Gravitational formfactors and **pressure** in elementary particles

FU AV CR, July 25, 2018

Oleg Teryaev,
Joint Institute for Nuclear Research,
Dubna, Russia

LETTER

NATURE | www.nature.com/nature

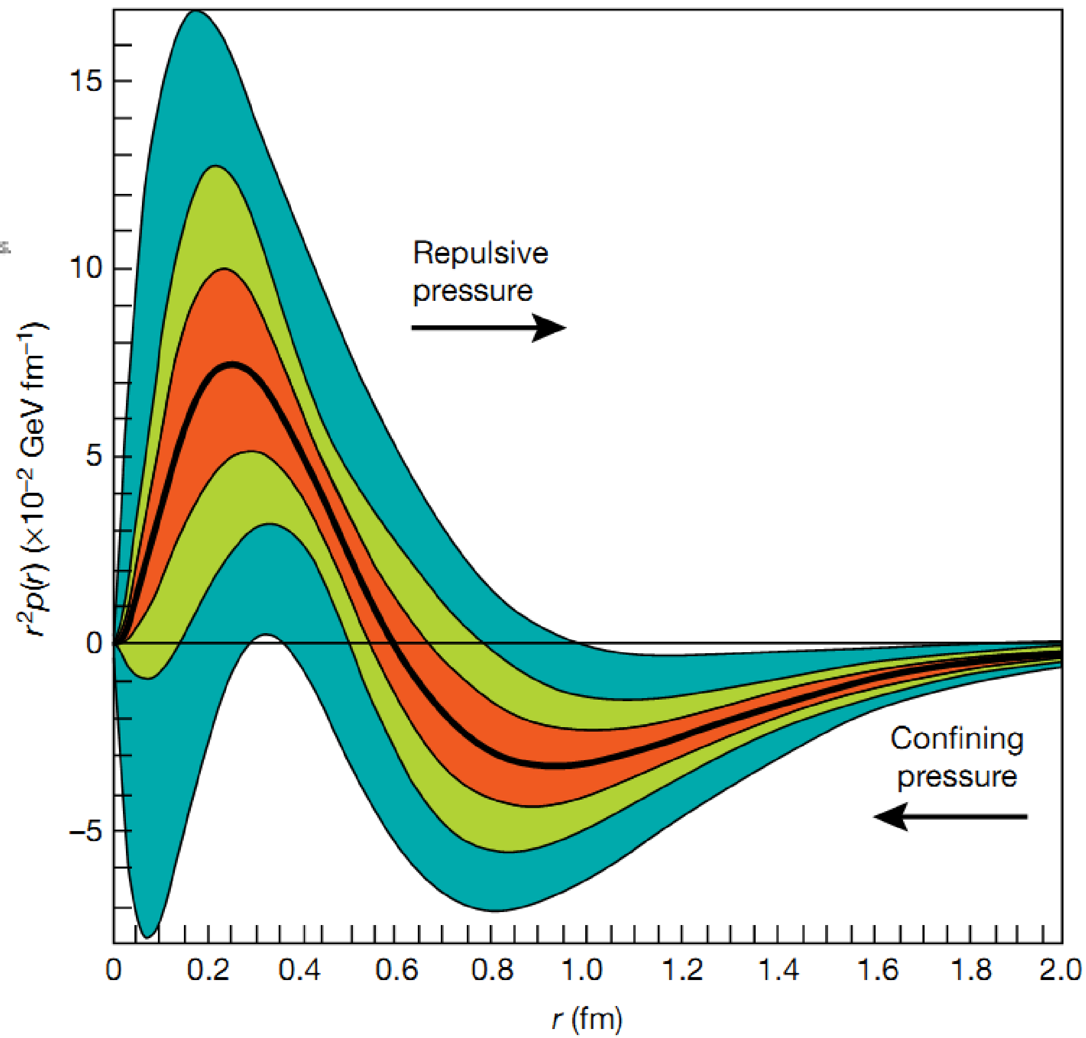
https://doi.org/10.1038/s41586-018-0060-z

The pressure distribution inside the proton

V. D. Burkert[1]*, L. Elouadrhiri[1] & F. X. Girod[1]

5. Teryaev, O. V. Gravitational form factors and nucleon spin structure. *Front. Phys.* **11**, 111207 (2016).

15. Anikin, I. V. & Teryaev, O. V. Dispersion relations and QCD factorization in hard reactions. *Fizika B* **17**, 151–158 (2008).



Main topics

- Energy momentum tensor (gravitational) formfactors
- Gravitomagnetism and post-Newtonian equivalence principle
- Spin-gravity interactions: anisotropic Universe
- Quadrupole FF, pressure and stability
- Holographic sum rule and pressure from subtraction
- Proton data and pressure distribution
- Photons as stable macroscopic objects
- What else (viscous protons etc.)
- Conclusions

Gravitational Formfactors

$$\langle p'|T^{\mu\nu}_{q,g}|p\rangle = \bar{u}(p')\Big[A_{q,g}(\Delta^2)\gamma^{(\mu}p^{\nu)} + B_{q,g}(\Delta^2)P^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha/2M]u(p)$$

- Conservation laws - zero Anomalous Gravitomagnetic Moment : $\mu_G = J$ (g=2)

$$P_{q,g} = A_{q,g}(0) \qquad A_q(0) + A_g(0) = 1$$

$$J_{q,g} = \frac{1}{2}\left[A_{q,g}(0) + B_{q,g}(0)\right] \qquad A_q(0) + B_q(0) + A_g(0) + B_g(0) = 1$$

- May be extracted from high-energy experiments/NPQCD calculations
- Describe the partition of angular momentum between quarks and gluons
- Describe interaction with both classical and TeV gravity

Generalized Parton Distributions (related to matrix elements of non local operators) – models for both EM and Gravitational Formfactors (Selyugin,OT ’09)

- Smaller mass square radius (attraction vs repulsion!?)

$$\rho(b) = \sum_q e_q \int dx q(x,b) \qquad = \int d^2q F_1(Q^2 = q^2) e^{i\vec{q}\,\vec{b}}$$

$$= \int_0^\infty \frac{q dq}{2\pi} J_0(qb) \frac{G_E(q^2) + \tau G_M(q^2)}{1+\tau}$$

$$\rho_0^{\rm Gr}(b) = \frac{1}{2\pi} \int_\infty^0 dq q J_0(qb) A(q^2)$$

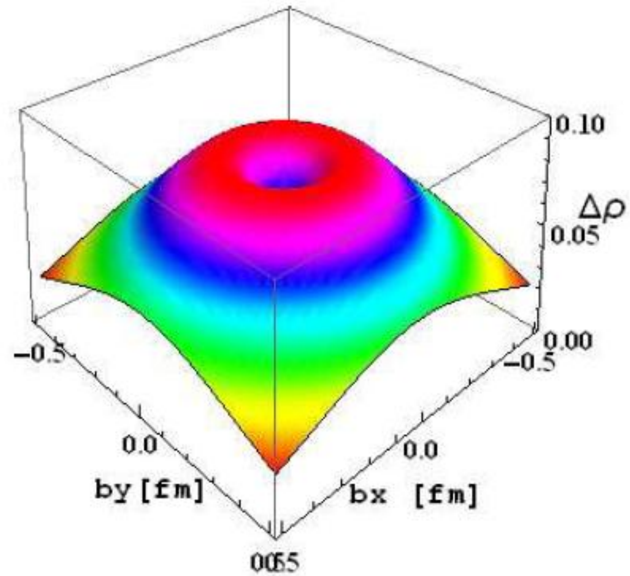


FIG. 17: Difference in the forms of charge density F_1^P and "matter" density (A)

Electromagnetism vs Gravity (OT'99)

- Interaction – field vs metric deviation

$$M = \langle P' | J_q^\mu | P \rangle A_\mu(q)$$

$$M = \frac{1}{2} \sum_{q,G} \langle P' | T_{q,G}^{\mu\nu} | P \rangle h_{\mu\nu}(q)$$

