D. R. Yennie, "Coulomb interference in high-energy scattering", Phys. Rev. **172**, 1413–1422 (1968)
  integral formula for relative phase (derived only for "small" values of $|t|$)

$$\alpha\phi(s,t) = \mp\alpha \left[\ln\left(\frac{-t}{s}\right) + \int_{-4p^2}^{0} \frac{\mathrm{d}t'}{|t-t'|} \left(1 - \frac{F^{\mathrm{N}}(s,t')}{F^{\mathrm{N}}(s,t)}\right)\right]. \tag{5}$$

- simplified interference formula of WY (1968)

$$F^{\mathrm{C+N}}_{\mathrm{WY}}(s,t) = \pm\frac{\alpha s}{t} G_1(t) G_2(t)\, \mathrm{e}^{\mathrm{i}\alpha\phi(s,t)} + \frac{\sigma^{\mathrm{tot,N}}(s)}{4\pi} p\sqrt{s}\,(\rho(s)+\mathrm{i})\, \mathrm{e}^{B(s)t/2} \tag{6}$$

  where (see also Locher 1967 [11])

$$\alpha\phi(s,t) = \mp\alpha\left[\ln\left(\frac{-B(s)t}{2}\right) + \gamma\right] \tag{7}$$

  assuming for all kinematically allowed values of t
  - t-independence of the phase of $F^{\mathrm{N}}(s,t)$, i.e., quantity $\rho(t) = \frac{\mathrm{Re}\, F^{\mathrm{N}}(t)}{\mathrm{Im}\, F^{\mathrm{N}}(t)} = \mathrm{const}$
  - purely exponential $\left|F^{\mathrm{N}}(s,t)\right|$ in t, i.e., diffractive slope $B(t) = \frac{2}{|F^{\mathrm{N}}(t)|}\frac{\mathrm{d}}{\mathrm{d}t}\left|F^{\mathrm{N}}(t)\right| = \mathrm{const}$
- used widely in the era of ISR for determination of $\sigma^{\mathrm{tot,N}}$, quantity $\rho(t=0)$ and $B(t=0)$

Problems and limitations involved in the WY approach

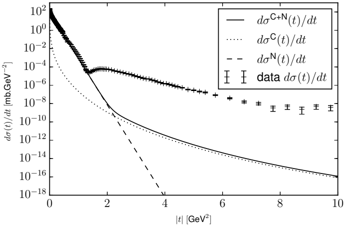
- relative phase $\phi(s,t)$ is real (defined as imaginary part of a complex function) $\Rightarrow$ the integral WY formula (5) consistent only with $\rho(t) = \text{const}$

  V. Kundrát, M. Lokajíček, and I. Vrkoč, "Limited validity of West and Yennie integral formula for elastic scattering of hadrons", Phys. Lett. **B656**, 182–185 (2007)

  J. Procházka and V. Kundrát, "Eikonal model analysis of elastic proton-proton collisions at 52.8 GeV and 8 TeV", arXiv:1606.09479 (2019)

- if $B(t)$ is t-independent $\Rightarrow$ contradiction to existence of observed dip-bump structure

  J. Kašpar, V. Kundrát, M. Lokajíček, and J. Procházka, "Phenomenological models of elastic nucleon scattering and predictions for LHC", Nucl.Phys. **B843**, 84–106 (2011)



- whole approach a priory limited and applied to data in region of only very small values of $|t|$ ($|t| \lesssim 0.01$ GeV^2 at 52.8 GeV)
- form factors $G_{1,2}(t)$ added by hand to the final interference formula(s)
- dependence of elastic hadronic collisions on impact parameter not considered
- ...

$\Rightarrow$ WY approach inapplicable for reliable data analysis; not usable for studying t-dependence of elastic hadronic amplitude and b-dependent characteristics; see detailed discussion in, e.g.,

J. Procházka and V. Kundrát, "Eikonal model analysis of elastic proton-proton collisions at 52.8 GeV and 8 TeV", arXiv:1606.09479 (2019)

Additional comments to the WY approach

many descriptions of elastic scattering negatively influenced by the simplified approach of WY

- quantity $\rho(t=0)$ and diffractive slope $B(t=0)$
  - unclear physical meaning (only very indirect relation to particle characteristics/interactions)
  - importance of these quantities overestimated in many contemporary hadronic models mainly under the influence of the WY approach where they are determining $F^{N}(s,t)$ at all values of t - both quantities assumed, without any reasoning, to be t-independent at *all* kinematically allowed values of t, see page 16
- measured $d\sigma/dt$ commonly divided into two parts
  1. region of very low values of $|t|$ (e.g., $|t| \lesssim 0.01$ GeV^2 at 52.8 GeV)
     analyzed with the help of the simplified WY interference formula assuming specific t-dependence of $F^{N}(s,t)$ at *all* values of t
  2. region of higher values of $|t|$ (containing dip-bump structure)
     described with the help of elastic hadronic models having different t-dependence of hadronic amplitude than the one assumed in the WY approach

  $\Rightarrow$ inconsistent dual description of data

$\Rightarrow$ one should look for different and more general description of (Coulomb-)hadronic elastic scattering; one should study transition from initial to final states, full physical picture

- t-dependence of $F^{N}(s,t)$? i.e., t-dependences of hadronic modulus and phase?
  or, equivalently, t-dependences of quantities $\rho(t)$ and $B(t)$?
- b-dependent characteristics of collisions taking into account that initial states corresponding given value of b have different frequencies (weights)? (one should not mix characteristics of collisions at different impact parameter values)
- corresponding physical properties of colliding particles?

Eikonal model approach

- introduces dependence of elastic collisions on impact parameter
- several authors started from it or have been developing it (Glauber, van Hove, Miettinen, Islam, Cahn,...); results on various level of sophistication
- Coulomb-hadronic interference formula derived by Kundrát and Lokajíček (1994)

$$F^{C+N}_{eik}(s,t) = \pm \frac{\alpha s}{t} G_1(t) G_2(t) + F^{N}(s,t)[1 \mp i\alpha \bar{G}(s,t)] \tag{8}$$

where

$$\bar{G}(s,t) = \int_{t_{min}}^{0} dt' \left\{ \ln\left(\frac{t'}{t}\right) \frac{d}{dt'}[G_1(t')G_2(t')] - \frac{1}{2\pi}\left[\frac{F^{N}(s,t')}{F^{N}(s,t)} - 1\right] I(t,t') \right\} \tag{9}$$

and

$$I(t,t') = \int_{0}^{2\pi} d\Phi'' \frac{G_1(t'')G_2(t'')}{t''}. \tag{10}$$

- derived for *any* value of t and s (high energy) with the aim *not* to impose any restriction on t-dependence of $F^{N}(s,t)$
- allows description of data in the *whole* measured t-range (NB: to calculate $F^{C+N}(s,t)$ at given value of t needs to be known $F^{N}(s,t)$ at *all* values of t - even *outside* measured t-range) $\Rightarrow$ consistent description of data (no duality)