- Static limit

$$\langle P | J_q^\mu | P \rangle = 2 e_q P^\mu$$

$$\sum_{q,G} \langle P | T_i^{\mu\nu} | P \rangle = 2 P^\mu P^\nu$$

$$h_{00} = 2\phi(x)$$

$$M_0 = \langle P | J_q^\mu | P \rangle A_\mu = 2 e_q M \phi(q)$$

$$M_0 = \frac{1}{2} \sum_{q,G} \langle P | T_i^{\mu\nu} | P \rangle h_{\mu\nu} = 2M \cdot M \phi(q)$$

- Mass as charge – equivalence principle

Gravitomagnetism

- Gravitomagnetic field (weak, except in gravity waves) – action on spin from $M = \frac{1}{2}\sum_{q,G}\langle P'|T^{\mu\nu}_{q,G}|P\rangle h_{\mu\nu}(q)$

$$\vec{H}_J = \frac{1}{2} rot\vec{g};\ \vec{g}_i \equiv g_{0i}$$

spin dragging twice smaller than EM

- Lorentz force – similar to EM case: factor ½ cancelled with 2 from $h_{00} = 2\phi(x)$ Larmor frequency same as EM

$$\omega_J = \frac{\mu_G}{J} H_J = \frac{H_L}{2} = \omega_L\ \vec{H}_L = rot\vec{g}$$

- Orbital and Spin momenta dragging – the same - Equivalence principle

Equivalence principle

- Newtonian – “Falling elevator” – well known and checked (also for elementary particles)
- Post-Newtonian – gravity action on SPIN – known since 1962 (Kobzarev and Okun’; ZhETF paper contains acknowledgment to Landau: probably his last contribution to theoretical physics before car accident); rederived from conservarion laws - Kobzarev and V.I. Zakharov
- Anomalous gravitomagnetic (and electric-CP-odd) moment iz ZERO or
- Classical and QUANTUM rotators behave in the SAME way

Experimental test of PNEP

- Reinterpretation of the data on G(EDM) search

PHYSICAL REVIEW LETTERS

VOLUME 68 | 13 JANUARY 1992 | NUMBER 2

Search for a Coupling of the Earth's Gravitational Field to Nuclear Spins in Atomic Mercury

B. J. Venema, P. K. Majumder, S. K. Lamoreaux, B. R. Heckel, and E. N. Fortson

Physics Department, FM-15, University of Washington, Seattle, Washington 98195

(Received 25 September 1991)

- If (CP-odd!) GEDM=0 -> constraint for AGM (Silenko, OT'07) from Earth rotation– was considered as obvious (but it is just EP! - quantum measurement in rotating frame crucial) background

$$|\chi(^{201}\mathrm{Hg}) + 0.369\chi(^{199}\mathrm{Hg})| < 0.042 \quad (95\%\mathrm{C.L.})$$

Indirect probe of spin-gravity coupling

- Matrix elements of energy-momentum tensors may be extracted from accurate high-energy experiments (“3D nucleon picture”)
- Allow to probe the couplings to quarks and gluons separately

Equivalence principle for moving particles

- Compare gravity and acceleration: gravity provides EXTRA space components of metrics $h_{zz} = h_{xx} = h_{yy} = h_{00}$
- Matrix elements DIFFER

$$\mathcal{M}_g = (\epsilon^2 + p^2)h_{00}(q), \qquad \mathcal{M}_a = \epsilon^2 h_{00}(q)$$

- Ratio of accelerations: $R = \frac{\epsilon^2 + p^2}{\epsilon^2}$ - confirmed by explicit solution of Dirac equation (Silenko, OT, ‘05)
- Arbitrary fields – Obukhov, Silenko, OT ’09,’11,’13

Gravity vs accelerated frame for spin and helicity

- Spin precession – well known factor 3 (Probe B; spin at satellite – probe of PNEP!) – smallness of relativistic correction (~$\mathbf{P}^2$) is compensated by 1/ $\mathbf{P}^2$ in the momentum direction precession frequency
- Helicity flip – the same!
- No helicity flip in gravitomagnetic field – another formulation of PNEP (OT'99) and
- Flip by "gravitoelectric" field: relic neutrino? Black hole?

$$\frac{d\sigma_{+-}}{d\sigma_{++}} = \frac{tg^2(\frac{\phi}{2})}{(2\gamma - \gamma^{-1})^2}$$

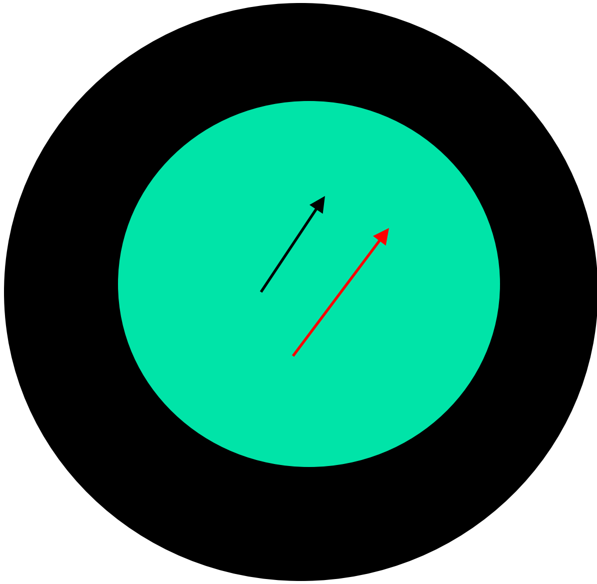
Gyromagnetic and Gravigyromagnetic ratios

- Free particles – coincide
- $\langle P+q|T^{mn}|P-q\rangle = P^{\{m}\langle P+q|J^{n\}}|P-q\rangle/e$ up to the terms linear in q
- Gravitomagnetic g=2 for any spin
- Special role of g=2 for ANY spin (asymptotic freedom for vector bosons)

- Should Einstein know about PNEP, the outcome of his and de Haas experiment would not be so surprising
- Recall also g=2 for Black Holes. Indication of “quantum” nature?!

Cosmological implications of PNEP

- Necessary condition for Mach's Principle (in the spirit of Weinberg's textbook) -
- Lense-Thirring inside massive rotating empty shell (=model of Universe)
- For flat "Universe" - precession frequency equal to that of shell rotation
- Simple observation-Must be the same for classical and quantum rotators – PNEP!
- More elaborate models - Tests for cosmology ?!



Yet another approach to rotation - Dirac Equation

- Metric of the type

$$ds^2 = V^2c^2dt^2 - \delta_{\hat{a}\hat{b}}W^{\hat{a}}{}_cW^{\hat{b}}{}_d(dx^c - K^c cdt)(dx^d - K^d cdt).$$

- Tetrads in Schwinger gauge

$$e_i^{\hat{0}} = V\delta_i^0, \qquad e_i^{\hat{a}} = W^{\hat{a}}{}_b(\delta_i^b - cK^b\delta_i^0),$$

$$e_{\hat{0}}^i = \frac{1}{V}(\delta^i{}_0 + \delta^i{}_a cK^a), \qquad e_{\hat{a}}^i = \delta^i{}_b W^b{}_{\hat{a}}, \qquad a = 1, 2, 3,$$

- Dirac eq $(i\hbar\gamma^\alpha D_\alpha - mc)\Psi = 0, \qquad \alpha = 0, 1, 2, 3.$

$$D_\alpha = e_\alpha^i D_i, \qquad D_i = \partial_i + \frac{iq}{\hbar}A_i + \frac{i}{4}\sigma^{\alpha\beta}\Gamma_{i\alpha\beta}.$$

Dirac hamiltonian

- Connection

$$\Gamma_{i\hat{a}\hat{0}} = \frac{c^2}{V} W^b{}_{\hat{a}} \partial_b V e_i{}^{\hat{0}} - \frac{c}{V} \mathcal{Q}_{(\hat{a}\hat{b})} e_i{}^{\hat{b}},$$

$$\Gamma_{i\hat{a}\hat{b}} = \frac{c}{V} \mathcal{Q}_{[\hat{a}\hat{b}]} e_i{}^{\hat{0}} + (\mathcal{C}_{\hat{a}\hat{b}\hat{c}} + \mathcal{C}_{\hat{a}\hat{c}\hat{b}} + \mathcal{C}_{\hat{c}\hat{b}\hat{a}}) e_i{}^{\hat{c}}.$$

$$\mathcal{Q}_{\hat{a}\hat{b}} = g_{\hat{a}\hat{c}} W^d{}_{\hat{b}} \left(\frac{1}{c} \dot{W}^{\hat{c}}{}_d + K^e \partial_e W^{\hat{c}}{}_d + W^{\hat{c}}{}_e \partial_d K^e \right),$$

$$\mathcal{C}_{\hat{a}\hat{b}}{}^{\hat{c}} = W^d{}_{\hat{a}} W^e{}_{\hat{b}} \partial_{[d} W^{\hat{c}}{}_{e]}, \qquad \mathcal{C}_{\hat{a}\hat{b}\hat{c}} = g_{\hat{c}\hat{d}} \mathcal{C}_{\hat{a}\hat{b}}{}^{\hat{d}}.$$

- Hermitian Hamiltonian

$$i\hbar \frac{\partial \psi}{\partial t} = \mathcal{H} \psi. \qquad \psi = (\sqrt{-g} e_{\hat{0}}^0)^{\frac{1}{2}} \Psi.$$

$$\mathcal{H} = \beta m c^2 V + q\Phi + \frac{c}{2}(\pi_b \mathcal{F}^b{}_a \alpha^a + \alpha^a \mathcal{F}^b{}_a \pi_b) + \frac{c}{2}(\boldsymbol{K} \cdot \boldsymbol{\pi} + \boldsymbol{\pi} \cdot \boldsymbol{K}) + \frac{\hbar c}{4}(\boldsymbol{\Xi} \cdot \boldsymbol{\Sigma} - \Upsilon \gamma_5).$$

$$\Upsilon = V \epsilon^{\hat{a}\hat{b}\hat{c}} \Gamma_{\hat{a}\hat{b}\hat{c}} = -V \epsilon^{\hat{a}\hat{b}\hat{c}} \mathcal{C}_{\hat{a}\hat{b}\hat{c}},$$

$$\Xi_{\hat{a}} = \frac{V}{c} \epsilon_{\hat{a}\hat{b}\hat{c}} \Gamma_{\hat{0}}{}^{\hat{b}\hat{c}} = \epsilon_{\hat{a}\hat{b}\hat{c}} \mathcal{Q}^{\hat{b}\hat{c}}.$$

Foldy-Wouthuysen transformation

- Even and odd parts

$$\mathcal{H} = \beta\mathcal{M} + \mathcal{E} + \mathcal{O}, \qquad \beta\mathcal{M} = \mathcal{M}\beta,$$

$$\beta\mathcal{E} = \mathcal{E}\beta, \qquad \beta\mathcal{O} = -\mathcal{O}\beta.$$

- FW transformation (Silenko '08)

$$U = \frac{\beta\epsilon + \beta\mathcal{M} - \mathcal{O}}{\sqrt{(\beta\epsilon + \beta\mathcal{M} - \mathcal{O})^2}}\beta, \qquad \psi_{\mathrm{FW}} = U\psi, \qquad \mathcal{H}_{\mathrm{FW}} = U\mathcal{H}U^{-1} - i\hbar U\partial_t U^{-1}.$$

$$U^{-1} = \beta\frac{\beta\epsilon + \beta\mathcal{M} - \mathcal{O}}{\sqrt{(\beta\epsilon + \beta\mathcal{M} - \mathcal{O})^2}}. \qquad \epsilon = \sqrt{\mathcal{M}^2 + \mathcal{O}^2}$$

$$\begin{aligned}\mathcal{H}' = \beta\epsilon + \mathcal{E} &+ \frac{1}{2T}([T,[T,(\beta\epsilon + Z)]] \\ &+ \beta[\mathcal{O},[\mathcal{O},\mathcal{M}]] - [\mathcal{O},[\mathcal{O},Z]] \\ &- [(\epsilon + \mathcal{M}),[(\epsilon + \mathcal{M}),Z]] - [(\epsilon + \mathcal{M}),[\mathcal{M},\mathcal{O}]] \\ &- \beta\{\mathcal{O},[(\epsilon + \mathcal{M}),Z]\} + \beta\{(\epsilon + \mathcal{M}),[\mathcal{O},Z]\})\frac{1}{T},\end{aligned}$$

$$T = \sqrt{(\beta\epsilon + \beta\mathcal{M} - \mathcal{O})^2}$$

$$Z = \mathcal{E} - i\hbar\tfrac{\partial}{\partial t}$$

$$\mathcal{H}' = \beta\epsilon + \mathcal{E}' + \mathcal{O}', \qquad \beta\mathcal{E}' = \mathcal{E}'\beta, \qquad \beta\mathcal{O}' = -\mathcal{O}'\beta,$$

$$\mathcal{H}_{\mathrm{FW}} = \beta\epsilon + \mathcal{E}' + \frac{1}{4}\beta\left\{\mathcal{O}'^2, \frac{1}{\epsilon}\right\}.$$

FW for arbitrary gravitational field (Obukhov,Silenko,OT'13)

- Result

$$\mathcal{H}_{\rm FW} = \mathcal{H}^{(1)}_{\rm FW} + \mathcal{H}^{(2)}_{\rm FW}.$$

$$\epsilon' = \sqrt{m^2c^4V^2 + \frac{c^2}{4}\delta^{ac}\{p_b, \mathcal{F}^b{}_a\}\{p_d, \mathcal{F}^d{}_c\}},$$

$$\mathcal{T} = 2\epsilon'^2 + \{\epsilon', mc^2V\}.$$

$$\mathcal{M} = mc^2V,$$

$$\mathcal{E} = q\Phi + \frac{c}{2}(\boldsymbol{K}\cdot\boldsymbol{\pi} + \boldsymbol{\pi}\cdot\boldsymbol{K}) + \frac{\hbar c}{4}\boldsymbol{\Xi}\cdot\boldsymbol{\Sigma},$$

$$\mathcal{O} = \frac{c}{2}(\pi_b\mathcal{F}^b{}_a\alpha^a + \alpha^a\mathcal{F}^b{}_a\pi_b) - \frac{\hbar c}{4}\Upsilon\gamma_5.$$

$$\begin{aligned}\mathcal{H}^{(1)}_{\rm FW} = {}& \beta\epsilon' + \frac{\hbar c^2}{16}\Big\{\frac{1}{\epsilon'}, (2\epsilon^{cae}\Pi_e\{p_b, \mathcal{F}^d{}_c\partial_d\mathcal{F}^b{}_a\} \\ & + \Pi^a\{p_b, \mathcal{F}^b{}_a\Upsilon\})\Big\} \\ & + \frac{\hbar mc^4}{4}\epsilon^{cae}\Pi_e\Big\{\frac{1}{\mathcal{T}}, \{p_d, \mathcal{F}^d{}_c\mathcal{F}^b{}_a\partial_bV\}\Big\},\end{aligned}$$

$$\begin{aligned}\mathcal{H}^{(2)}_{\rm FW} = {}& \frac{c}{2}(K^ap_a + p_aK^a) + \frac{\hbar c}{4}\Sigma_a\Xi^a \\ & + \frac{\hbar c^2}{16}\Big\{\frac{1}{\mathcal{T}}, \Big\{\Sigma_a\{p_e, \mathcal{F}^e{}_b\}, \Big\{p_f, \Big[\epsilon^{abc}\Big(\frac{1}{c}\dot{\mathcal{F}}^f{}_c \\ & - \mathcal{F}^d{}_c\partial_dK^f + K^d\partial_d\mathcal{F}^f{}_c\Big) \\ & - \frac{1}{2}\mathcal{F}^f{}_d(\delta^{db}\Xi^a - \delta^{da}\Xi^b)\Big]\Big\}\Big\}\Big\},\end{aligned}$$

Operator EOM

- Polarization operator $\mathbf{\Pi} = \beta \mathbf{\Sigma}$

$$\frac{d\mathbf{\Pi}}{dt} = \frac{i}{\hbar}[\mathcal{H}_{\mathrm{FW}}, \mathbf{\Pi}] = \mathbf{\Omega}_{(1)} \times \mathbf{\Sigma} + \mathbf{\Omega}_{(2)} \times \mathbf{\Pi}.$$