Comparison of proton electromagnetic form factors

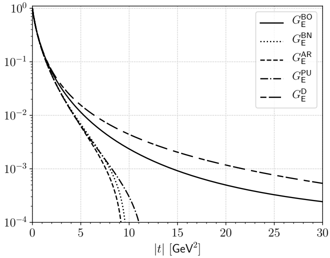


Figure: electric form factors

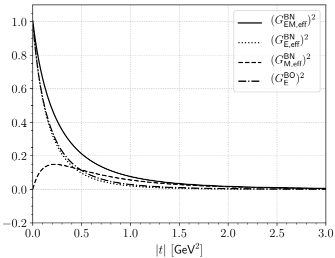


Figure: effective form factors

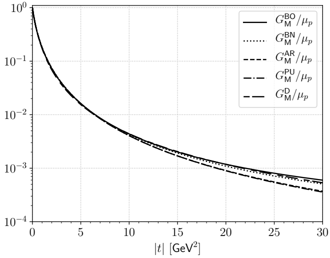


Figure: magnetic form factors

- determined from elastic ep scattering
- different authors/analyses - different t-dependences
- electric form factors
  - differences visible at $|t| > 2.5$ GeV^2
- electric vs. effective electromagnetic form factors
  - very significant differences already at lower $|t|$ values
  - originally only the electric form factors used in the eikonal interference formula
  - impact on determination of $F^N(s,t)$ if effective electromagnetic form factors are used instead of electric form factors?

Hadronic quantities I

- modulus and phase of hadronic amplitude may be defined as

$$F^{\mathrm{N}}(s,t) = \mathrm{i}\left|F^{\mathrm{N}}(s,t)\right| \mathrm{e}^{-\mathrm{i}\zeta^{\mathrm{N}}(s,t)} \tag{11}$$

  it means

$$\tan\zeta^{\mathrm{N}}(s,t) = \rho(s,t) \tag{12}$$

- t-dependent quantities may be introduced

$$B(s,t) = \frac{\mathrm{d}}{\mathrm{d}t}\left[\ln\frac{\mathrm{d}\sigma^{\mathrm{N}}}{\mathrm{d}t}(s,t)\right] = \frac{2}{\left|F^{\mathrm{N}}(s,t)\right|}\frac{\mathrm{d}}{\mathrm{d}t}\left|F^{\mathrm{N}}(s,t)\right| \tag{13}$$

$$\rho(s,t) = \frac{\mathrm{Re}\,F^{\mathrm{N}}(s,t)}{\mathrm{Im}\,F^{\mathrm{N}}(s,t)} \tag{14}$$

Several physically interesting quantities derived from the hadronic amplitude $F^{\mathrm{N}}(s,t)$

- total cross section (optical theorem)

$$\sigma^{\mathrm{tot,N}}(s) = \frac{4\pi}{p\sqrt{s}}\,\mathrm{Im}\,F^{\mathrm{N}}(s,t=0) \tag{15}$$

- elastic and inelastic cross sections

$$\sigma^{\mathrm{el,N}} = \int \frac{\mathrm{d}\sigma^{\mathrm{el,N}}}{\mathrm{d}t} = \int \frac{\pi}{sp^2}\left|F^{\mathrm{N}}(s,t)\right|^2; \qquad \sigma^{\mathrm{inel}} = \sigma^{\mathrm{tot,N}} - \sigma^{\mathrm{el,N}} \tag{16}$$

Hadronic quantities II

- elastic hadronic amplitude in b-space - Fourier-Bessel (FB) transformation (Adachi, Kotani, Takeda, Islam, ...)

$$h_{\text{el}}(s,b) = h_1(s,b) + h_2(s,b)$$
$$= \frac{1}{4p\sqrt{s}} \int_{t_{\min}}^{0} F^{\text{N}}(s,t) J_0(b\sqrt{-t})\,\text{d}t + \frac{1}{4p\sqrt{s}} \int_{-\infty}^{t_{\min}} \lambda(s,t) J_0(b\sqrt{-t})\,\text{d}t \tag{17}$$

- unitarity condition at *finite* energies

$$\text{Im}\, h_1(s,b) + c(s,b) = |h_1(s,b)|^2 + g_1(s,b) + K(s,b) + c(s,b) \tag{18}$$

- profile functions
  - main b-dependent characteristics of collisions, introduced in analogy to description of some optics phenomena (light meeting an obstacle of a given profile which describes its absorptive properties)
  - sometimes interpreted as probabilities of total, elastic or inelastic collision at given value of impact parameter
  -

$$D^{\text{el}}(s,b) \equiv 4\,|h_1(s,b)|^2, \tag{19}$$
$$D^{\text{tot}}(s,b) \equiv 4\,(\text{Im}\, h_1(s,b) + c(s,b)), \tag{20}$$
$$D^{\text{inel}}(s,b) \equiv 4\,(g_1(s,b) + K(s,b) + c(s,b)) \tag{21}$$

- cross sections determined on the basis of the profile functions (X=tot, el, inel)

$$\sigma^{\text{X}}(s) = 2\pi \int_0^{\infty} b\text{d}b\, D^{\text{X}}(s,b). \tag{22}$$

Hadronic quantities III

mean-square values of impact parameter b

- definition ($n = 2$ and $w(b) = 2\pi b$)

$$\langle b^n \rangle^{\mathrm{X}} = \frac{\int\limits_0^\infty b^n \, w(b) D^{\mathrm{X}}(s,b) \mathrm{d}b}{\int\limits_0^\infty w(b) D^{\mathrm{X}}(s,b) \mathrm{d}b} \tag{23}$$

- expressions of the mean-square values in terms of $F^{\mathrm{N}}(s,t)$

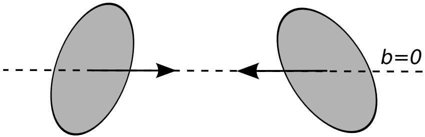
  V. Kundrát, M. V. Lokajíček, and D. Krupa, "Impact parameter structure derived from elastic collisions", Phys. Lett. B **544**, 132–138 (2002)

$$\begin{aligned}
\langle b^2 \rangle^{\mathrm{el}} =& \langle b^2 \rangle^{\mathrm{mod}} + \langle b^2 \rangle^{\mathrm{ph}} \\
=& \frac{4 \int\limits_{t_{\min}}^0 \mathrm{d}t |t| \left(\frac{\mathrm{d}}{\mathrm{d}t} \left| F^{\mathrm{N}}(s,t) \right| \right)^2}{\int\limits_{t_{\min}}^0 \mathrm{d}t \left| F^{\mathrm{N}}(s,t) \right|^2} + \frac{4 \int\limits_{t_{\min}}^0 \mathrm{d}t \left| F^{\mathrm{N}}(s,t) \right|^2 |t| \left(\frac{\mathrm{d}}{\mathrm{d}t} \zeta^{\mathrm{N}}(s,t) \right)^2}{\int\limits_{t_{\min}}^0 \mathrm{d}t \left| F^{\mathrm{N}}(s,t) \right|^2} \\
\langle b^2 \rangle^{\mathrm{tot}} =& 4 \left(\frac{\frac{\mathrm{d}}{\mathrm{d}t} \left| F^{\mathrm{N}}(s,t) \right|}{\left| F^{\mathrm{N}}(s,t) \right|} - \tan \zeta^{\mathrm{N}}(s,t) \frac{\mathrm{d}}{\mathrm{d}t} \zeta^{\mathrm{N}}(s,t) \right) \Bigg|_{t=0} \\
\langle b^2 \rangle^{\mathrm{inel}} =& \frac{\sigma^{\mathrm{tot,N}}(s) \langle b^2 \rangle^{\mathrm{tot}} - \sigma^{\mathrm{el,N}}(s) \langle b^2 \rangle^{\mathrm{el}}}{\sigma^{\mathrm{inel}}(s)}
\end{aligned} \tag{24}$$

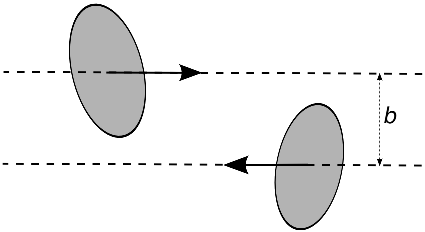
Definition: central vs. peripheral behaviour of elastic collisions

Two basic types of behaviour of elastic hadron collisions (models) in dependence on impact parameter may be distinguished

1. peripheral: $\sqrt{\langle b^2 \rangle^{\text{el}}} > \sqrt{\langle b^2 \rangle^{\text{inel}}}$
   i.e., if elastic collisions correspond in average to higher impact parameter b then the inelastic ones; corresponds to usual ideas of collisions of two matter objects
2. central: $\sqrt{\langle b^2 \rangle^{\text{el}}} < \sqrt{\langle b^2 \rangle^{\text{inel}}}$
   the opposite; anti-ontological behaviour; some kind of transparency of colliding particles; corresponding particle structure never sufficiently explained in the literature



(a) central collision: small values of b



(b) peripheral collision: higher values of b

Contemporary phenomenological models of ES

- J. Kašpar, V. Kundrát, M. Lokajíček, and J. Procházka, "Phenomenological models of elastic nucleon scattering and predictions for LHC", Nucl.Phys. **B843**, 84–106 (2011)

Model	theoretical framework	$\sqrt{\langle b^2\rangle^{\text{tot}}}$	$\sqrt{\langle b^2\rangle^{\text{el}}}$	$\sqrt{\langle b^2\rangle^{\text{inel}}}$
Bourelly et al.	eikonal	1.249	0.876	1.399
Petrov et al. (2P)	eikonal+Regge	1.227	0.875	1.324
Petrov et al. (3P)	eikonal+Regge	1.263	0.901	1.375
Block et al.	eikonal (QCD-inspired)	1.223	0.883	1.336
Islam et al.	eikonal	1.552	1.048	1.659

Table: Values of root-mean-squares of impact parameter (in femtometers) predicted by several contemporary phenomenological models of pp collisions at collision energy of 14 TeV [14, 16].

according to these models $\sqrt{\langle b^2\rangle^{\text{el}}} < \sqrt{\langle b^2\rangle^{\text{inel}}}$, i.e., elastic hadronic collisions should be in average more central then inelastic - which is puzzling

- central behavior of elastic collisions was obtained on the basis of several very doubtful assumptions of unclear physical meaning

Elastic hadronic amplitude $F^{N}(s,t)$

- elastic hadronic amplitude in many contemporary models is a priory strongly constrained, without sufficient reasoning, by requiring ("standard" case):
  1. dominance of the imaginary part of $F^{N}(s,t)$ in quite broad interval of t in forward region
  2. vanishing of the imaginary part of $F^{N}(s,t)$ at (or around) $t = t_{\text{dip}}$ (wrongly reasoned as a consequence of the minimum of $\mathrm{d}\sigma/\mathrm{d}t$ at t_{dip})
  3. change of sign of the real part of $F^{N}(s,t)$ at "low" value of $|t|$ (required by Martin's theorem [17] derived under certain conditions)
  4. values of $\sigma^{\text{tot,N}}$, $B(t=0)$ and $\rho(t=0)$ obtained from the simplified formula of WY (misleadingly denoted as "measurement", see page (52))
  5. (nearly) exponential $\left|F^{N}(s,t)\right|$ in t at low values of $|t|$ close to $t=0$ ($\Rightarrow$ maximal value of $\frac{\mathrm{d}\sigma^{N}}{\mathrm{d}t}(s,t)$ at $t=0$)

  the corresponding t-dependence of $F^{N}(s,t)$ (its phase) is strongly constrained by these requirements and it may be shown that mainly the first requirement leads to central behaviour of elastic collisions

- one may ask if it is possible to obtain description of data which would lead to peripheral behaviour of elastic collisions (possibly without imposing the unreasoned constrains above);
  1981 - peripheral solution of the scattering problem may be obtained if hadronic phase has specific t-dependence
  V. Kundrát, M. Lokajíček, and M. V. Lokajíček, "Are elastic collisions central or peripheral?", Czech. J. Phys. **B31**, 1334 (1981)

$\Rightarrow$ one may try to determine $F^{N}(s,t)$ on the basis of experimental data under given set of assumptions (constrains) and study their impact on values of determined hadronic quantities

Fitting procedure I

1. known form factors used (determined from ep scattering, see page (20))
2. $F^{\mathrm{N}}(s,t)$ parameterized

$$F^{\mathrm{N}}(s,t) = \mathrm{i}\left|F^{\mathrm{N}}(s,t)\right| \mathrm{e}^{-\mathrm{i}\zeta^{\mathrm{N}}(s,t)} \tag{25}$$

modulus

$$\left|F^{\mathrm{N}}(s,t)\right| = (a_1 + a_2 t)\,\mathrm{e}^{b_1 t + b_2 t^2 + b_3 t^3} + (c_1 + c_2 t)\,\mathrm{e}^{d_1 t + d_2 t^2 + d_3 t^3} \tag{26}$$

phase (analytic if κ is positive integer)

$$\zeta^{\mathrm{N}}(s,t) = \zeta_0 + \zeta_1 \left(\frac{t}{t_0}\right)^{\kappa} \mathrm{e}^{\nu t} \tag{27}$$

this very general parameterization of the modulus and phase may reproduce various t shapes in dependence on values of the free parameters (according to additional constrains)