- Angular velocities

$$\Omega^a_{(1)} = \frac{mc^4}{2}\left\{\frac{1}{\mathcal{T}}, \{p_e, \epsilon^{abc}\mathcal{F}^e{}_b\mathcal{F}^d{}_c\partial_d V\}\right\} + \frac{c^2}{8}\left\{\frac{1}{\epsilon'}, \{p_e, (2\epsilon^{abc}\mathcal{F}^d{}_b\partial_d\mathcal{F}^e{}_c + \delta^{ab}\mathcal{F}^e{}_b\Upsilon)\}\right\},$$

$$\Omega^a_{(2)} = \frac{\hbar c^2}{8}\left\{\frac{1}{\mathcal{T}}, \left\{\{p_e, \mathcal{F}^e{}_b\}, \left\{p_f, \left[\epsilon^{abc}\left(\frac{1}{c}\dot{\mathcal{F}}^f{}_c - \mathcal{F}^d{}_c\partial_d K^f + K^d\partial_d\mathcal{F}^f{}_c\right) - \frac{1}{2}\mathcal{F}^f{}_d(\delta^{db}\Xi^a - \delta^{da}\Xi^b)\right]\right\}\right\}\right\} + \frac{c}{2}\Xi^a.$$

Semi-classical limit

- Average spin

$$\frac{d\mathbf{s}}{dt} = \mathbf{\Omega} \times \mathbf{s} = (\mathbf{\Omega}_{(1)} + \mathbf{\Omega}_{(2)}) \times \mathbf{s},$$

$$\Omega^a_{(1)} = \frac{c^2}{\epsilon'} \mathcal{F}^d{}_c p_d \Big(\frac{1}{2} \Upsilon \delta^{ac} - \epsilon^{aef} V C_{ef}{}^c + \frac{\epsilon'}{\epsilon' + mc^2 V} \epsilon^{abc} W^e{}_{\hat{b}} \partial_e V \Big),$$

$$\Omega^a_{(2)} = \frac{c}{2} \Xi^a - \frac{c^3}{\epsilon'(\epsilon' + mc^2 V)} \epsilon^{abc} Q_{(bd)} \delta^{dn} \mathcal{F}^k{}_n p_k \mathcal{F}^l{}_c p_l,$$

Application to anisotropic universe (Kamenshchik,OT'16) – no suppression ~ G M/Rc^2

- Bianchi-1 Universe

$$ds^2 = dt^2 - a^2(t)(dx^1)^2 - b^2(t)(dx^2)^2 - c^2(t)(dx^3)^2.$$

- Particular case $W^{\hat{1}}_1 = a(t),\ W^{\hat{2}}_2 = b(t),\ W^{\hat{3}}_3 = c(t).$

$$W^1_{\hat{1}} = \frac{1}{a(t)},\ W^2_{\hat{2}} = \frac{1}{b(t)},\ W^3_{\hat{3}} = \frac{1}{c(t)}.$$

- No anholonomity $\Upsilon = 0$

$$\Omega^{\hat{1}}_{(2)} = \frac{\gamma}{\gamma+1} v_{\hat{2}} v_{\hat{3}} \left(\frac{\dot{b}}{b} - \frac{\dot{c}}{c}\right).$$

$$Q_{\hat{1}\hat{1}} = -\frac{\dot{a}}{a},\ Q_{\hat{2}\hat{2}} = -\frac{\dot{b}}{b},\ Q_{\hat{3}\hat{3}} = -\frac{\dot{c}}{c}.$$

Kasner solution

- t-dependence

$$a(t) = a_0 t^{p_1},\ b(t) = b_0 t^{p_2},\ c(t) = c_0 t^{p_3},$$

$$p_1 + p_2 + p_3 = 1,\ \ p_1^2 + p_2^2 + p_3^2 = 1.$$

- Euler-type expressions

$$\Omega^{\bar{1}}_{(2)} = \frac{\gamma}{\gamma + 1} v_{\bar{2}} v_{\bar{3}} \left(\frac{p_2 - p_3}{t} \right)$$

Heckmann-Schucking solution

- Dust admixture

$$a(t) = a_0 t^{p_1}(t_0+t)^{\frac{2}{3}-p_1},\ b(t) = b_0 t^{p_2}(t_0+t)^{\frac{2}{3}-p_2},$$
$$c(t) = c_0 t^{p_3}(t_0+t)^{\frac{2}{3}-p_3}.$$

- Modification:

$$\Omega^1_{(2)} = \frac{\gamma}{\gamma+1} v_2 v_3 \frac{(p_2-p_3)t_0}{t(t_0+t)}$$

$$= \frac{\gamma}{\gamma+1} v_2 v_3 \frac{(p_2-p_3)t_0}{t^2}\left(1+o\left(\frac{t_0}{t}\right)\right)$$

Biancki-IX Universe

- Metric $W^{\hat{b}}{}_{a} = \begin{pmatrix} -a\sin x^3 & a\sin x^1 \cos x^3 & 0 \\ b\cos x^3 & b\sin x^1 \sin x^3 & 0 \\ 0 & c\cos x^1 & c \end{pmatrix}$ $W^{c}{}_{\hat{b}} = \begin{pmatrix} -\frac{1}{a}\sin x^3 & \frac{1}{b}\cos x^3 & 0 \\ \frac{1}{a}\frac{\cos x^3}{\sin x^1} & \frac{1}{b}\frac{\sin x^3}{\sin x^1} & 0 \\ -\frac{1}{a}\frac{\cos x^1 \cos x^3}{\sin x^1} & -\frac{1}{b}\frac{\sin x^3 \cos x^1}{\sin x^1} & \frac{1}{c} \end{pmatrix}$

- Anholonomity coefficients
- $C^{\hat{3}}_{\hat{1}\hat{2}} = \frac{c}{ab}$ + cyclic permutations
- -> non-zero $\Upsilon = 2\left(\frac{c}{ab} + \frac{b}{ac} + \frac{a}{bc}\right)$

$$\Omega^{\hat{1}}_{(1)} = v^{\hat{1}}\left(\frac{c}{ab} + \frac{b}{ac} - \frac{a}{bc}\right)$$

Approach to singularity

- Chaotic oscillations – sequence of Kasner regimes $p_1 = -\frac{u}{1+u+u^2},\ p_2 = \frac{1+u}{1+u+u^2},\ p_3 = \frac{u(1+u)}{1+u+u^2}$
- If Lifshitz-Khalatnıkov parameter u >1 – "epochs" $p_1' = p_2(u-1),\ p_2' = p_1(u-1),\ p_3' = p_3(u-1)$

$$p_1' = p_1\left(\frac{1}{u}\right),\ p_2' = p_3\left(\frac{1}{u}\right),\ p_3' = p_2\left(\frac{1}{u}\right)$$

- If u< 1 – "eras"
- Change of eras – chaotic mapping of [0,1]interval

$$Tx = \left\{\frac{1}{x}\right\},\quad x_{s+1} = \left\{\frac{1}{x_s}\right\}$$

Angular velocities

- New epoch: u -> -u
- New era – changed sign

$$\Omega^{\hat{1}}_{(2)} = \frac{\gamma}{(\gamma+1)t} v_{\hat{2}} v_{\hat{3}} \cdot \frac{1-u^2}{1+u+u^2},$$
$$\Omega^{\hat{2}}_{(2)} = \frac{\gamma}{(\gamma+1)t} v_{\hat{1}} v_{\hat{3}} \cdot \frac{2u+u^2}{1+u+u^2},$$
$$\Omega^{\hat{3}}_{(2)} = -\frac{\gamma}{(\gamma+1)t} v_{\hat{1}} v_{\hat{2}} \cdot \frac{1+2u}{1+u+u^2}.$$

- Odd velocity

$$\Omega^{\hat{1}}_{(1)} \sim -v^{\hat{1}}(t)^{\left(-1-\frac{2u}{1+u+u^2}\right)},$$
$$\Omega^{\hat{b}}_{(1)} \sim v^{\hat{b}}(t)^{\left(-1-\frac{2u}{1+u+u^2}\right)}, \; b = 2, 3.$$