3. eikonal Coulomb-hadronic interference formula (8)

$$\frac{\mathrm{d}\sigma(s,t)}{\mathrm{d}t} = \frac{\pi}{sp^2}\left|F^{C+N}_{eik}(s,t)\right|^2 \tag{28}$$

Fitting procedure II

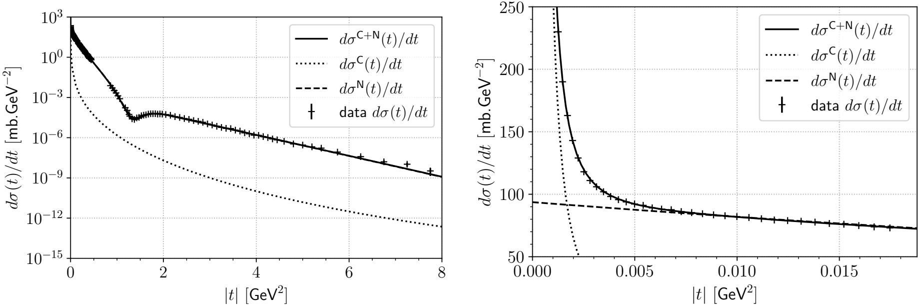
several fits of measured $d\sigma/dt$ at 52.8 GeV and 8 TeV performed under different assumptions:

- different constrains on $F^N(s,t)$, two main types of fits:
  1. Fit 1: "standard" case - reproducing widely used strong constrains imposed on $F^N(s,t)$
     - possible t-dependences of hadronic phase a priory strongly limited
       $\Rightarrow$ unique solution of the t-dependence of $F^N(s,t)$ determined from data
     - mainly the required dominance of the imaginary part of $F^N(s,t)$ leads to centrality of elastic collisions;
  2. Fit 2: alternative peripheral case
     - used parameterization of the phase (27) allowing very different t-dependences and required peripherality
     - no unique solution of the t-dependence of $F^N(s,t)$ determined from data
       $\Rightarrow$ added additional constrain on value of $\sqrt{\langle b^2 \rangle^{el}}$
     - needed to solve complicated problem of bounded extrema (non-trivial optimization problem)
- different types of form factors (effective electric vs. effective electromagnetic). However, impact of different form factors on determination of hadronic quantities small or negligible $\Rightarrow$ results shown only for *effective electromagnetic* form factors

2 different fits/models of data at each energy showed in the following (1 central and 1 peripheral)

Particle types		pp	pp	pp	pp
$\sqrt{s}$	[GeV]	52.8	52.8	8000	8000
Fit		1	2	1	2
Case		central	peripheral	central	peripheral
Form factor		effective electromagnetic	effective electromagnetic	effective electromagnetic	effective electromagnetic
ζ_0		0.0762 ± 0.0017	0.0825 ± 0.0017	0.121 ± 0.018	0.148 ± 0.016
ζ_1		-2.605	1974 ± 37	-12.02	281 ± 11
κ		3	3	2	2
ν	[GeV^{-2}]	1.028	8.23 ± 0.14	1.304	5.68 ± 0.20
a_1		12149.8 ± 9.2	12202.3 ± 9.3	66.58 ± 0.12	66.79 ± 0.11
a_2	[GeV^{-2}]	10705 ± 29	10767 ± 33	163.06 ± 0.73	170.39 ± 0.39
b_1	[GeV^{-2}]	5.905 ± 0.017	5.868 ± 0.017	8.291 ± 0.038	8.137 ± 0.026
b_2	[GeV^{-4}]	3.677 ± 0.063	3.445 ± 0.060	9.27 ± 0.23	7.58 ± 0.16
b_3	[GeV^{-6}]	1.678 ± 0.041	1.520 ± 0.038	14.85 ± 0.34	12.15 ± 0.25
c_1		58.8 ± 1.4	60.4 ± 1.9	1.57 ± 0.14	2.047 ± 0.067
c_2	[GeV^{-2}]	-5.4e-6 ± 2.9	-6.3e-8 ± 2.3	-3.14 ± 0.33	-2.46 ± 0.14
d_1	[GeV^{-2}]	0.901 ± 0.050	0.907 ± 0.041	2.75 ± 0.077	2.688 ± 0.019
$\rho(t=0)$		0.0763 ± 0.0017	0.0827 ± 0.0016	0.122 ± 0.018	0.149 ± 0.016
$B(t=0)$	[GeV^{-2}]	13.515 ± 0.035	13.444 ± 0.036	21.021 ± 0.085	20.829 ± 0.055
$\sigma^{\mathrm{tot,N}}$	[mb]	42.694 ± 0.033	42.861 ± 0.034	103.44 ± 0.35	104.12 ± 0.31
$\sigma^{\mathrm{el,N}}$	[mb]	7.469	7.539	27.6	28.0
σ^{inel}	[mb]	35.22	35.32	75.9	76.1
$\sigma^{\mathrm{el,N}}/\sigma^{\mathrm{tot,N}}$		0.1750	0.1759	0.267	0.269
$\mathrm{d}\sigma^{\mathrm{N}}/\mathrm{d}t(t=0)$	[mb.GeV^{-2}]	93.67	94.51	555	566
$\sqrt{\langle b^2\rangle^{\mathrm{tot}}}$	[fm]	1.026	1.023	1.28	1.27
$\sqrt{\langle b^2\rangle^{\mathrm{el}}}$	[fm]	0.6778	1.959	0.896	1.86
$\sqrt{\langle b^2\rangle^{\mathrm{inel}}}$	[fm]	1.085	0.671	1.39	0.970
$D^{\mathrm{tot}}(b=0)$		1.29	1.30	2.01	2.04
$D^{\mathrm{el}}(b=0)$		0.530	0.0342	0.980	0.205
$D^{\mathrm{inel}}(b=0)$		0.762	1.27	1.03	1.84

pp 52.8 GeV - differential cross sections

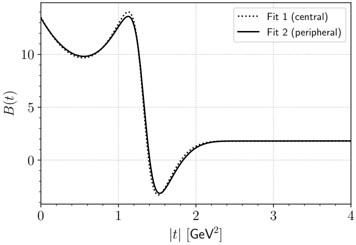


(a) full available $|t|$-range of measured data

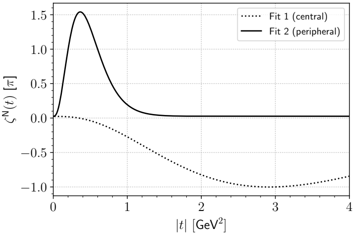
(b) region of very low values of $|t|$

- fits in the very broad interval $|t| \in \langle 0.00126, 7.75\rangle$ GeV^2 including *both* peak at the lowest measured values of $|t|$ and dip-bump structure at higher values of $|t|$
- all the performed fits at 52.8 GeV lead to similar t-dependences of hadronic $d\sigma/dt$

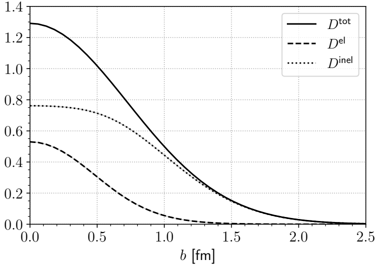
pp 52.8 GeV - t and b dependent characteristics



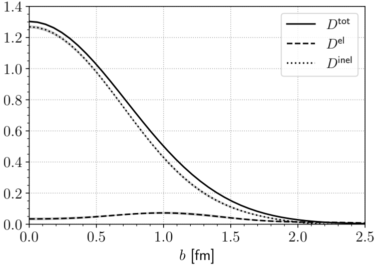
(a) diffractive slopes



(b) hadronic phases

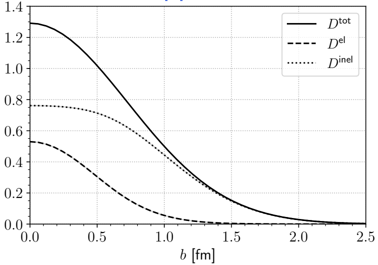


(c) profile functions: Fit 1 (central)

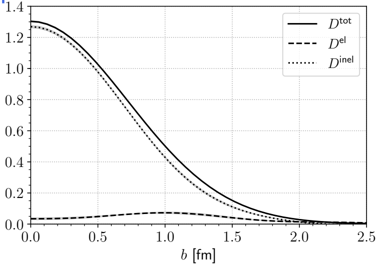


(d) profile functions: Fit 2 (peripheral)

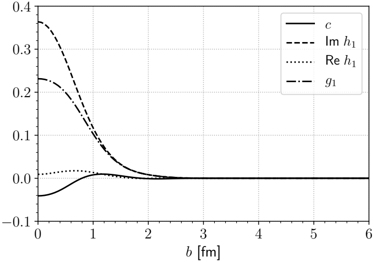
pp 52.8 GeV - *b*-dependent functions



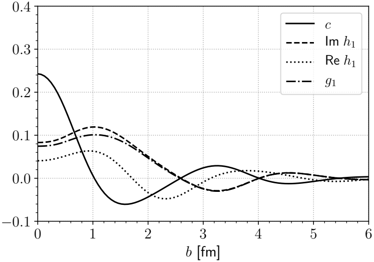
(a) profile functions: Fit 1 (central)



(b) profile functions: Fit 2 (peripheral)



(c) other *b*-dep. functions: Fit 1 (central)



(d) other *b*-dep. functions: Fit 2 (peripheral)

pp 52.8 GeV - real and imaginary parts of $F^{N}(s,t)$

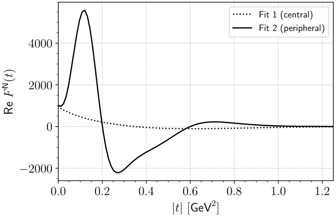


Figure: real parts - Fits 1 and 2

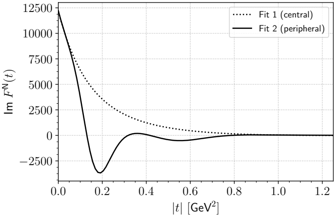


Figure: imaginary parts - Fits 1 and 2

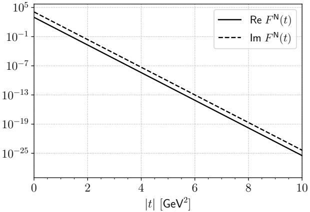


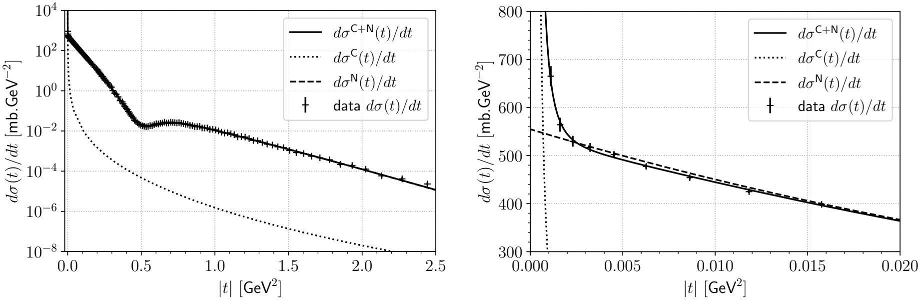
Figure: WY - real and imaginary parts (corresponding free parameters of $F^{N}(s,t)$ taken from [19])

- Eikonal model
  - Fit 1 (central) - real part of $F^{N}(s,t)$ changes sign at $|t| \approx 0.35$ GeV^2
  - Fit 2 (peripheral) - real part of $F^{N}(s,t)$ changes sign at $|t| \approx 0.2$ GeV^2

  $\Rightarrow$ conclusion of the Martin's theorem *fulfilled*
- WY
  - real part of $F^{N}(s,t)$ does not change sign at any t value (hadronic phase assumed to be t-independent)

  $\Rightarrow$ conclusion of the Martin's theorem *not fulfilled*

pp 8 TeV - differential cross sections

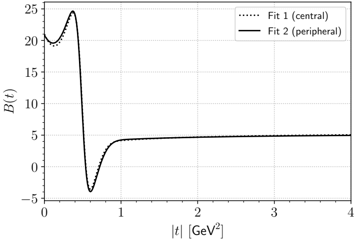


(a) full available $|t|$-range of measured data

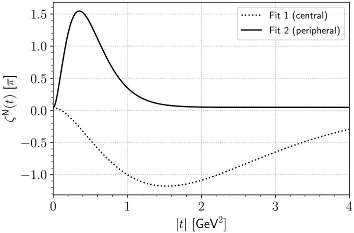
(b) region of very low values of $|t|$

- TOTEM 8 TeV data (1000m and 90m optics data up to $|t| = 0.2$ GeV^2, see [4, 5]); extended by renormalized TOTEM 7 TeV data [3] up to 2.5 GeV^2 to obtain data in wider t-region ($|t| \in \langle 6 \times 10^{-4}, 2.5\rangle$ GeV^2) including both region of peak at very low values of $|t|$ and dip-bump structure region ⇒ approximate data denoted as "8 TeV data"
- all the performed fits at 8 TeV leads to similar t-dependences of hadronic dσ/dt