- New epoch
- New era - preserved

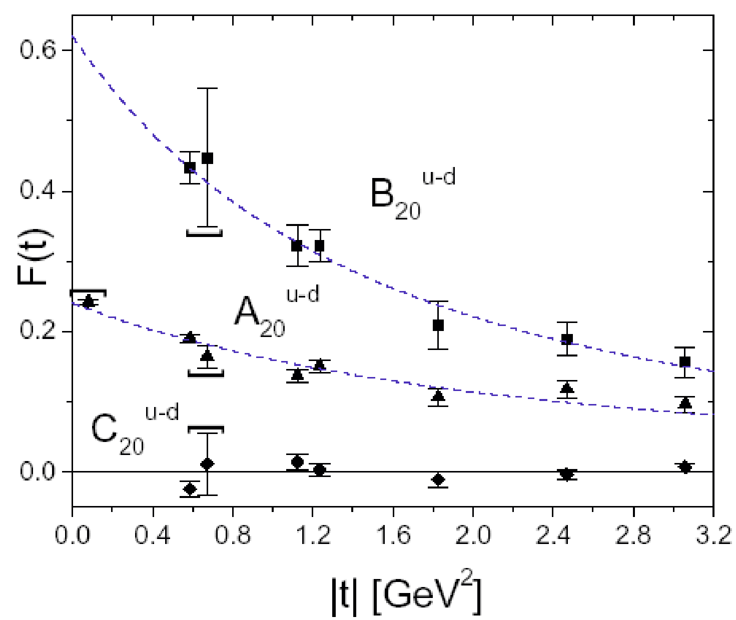
$$\Omega^{\hat{2}}_{(1)} \sim -v^{\hat{2}}(t)^{\left(-1-\frac{2u-2}{1-u+u^2}\right)},$$
$$\Omega^{\hat{a}}_{(1)} \sim v^{\hat{a}}(t)^{\left(-1-\frac{2u-2}{1-u+u^2}\right)}, \; a = 1, 3.$$

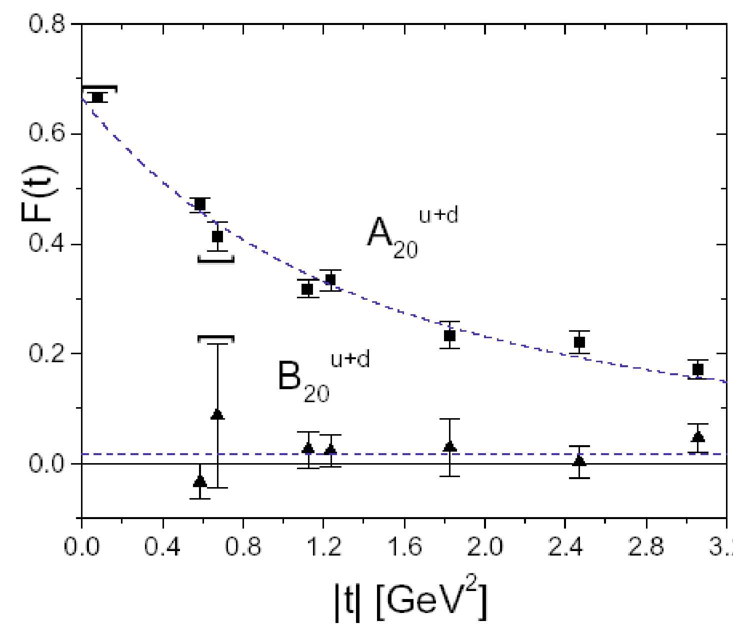
Possible applications

- Anisotropy (c.f. crystals) ~ magnetic field
- Spin precession + equivalence principle = helicity flip (~AMM effect)
- Dirac neutrino – transformed to sterile component in early (bounced) Universe
- Angular velocity ~ 1/t -> amount of decoupled ~ 1
- Possible new candidate for dark matter?!
- Other fields AFTER inflation?

Generalization of Equivalence principle

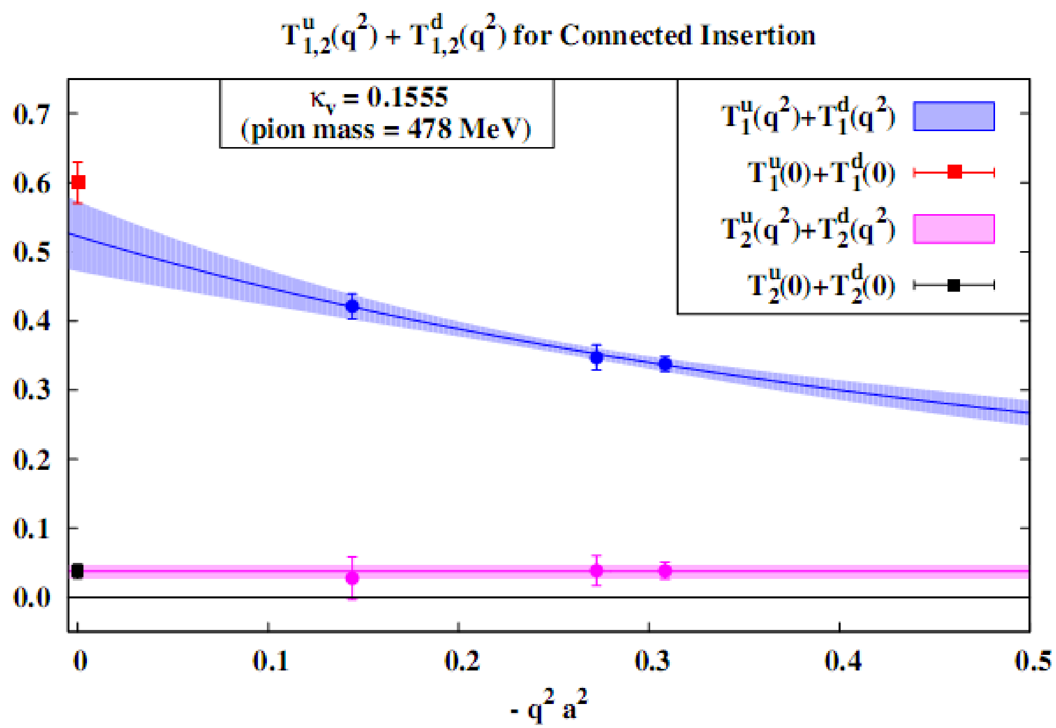
- Various arguments: AGM ≈ 0 separately for quarks and gluons – most clear from the lattice (LHPC/SESAM)





Recent lattice study (M. Deka et al. Phys.Rev. D91 (2015) no.1, 014505)

- Sum of u and d for Dirac (T1) and Pauli (T2) FFs



Extended Equivalence Principle=Exact EquiPartition

- In pQCD – violated
- Reason – in the case of ExEP- no smooth transition for zero fermion mass limit (Milton, 73)
- Conjecture (O.T., 2001 – prior to lattice data) – valid in NP QCD – zero quark mass limit is safe due to chiral symmetry breaking
- Gravityproof confinement? Nucleons do not break even by black holes?

One more gravitational formfactor

- Quadrupole

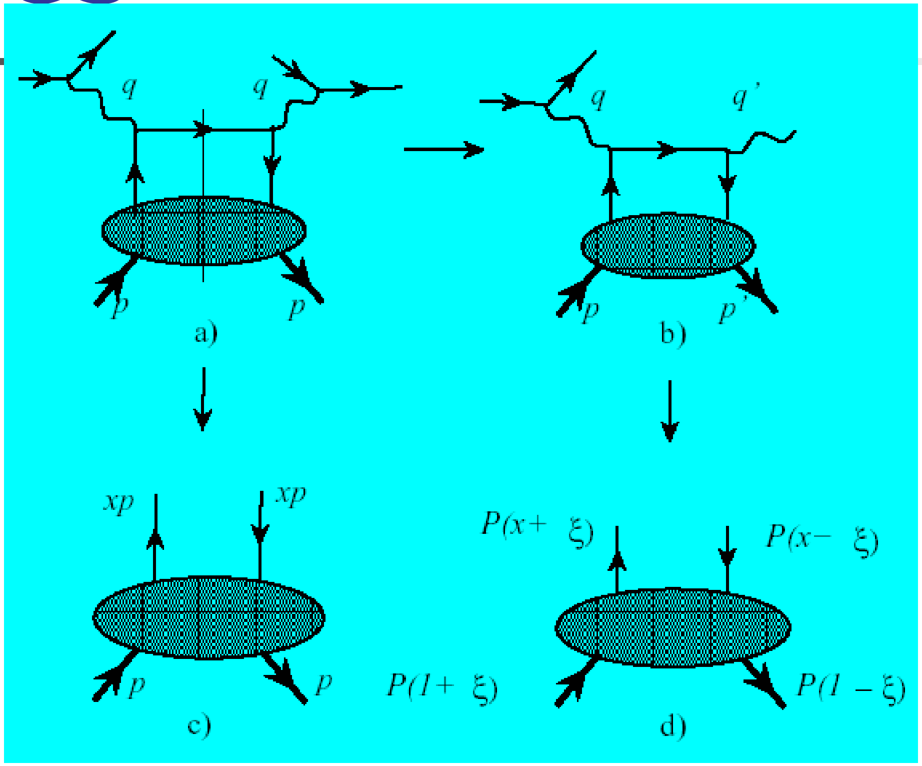
$$\langle P+q/2|T^{\mu\nu}|P-q/2\rangle = C(q^2)(g^{\mu\nu}q^2 - q^\mu q^\nu) + \ldots$$