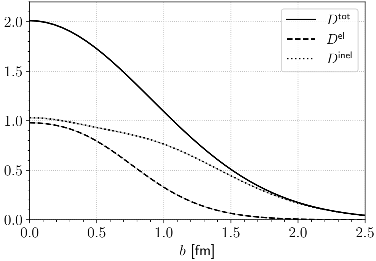
pp 8 TeV - t and b dependent characteristics



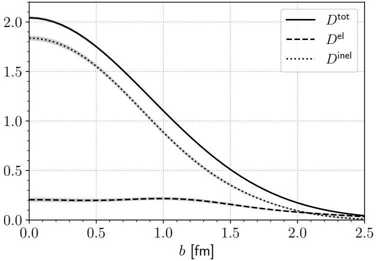
(a) diffractive slopes



(b) hadronic phases

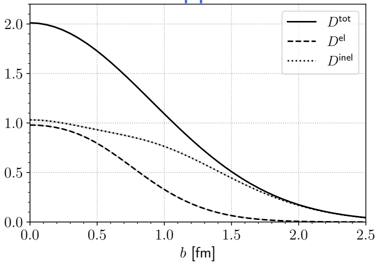


(c) profile functions: Fit 1 (central)

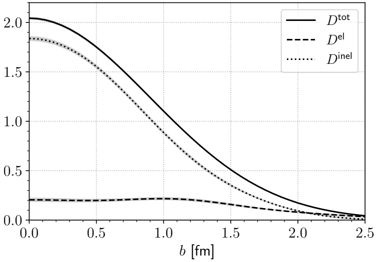


(d) profile functions: Fit 2 (peripheral)

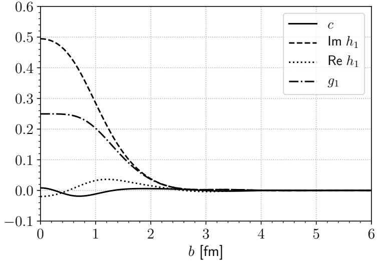
pp 8 TeV - *b*-dependent functions



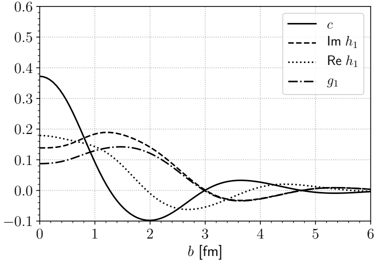
(a) profile functions: Fit 1 (central)



(b) profile functions: Fit 2 (peripheral)



(c) other *b*-dep. functions: Fit 1 (central)



(d) other *b*-dep. functions: Fit 2 (peripheral)

pp 8 TeV - real and imaginary parts of $F^{\mathrm{N}}(s,t)$

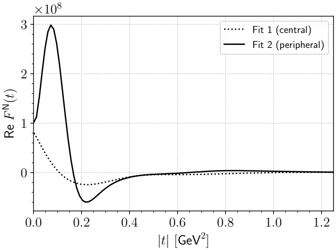


Figure: real parts - Fits 1 and 2

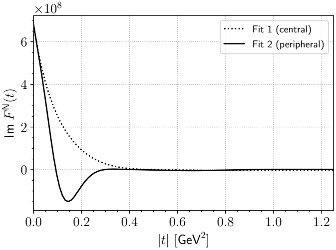


Figure: imaginary parts - Fits 1 and 2

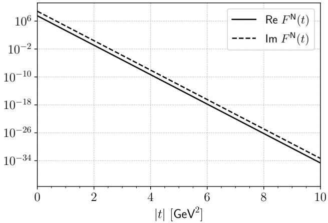


Figure: WY - real and imaginary parts (corresponding free parameters of $F^{\mathrm{N}}(s,t)$ taken from [5])

- Eikonal model
  - Fit 1 (central) - real part of $F^{\mathrm{N}}(s,t)$ changes sign at $|t| \approx 0.1$ GeV^2
  - Fit 2 (peripheral) - real part of $F^{\mathrm{N}}(s,t)$ changes sign at $|t| \approx 0.18$ GeV^2

$\Rightarrow$ conclusion of the Martin's theorem *fulfilled*

- WY
  - real part of $F^{\mathrm{N}}(s,t)$ does not change sign at any $|t|$ value (hadronic phase assumed to be t-independent)

$\Rightarrow$ conclusion of the Martin's theorem *not fulfilled*

Eikonal model - summary of the obtained results

1. measured $d\sigma/dt$ at two very different energies 52.8 GeV and 8 TeV analyzed in broad interval of $|t|$ values (including both peak at low values of $|t|$ and also region of dip-bump structure at higher $|t|$ values) under different assumptions (constrains) to determine their impact on hadronic quantities (and overall relevance of the given description)
2. for each description the corresponding t-dependence of $F^{N}(s,t)$ at *all* values of t has been determined from data and several quantities characterizing the collision process in t and b space have been calculated - to study the whole physical picture at given energy under the given set of assumptions
3. the results show:
   - choice of form factor (effective electric vs. effective electromagnetic) - *small or negligible* impact on determination of $F^{N}(s,t)$
   - t-dependence of hadronic phase - may *completely* change behaviour of collisions in b-space; the phase is only weakly constrained by the eikonal interference formula itself
   - t-dependence of $\left|F^{N}(s,t)\right|$ - *strongly* constrained by measured $d\sigma/dt$
4. hadronic amplitude in many models of elastic hadronic collisions is strongly a priory constrained without sufficient reasoning - our results show that these models then leads to *central* behaviour of elastic collisions (mainly due to required dominance of the imaginary part of $F^{N}(s,t)$ in forward region), corresponding particle structure has never been sufficiently explained in the literature
5. elastic collisions may be interpreted as *peripheral* processes (in agreement with usual ideas corresponding to collisions of two matter objects) just by allowing hadronic phase to be strongly t-dependent already at low values of $|t|$
6. hadronic amplitudes in all the studied cases are analytic, satisfy condition of unitarity and conclusion of Martin's theorem

Studies of assumptions in models of elastic hadron scattering

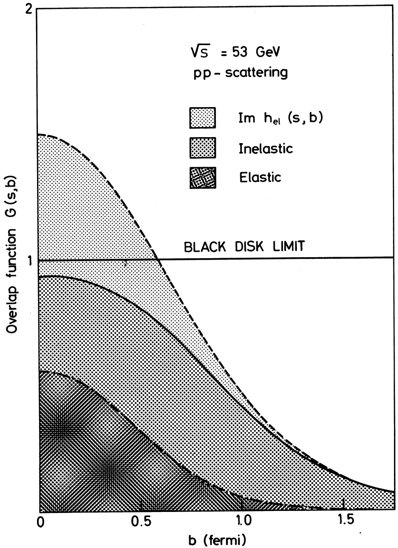
- many (all widely discussed) historical as well as contemporary models concerning description of ES have been reviewed
- many papers devoted to ES do not mention important assumptions and systematically study all their consequences, focus often on quantities of unclear physical meaning
- the "standardly" constrained elastic hadronic amplitude (leading to central behaviour of ES, see page (26)) was considered already by Miettinen several decades ago (in 1973-1975, see page (42)) - many contemporary models essentially repeat the same calculations without knowing and quoting his work
- centrality of ES (corresponding particles stucture)
  - never sufficiently explained in the literature
  - recently "rediscovered" in the literature under term *hollowness* - still the same story corresponding to the "standardly" constrained $F^{N}(s,t)$
- various fits of data under different assumptions in order to better understand strongly interacting particle processes have been performed
- the peripheral solution obtained within the eikonal framework solved many problems contained in other theoretical descriptions but some problems remained
- on the basis of these studies some deeper problems and open questions in *all* models and theoretical frameworks used in description of ES (WY model, eikonal model approach, Regge model approach, QCD-inspired models,...) have been identified, see page (43)

Miettinen's results - profile functions at 52.8 GeV

Miettinen [20–22] (1973-1975) - **one of the first discussions of behaviour of pp collisions in *b*-space** (many contemporary models are based on very similar calculations)

- profile functions (overlap functions $G(s,b)$ in Miettinen's terminology) established from pp data at 52.8 GeV and identified with probability functions
- it has been concluded from inelastic overlap (probability) function that $P^{\rm el}(b=0) = 1 - P^{\rm inel}(b=0) = 6\%$ (subtraction from "black disc limit")
- why the elastic probability $P^{\rm el}(b=0)$ is not taken directly from the elastic overlap (probability) function which is $\approx 50\%$, i.e., completely different?
- why $\mathrm{Im}\, h_{\rm el}(s,b)$, which is often identified with total profile function, i.e., probability in this case, is significantly greater then 1 for certain values of b?

⇒ the results suggesting transparency of colliding particles during collisions (+meaning of the profile functions) should be strongly questioned or even doubted, see detailed discussion in J. Procházka, M. V. Lokajíček, and V. Kundrát, "Dependence of elastic hadron collisions on impact parameter", Eur. Phys. J. Plus **131**, 147 (2016), see also arXiv:1509.05343 (2015)



Open questions and problems

Identified deeper problems and open questions in all contemporary descriptions of el. scattering:

1. Coulomb interaction and experimental conditions
   1.1 (non-)divergence at $t = 0$
   1.2 multiple collisions
   1.3 electromagnetic form factors
2. different mechanism of Coulomb and strong forces
3. different types of short-ranged (contact) interactions
4. properties of S matrix and structure of Hilbert space
5. optical theorem
6. determination of b-dependent probability functions of hadron collisions
7. distribution of elastic scattering angles for a given value of impact parameter
8. increase of integrated total, elastic and inelastic cross sections and dimensions of colliding particles in dependence on collision energy
9. extrapolations outside measured regions

- main results concerning b-dependence of elastic collisions and the open problems 1.-7. (including historical context) published

  J. Procházka, M. V. Lokajíček, and V. Kundrát, "Dependence of elastic hadron collisions on impact parameter", Eur. Phys. J. Plus **131**, 147 (2016), see also arXiv:1509.05343 (2015)
- problems related specifically to optical theorem discussed in

  J. Procházka, V. Kundrát, and M. V. Lokajíček, "Elastic scattering of hadrons without optical theorem", arXiv: 1502.00468 (2015)
- points 8. and 9. discussed in

  J. Procházka and V. Kundrát, "Eikonal model analysis of elastic proton-proton collisions at 52.8 GeV and 8 TeV", arXiv:1606.09479 (2019)

Proposed ways how to solve the problems

- new impact parameter analysis of pp collisions based on b-dependent probabilities and preliminary results at 52.8 GeV
  1. J. Procházka, M. V. Lokajíček, and V. Kundrát, "Dependence of elastic hadron collisions on impact parameter", Eur. Phys. J. Plus **131**, 147 (2016), see also arXiv:1509.05343 (2015)
  2. M. V. Lokajíček, V. Kundrát, and J. Procházka, "Schrödinger equation and (future) quantum physics", in *Advances in quantum mechanics*, edited by P. Bracken (InTech Publisher, 2013), pp. 105–132
- contemporary state of fundamental physical research (+historical context)

  M. V. Lokajíček and J. Procházka, "The contemporary state of fundamental physical research and the future path to scientific knowledge", arXiv:1610.08331 (2019)

  - identified main conceptual problems and other mistaken assumptions + proposed ways how to solve them
  - phenomenological approach has impeded real scientific progress
  - to make progress in physics one should return to
    - causal ontology
    - falsification approach (logic, consistence, systematical analysis of involved assumptions, testing, ...)

Summary and conclusion I

1. WY approach
   - used widely at ISR for "measurement" of $\sigma^{\text{tot,N}}$, $B(t=0)$ and $\rho(t=0)$
   - many problems and limitations identified have been later (several papers exist)
   - ⇒ WY approach should be abandoned as it may lead to wrong physical conclusions; it should not be used for constraining hadronic models based on assumptions inconsistent with the simplified model of WY
   - ⇒ one should look for other description of el. scattering of (charged) hadrons
2. eikonal model approach
   - more general and relevant for analysis of el. data than the (over)simplified approach of WY
   - more fundamental then other contemporary models of el. scattering as it may be used for description of Coulomb-hadronic interference *and* take into account also dependence of collisions on impact parameter (with the aim not to mix collisions corresponding to different values of impact parameter)
   - ES data at 52.8 GeV and 8 TeV analyzed under different assumptions consistently in the whole measured t-range and the results compared
   - the results obtained with the help of the eikonal model represent the most detailed and elaborated impact parameter analysis of elastic pp collision data which has ever been performed
   - ⇒ transparency of protons during elastic collisions based on unreasoned assumptions of unclear physical meaning; corresponding structure of protons never sufficiently explained in the literature
   - ⇒ elastic collision process may be interpreted as peripheral and protons as compact (non-transparent) particles

- J. Procházka and V. Kundrát, "Eikonal model analysis of elastic proton-proton collisions at 52.8 GeV and 8 TeV", arXiv:1606.09479 (2019)
- J. Procházka, "Elastic proton-proton collisions at high energies", CERN-THESIS-2018-294, PhD thesis (Feb. 2018)
- TOTEM Collaboration, "Measurement of elastic pp scattering at $\sqrt{s} = 8$ TeV in the Coulomb-nuclear interference region – determination of the ρ-parameter and the total cross-section", Eur. Phys. J. C **76**, 661 (2016), see also CERN-PH-EP-2015-325, arXiv:1610.00603