- Cf vacuum matrix element – cosmological constant $\langle 0|T^{\mu\nu}|0\rangle = \Lambda g^{\mu\nu}$

$$\Lambda = C(q^2)q^2$$

- Inflation ~ annihilation ($q^2>0$) OT'15
- How to measure experimentally – Deeply Virtual Compton Scattering

QCD Factorization for DIS and DVCS



- Manifestly spectral

$$\mathcal{H}(x_B) = \int_{-1}^{1} dx \frac{H(x)}{x - x_B + i\epsilon}$$

- Extra dependence on ξ

$$\mathcal{H}(\xi) = \int_{-1}^{1} dx \frac{H(x,\xi)}{x - \xi + i\epsilon},$$

Unphysical regions

- DIS : Analytical function – $\mathrm{J} \backslash \mathrm{X}^{\mathrm{B}}$ polynomial in if $1 \leq |X_B|$

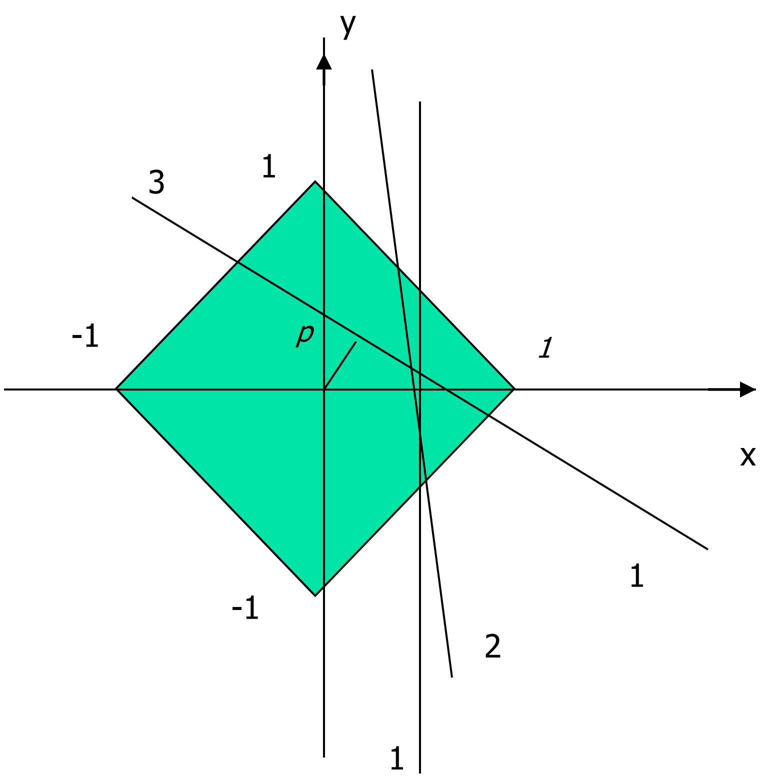
$$H(x_B) = -\int_{-1}^{1} dx \sum_{n=0}^{\infty} H(x) \frac{x^n}{x_B^{n+1}}$$

- DVCS – additional problem of analytical continuation of $H(x,\xi)$
- Solved by using of Double Distributions Radon transform

$$H(z,\xi) = \int_{-1}^{1} dx \int_{|x|-1}^{1-|x|} dy (F(x,y) + \xi G(x,y)) \delta(z - x - \xi y)$$

Double distributions and their integration

- Slope of the integration line-skewness
- Kinematics of DIS: $\xi = 0$ ("forward") - vertical line (1)
- Kinematics of DVCS: $\xi < 1$ - line 2
- Line 3: $\xi > 1$ unphysical region - required to restore DD by inverse Radon transform: tomography

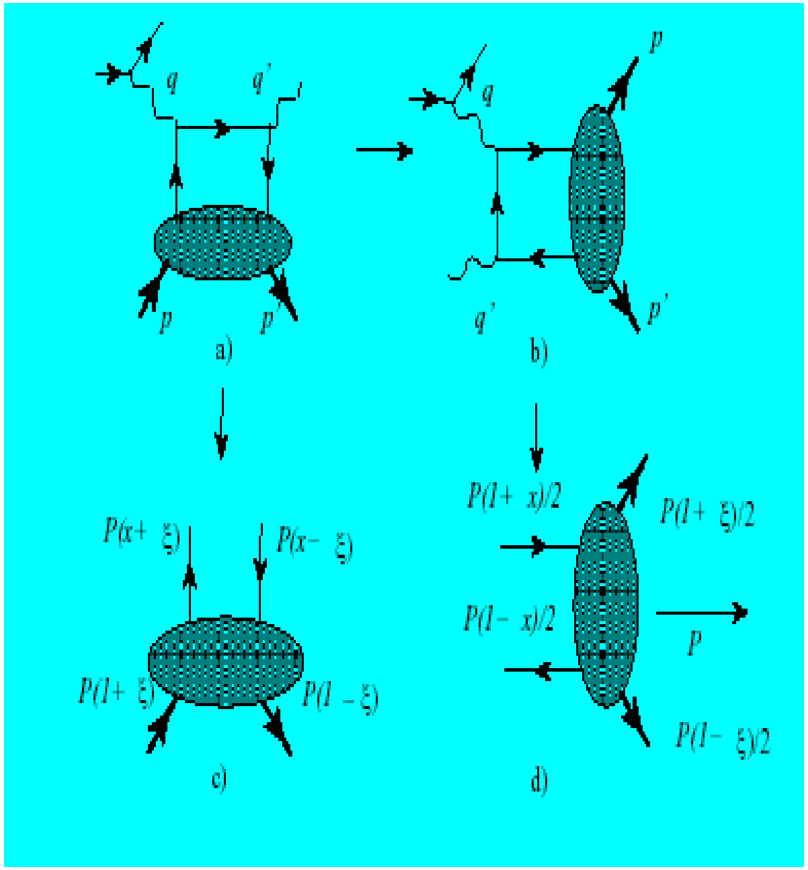


$$f(x,y) = -\frac{1}{2\pi^2}\int_0^\infty \frac{dp}{p^2}\int_0^{2\pi} d\phi |cos\phi| (H(p/cos\phi + x + ytg\phi, tg\phi) - H(x + ytg\phi, tg\phi)) =$$

$$= -\frac{1}{2\pi^2}\int_{-\infty}^\infty \frac{dz}{z^2}\int_{-\infty}^\infty d\xi (H(z + x + y\xi, \xi) - H(x + y\xi, \xi))$$

Crossing for DVCS and GPD

- DVCS -> hadron pair production in the collisions of real and virtual photons
- GPD -> Generalized Distribution Amplitudes
- Duality between s and t channels (Polyakov,Shuvaev, Guzey, Vanderhaeghen)



GDA -> back to unphysical regions for DIS and DVCS

- Recall DIS

$$H(x_B) = -\int_{-1}^{1} dx \sum_{n=0}^{\infty} H(x) \frac{x^n}{x_B^{n+1}}$$

- Non-positive powers of x_B

- DVCS

$$H(\xi) = -\int_{-1}^{1} dx \sum_{n=0}^{\infty} H(x,\xi) \frac{x^n}{\xi^{n+1}}$$

- Polynomiality (general property of Radon transforms): moments - integrals in *x* weighted with x^n - are polynomials in $1/\xi$ of power *n+1*
- As a result, analyticity is preserved: only non-positive powers of ξ appear

Holographic property (OT'05)

- Factorization Formula ->

$$\mathcal{H}(\xi) = \int_{-1}^{1} dx \frac{H(x,\xi)}{x-\xi+i\epsilon}$$

$$\Delta\mathcal{H}(\xi) \equiv \int_{-1}^{1} dx \frac{H(x,x)-H(x,\xi)}{x-\xi+i\epsilon}$$

$$= \sum_{n=1}^{\infty} \frac{1}{n!} \frac{\partial^n}{\partial \xi^n} \int_{-1}^{1} H(x,\xi) dx (x-\xi)^{n-1} = const$$

- Analyticity -> Imaginary part -> Dispersion relation:

$$\mathcal{H}(\xi) = \int_{-1}^{1} dx \frac{H(x,x)}{x-\xi+i\epsilon}$$

- "Holographic" equation

Holographic property - II

- Directly follows from double distributions

$$H(z,\xi) = \int_{-1}^{1} dx \int_{|x|-1}^{1-|x|} dy (F(x,y) + \xi G(x,y)) \delta(z - x - \xi y)$$

- Constant is the SUBTRACTION one - due to the (generalized) Polyakov-Weiss term G(x,y)

$$\Delta\mathcal{H}(\xi) = \int_{-1}^{1} dx \int_{|x|-1}^{1-|x|} dy \frac{G(x,y)}{1-y}$$

$$= \int_{-\xi}^{\xi} dx \frac{D(x/\xi)}{x - \xi + i\epsilon} = \int_{-1}^{1} dz \frac{D(z)}{z-1} = const$$

Analyticity of Compton amplitudes in energy plane (Anikin,OT'07)

- Finite subtraction implied

$$\mathrm{Re}\mathcal{A}(\nu, Q^2) = \frac{\nu^2}{\pi}\mathcal{P}\int_{\nu_0}^{\infty}\frac{d\nu'^2}{\nu'^2}\frac{\mathrm{Im}\mathcal{A}(\nu', Q^2)}{(\nu'^2-\nu^2)} + \Delta \qquad \Delta = 2\int_{-1}^{1} d\beta \frac{D(\beta)}{\beta - 1}$$

$$\Delta^p_{\mathrm{CQM}}(2) = \Delta^n_{\mathrm{CQM}}(2) \approx 4.4, \qquad \Delta^p_{\mathrm{latt}} \approx \Delta^n_{\mathrm{latt}} \approx 1.1$$

- Numerically close to Thomson term for real proton (but NOT neutron) Compton Scattering!

- Duality (sum of squares vs square of sum; proton: 4/9+4/9+1/9=1)?!

From D-term to pressure

- Inverse -> 1st moment (model)
- Kinematical factor – moment of pressure $C \sim <p\, r^4>$ ($<p\, r^2> =0$)

M.Polyakov'03

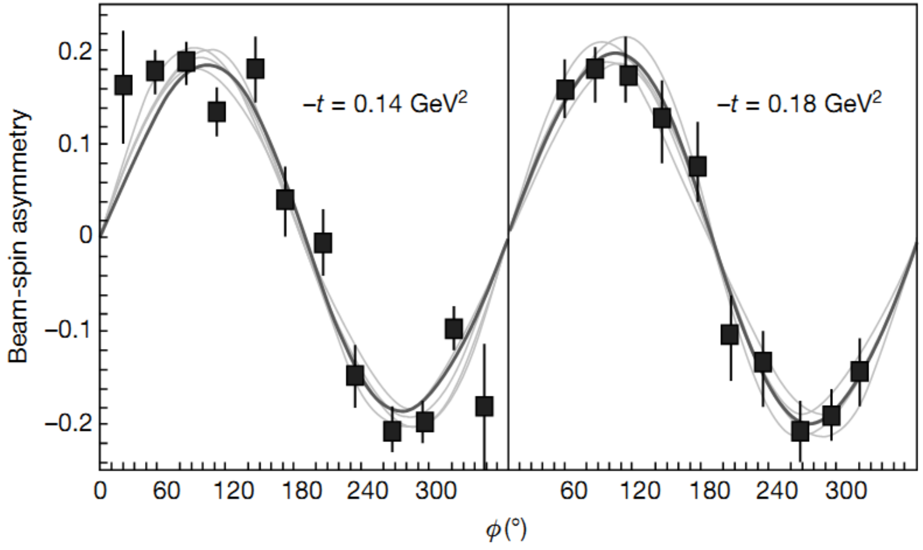
$$T^Q_{\mu\nu}(\vec{r},\vec{s}) = \frac{1}{2E}\int \frac{d^3\Delta}{(2\pi)^3}\, e^{i\vec{r}\cdot\vec{\Delta}}\, \langle p', S'|\hat{T}^Q_{\mu\nu}(0)|p, S\rangle$$

$$T_{ij}(\vec{r}) = s(r)\left(\frac{r_i r_j}{r^2} - \frac{1}{3}\,\delta_{ij}\right) + p(r)\delta_{ij}$$

- Stable equilibrium C>0:

Experiment

- Jlab, TJNAF, CEBAF
- Very accurate data
- Imaginary part from Single Spin Asymmetry