Summary and conclusion II

Measurement of diffractive pp collisions

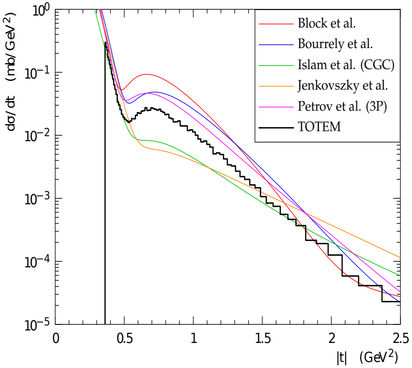
- elastic pp data available at several energies - including LHC energies, see TOTEM results
- CMS-PPS project designed to provide further important experimental data at the LHC concerning diffractive inelastic collisions [2]

Description of diffractive pp collisions

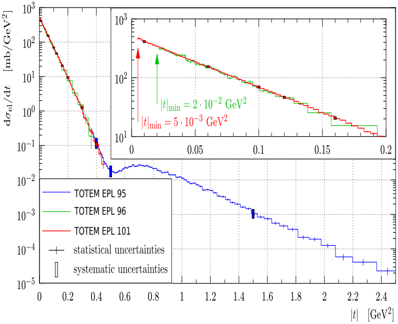
- proper analysis of hadron collisions in dependence on impact parameter may provide important insight concerning spacial characteristics (and other properties) of colliding particles which can be hardly obtained in a different way
- however, to move forward, one should first solve the known problems and open questions in *all* contemporary descriptions of elastic pp scattering (WY model, eikonal model, Regge approach, QCD-inspired models, ...), see page (43), before making far-reaching conclusions concerning structure and properties of colliding particles
- ways how to solve the problems have been proposed

1. Measurement of elastic pp collisions (TOTEM experiment)
2. Theoretical description of elastic pp collisions
    - West and Yennie (WY) approach
    - Eikonal model approach
3. Application of the eikonal model to elastic pp data
4. Open questions and problems + proposed ways how to solve them
5. Summary and conclusion
6. Backup

Measured elastic $d\sigma/dt$ at $\sqrt{s} = 7$ TeV by TOTEM



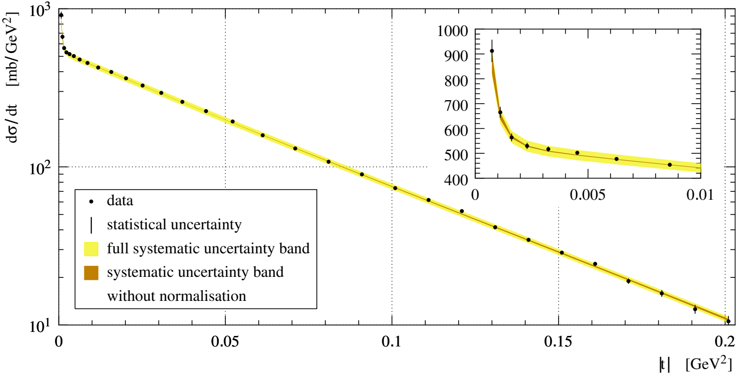
(a) first measurement of pp elastic differential cross section by TOTEM and predictions of different phenomenological models [27]



(b) three (independent) measurements by TOTEM at different settings [3]

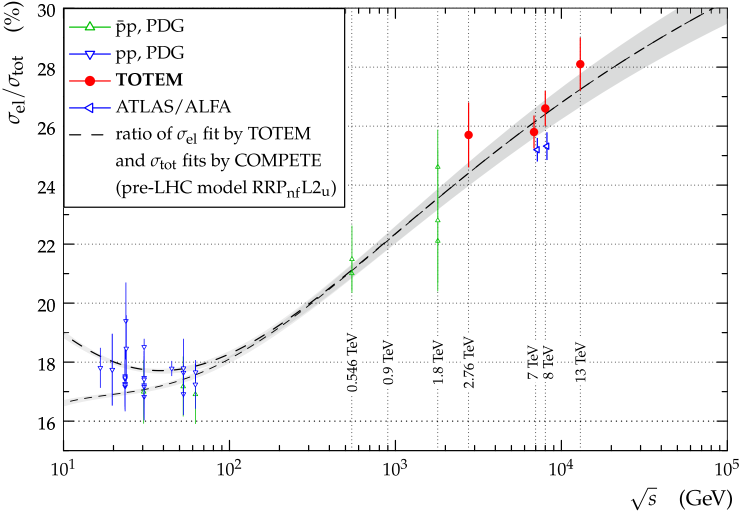
- elastic pp differential cross section at energy of $\sqrt{s} = 7$ TeV has similar t-dependence as at the ISR energies
- dip-bump structure in pp data observed again by TOTEM at the LHC since the era of the ISR

Measured elastic $d\sigma/dt$ at $\sqrt{s} = 8$ TeV by TOTEM



- significantly non-exponential part (see the points corresponding to the lowest measured values of $|t|$) attributed standardly to Coulomb-hadronic interference; special beam optics ($\beta^* = 1000$ m) has been needed to reach such low values of $|t|$ [5]

Elastic to total cross section ratio for pp and $\bar{p}p$



- significant increase with increasing collision energy $\sqrt{s}$, see TOTEM overview [7]

Fitting of observed $d\sigma/dt(s,t)$ and "measured" $\sigma^{tot,N}(s)$, $B(s,t=0)$, $\rho(s,t=0)$,...

- $d\sigma/dt(s,t)$ represents experimental data in the case of elastic scattering (ES)
- hadronic quantities like $\sigma^{tot,N}$, $B(t=0)$, $\rho(t=0)$, ... are determined with the help of a model of ES applied to measured $d\sigma/dt(s,t)$ under several strong assumptions $\Rightarrow$ speaking about "measurement" of these quantities may be very misleading
- a model of ES ($F^N(s,t)$), which is trying to describe given set of "measured" values, should be consistent with the method/model/assumptions used to produce them (this is often not the case)
- it is typically not a priory clear how much given principle, theorem or other assumption constrain $F^N(s,t)$, it may or may not be possible to describe experimental data under given set of assumptions - it needs to be study
- it is not sufficient to fit data $d\sigma/dt$ only at low values of $|t|$; one should study full physical picture (see also any contemporary Coulomb-hadronic formula and the FB transformation connecting t and b dependent elastic amplitudes)

studies of method of measurement of elastic pp scattering ($d\sigma/dt(s,t)$) and various models of ES together with their assumptions are essential for determination of several hadronic characteristics which can be hardly obtained in a different way

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