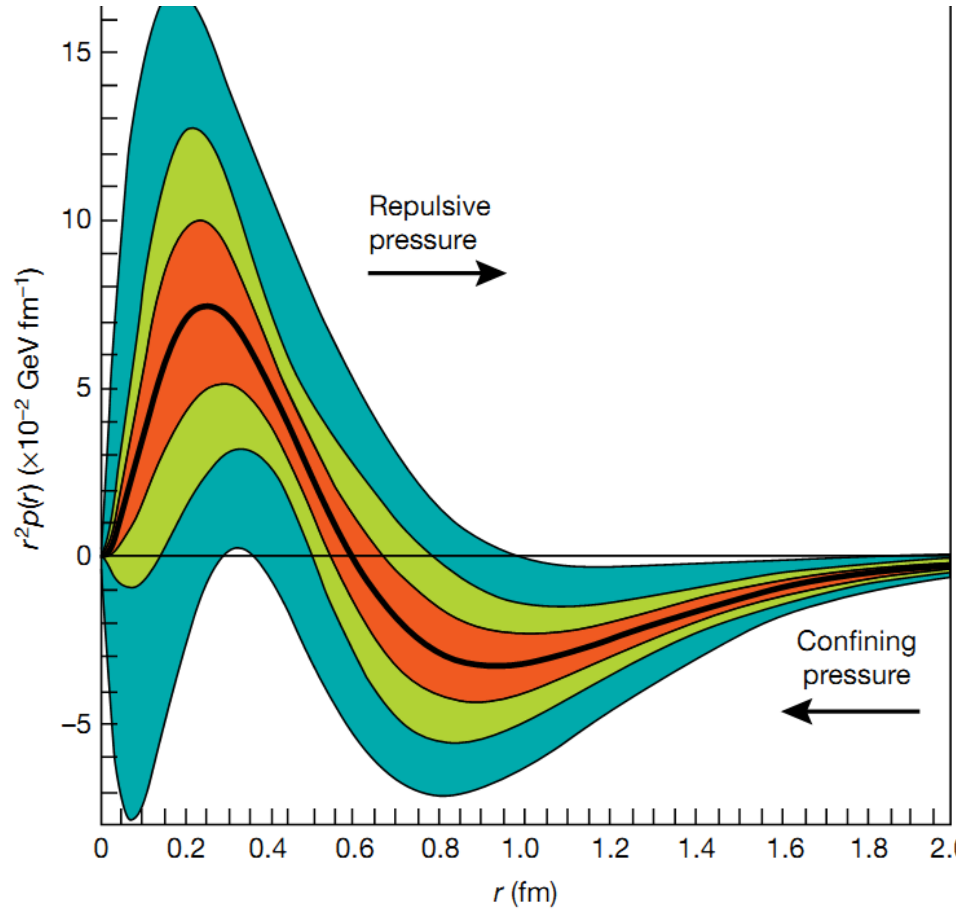
LETTER

https://doi.org/10.1038/s41586-018-0060-z

The pressure distribution inside the proton

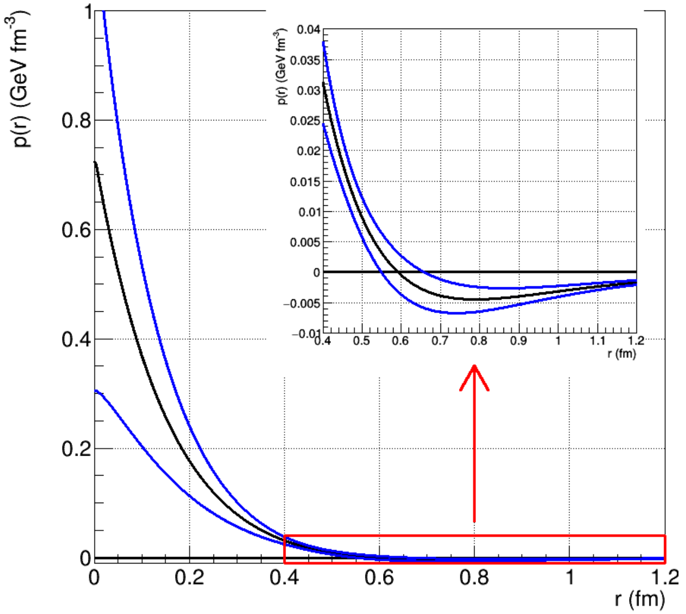
V. D. Burkert[1]*, L. Elouadrhiri[1] & F. X. Girod[1]

- Largest ever ($\sim\Lambda^4_{QCD}$) $\sim 10^{35}$ pascals
- Cosmological constant " natural" scale



Details of coordinate dependence

- Follows from t-dependence
- No term without quadrupole structure (balance of quarks and gluons separate – to be checked!)



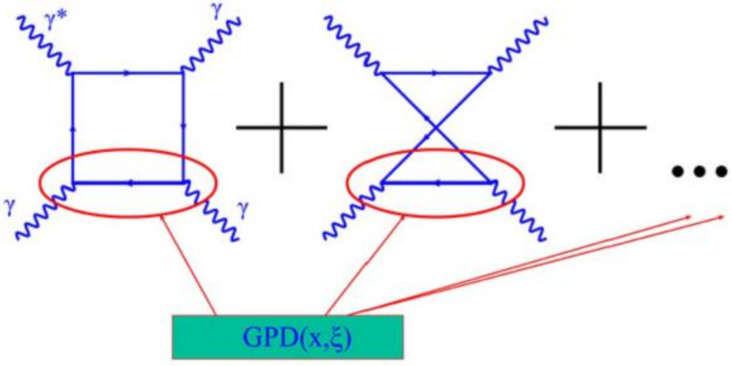
Stability

- All the known cases (hadrons, Q-balls) Schweitzer e.a.
  – stable objects

- Photon (but no rest frame!): C ~ln2 Gabdrakhmanov, OT '12

GPD for photon

- QED+factorization



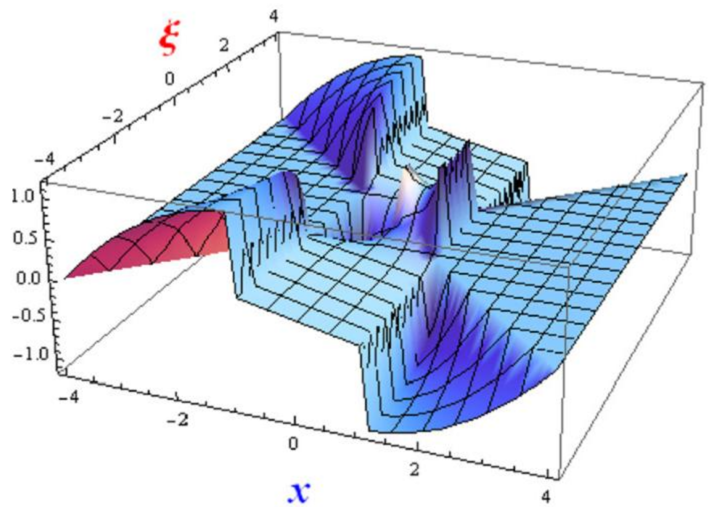
$$T^{\mu\nu\alpha\beta}(\Delta_T = 0) = \frac{1}{4} g_T^{\mu\nu} g_T^{\alpha\beta} A_1$$

$$A_1(\xi) = \int_{-1}^{1} dx C_V(x,\xi) H_1(x,\xi,0) \frac{N_C e_q^2}{4\pi^2} \ln \frac{Q^2}{m^2}$$

$$C_{V/A}^q(x,\xi) = -2e_q^2\left(\frac{1}{x-\xi+i\eta} \pm \frac{1}{x+\xi-i\eta}\right)$$

$$H_1^q(x,\xi,0) = \theta(x-\xi)\frac{x^2+(1-x)^2-\xi^2}{1-\xi^2} + \theta(\xi-x)\theta(x+\xi)\frac{x(1-|\xi|)}{|\xi|(|\xi|+1)} - \theta(-x-\xi)\frac{x^2+(1+x)^2-\xi^2}{1-\xi^2}$$

- Calculable



Pressure of quarks in photon

- Holographic sum rule

$$\int_{-1}^{1} \frac{H_1(x,\xi) - H_1(x,x)}{x-\xi}\, dx = 2\ln 2$$

- Positive sign – stability

- Pressure – requires t-dependence (Gabrakhmanov,OT, in progress)

Some further development

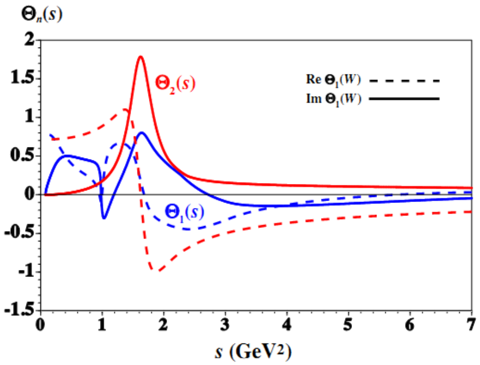
- Not pressure of photons gas (=e/3)
- Pressure of quarks in photon (at rest?!-slightly virtual)
- 2d integration –pressure of moving particle – contact with HIC
- EOS – expressed via GPDs
- Viscosity – T-odd GPDs (Polyakov,OT, in progress)

Pions

- No target – but crossed channel
- Gravitational FFs and radii – GDAs from BELLE data Kumano,Song,OT'17

$$\langle \pi^a(p') \,|\, T_q^{\mu\nu}(0) \,|\, \pi^b(p) \rangle$$
$$= \frac{\delta^{ab}}{2} \left[(t\, g^{\mu\nu} - q^\mu q^\nu)\, \Theta_{1,q}(t) + P^\mu P^\nu\, \Theta_{2,q}(t) \right]$$

$$\langle \pi^a(p)\, \pi^b(p') \,|\, T_q^{\mu\nu}(0) \,|\, 0 \rangle$$
$$= \frac{\delta^{ab}}{2} \left[(s\, g^{\mu\nu} - P^\mu P^\nu)\, \Theta_{1,q}(s) + \Delta^\mu \Delta^\nu\, \Theta_{2,q}(s) \right]$$



- Pressure distribution may be extracted from W dependence (in progress)

To be done

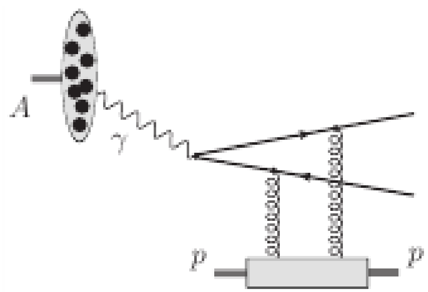
- Weighted (with quark charge squared) pressure measured – flavor separation
- Gluons: are quarks and gluons stable separately or together (terms ~ $g^{\mu\gamma}$)
- Scale dependence?
- Errors?

CONCLUSIONS

- Gravitational formfactors – extra probe of hadron structure
- Way to pressure – universality at all scales
- Similarity to stable macroscopic objects in all known cases
- Transition to HIC – similarity to hadronic physics (c.f. "Ridge")

Measurement of Wigner (GTMD) function

- Small x – lp (Hatta,Xiao,Yuan'16) or Ap UP (Hagiwara, Hatta, Pasechnik, Tasevsky, OT'17) collisions



- Complementary description of elliptic flow – another interplay of hadronic/HIC