The three-dimensional holographic universe

Gravity, higher spins and strings

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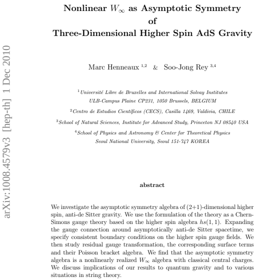
3D HIGHER SPINS AND HOLOGRAPHY

3D gravity with negative cosmological constant coupled to higher spins: simple(st) candidate quantum gravity theories



arXiv:1008.4579v3 [hep-th] 1 Dec 2010

Nonlinear W_∞ as Asymptotic Symmetry of Three-Dimensional Higher Spin AdS Gravity

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abstract

We investigate the asymptotic symmetry algebra of (2+1)-dimensional higher spin, anti-de Sitter gravity. We use the formulation of the theory as a Chern-Simons gauge theory based on the higher spin algebra $hs(1,1)$. Expanding the gauge connection around asymptotically anti-de Sitter spacetime, we specify consistent boundary conditions on the higher spin gauge fields. We then study residual gauge transformation, the corresponding surface terms and their Poisson bracket algebra. We find that the asymptotic symmetry algebra is a nonlinearly realized W_∞ algebra with classical central charges. We discuss implications of our results to quantum gravity and to various situations in string theory.

AEI-2010-140

arXiv:1008.4744v2 [hep-th] 1 Nov 2010

ASYMPTOTIC SYMMETRIES OF THREE-DIMENSIONAL GRAVITY COUPLED TO HIGHER-SPIN FIELDS

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ABSTRACT

We discuss the emergence of $\mathcal{W}$-algebras as asymptotic symmetries of higher-spin gauge theories coupled to three-dimensional Einstein gravity with a negative cosmological constant. We focus on models involving a finite number of bosonic higher-spin fields, and especially on the example provided by the coupling of a spin-3 field to gravity. It is described by a $SL(3) \times SL(3)$ Chern-Simons theory and its asymptotic symmetry algebra is given by two copies of the classical $\mathcal{W}_3$-algebra with central charge the one computed by Brown and Henneaux in pure gravity with negative cosmological constant.

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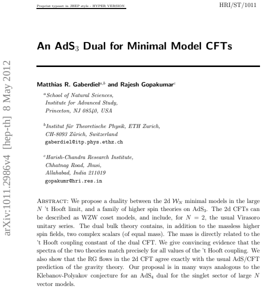

Preprint typeset in JHEP style - HYPER VERSION

HRI/ST/1011

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An AdS_3 Dual for Minimal Model CFTs

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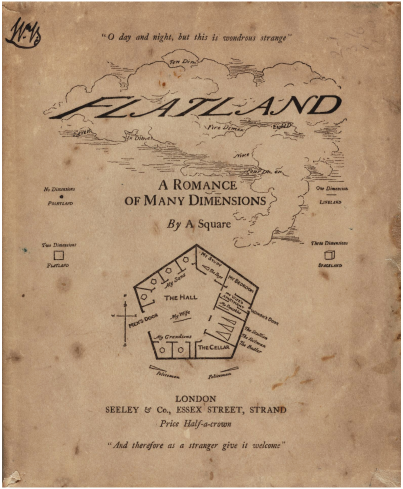
ABSTRACT: We propose a duality between the 2d $\mathcal{W}_N$ minimal models in the large N 't Hooft limit, and a family of higher spin theories on AdS_3. The 2d CFTs can be described as WZW coset models, and include, for $N = 2$, the usual Virasoro unitary series. The dual bulk theory contains, in addition to the massless higher spin fields, two complex scalars (of equal mass). The mass is directly related to the 't Hooft coupling constant of the dual CFT. We give convincing evidence that the spectra of the two theories match precisely for all values of the 't Hooft coupling. We also show that the RG flows in the 2d CFT agree exactly with the usual AdS/CFT prediction of the gravity theory. Our proposal is in many ways analogous to the Klebanov-Polyakov conjecture for an AdS_4 dual for the singlet sector of large N vector models.

- A. Castro, R. Gopakumar, M. Gutperle, J.R., *Conical Defects in Higher Spin Theories*, JHEP **1202**, 096 (2012) [arXiv:1111.3381 [hep-th]].
- E. Perlmutter, T. Procházka, J.R., *The semiclassical limit of W_N CFTs and Vasiliev theory*, JHEP **1305**, 007 (2013) [arXiv:1210.8452 [hep-th]].
- A. Campoleoni, T. Procházka, J.R., *A note on conical solutions in 3D Vasiliev theory*, JHEP **1305**, 052 (2013) [arXiv:1303.0880 [hep-th]].
- J.R., *Quantization of conical spaces in 3D gravity*, JHEP **1503**, 060 (2015) [arXiv:1412.0278 [hep-th]].
- C. Iazeolla, J.R., *On big crunch solutions in Prokushkin-Vasiliev theory*, JHEP **1601**, 177 (2016) [arXiv:1510.08835 [hep-th]].
- J.R., *On matter coupled to the higher spin square*, arXiv:1603.07845 [hep-th].

OUTLINE

- 3D gravity and holography
- Adding higher spins
- String theory as a higher spin theory

What would gravity be like in a 2+1 dimensional flatland?



3D GRAVITY: TRIVIAL...

- Einstein gravity in D dimensions

$$R_{\mu\nu} = 0$$

- Linearized approximation

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

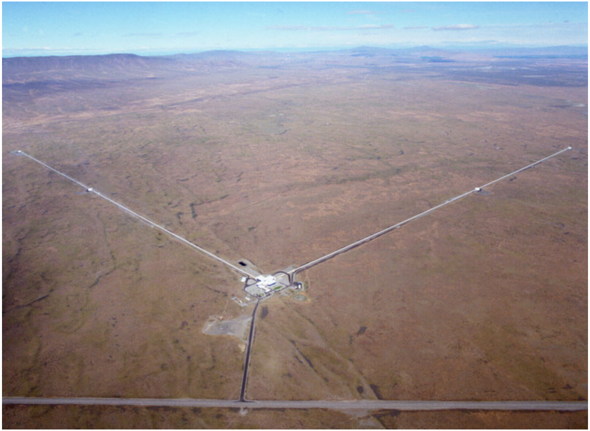
- Physical polarizations: traceless symmetric $(D-2) \times (D-2)$-matrix

$$h^{TT}_{ij}, \qquad h^{TT}_{ii} = 0 \qquad i = 1, \ldots, D-2$$

- No physical polarizations in $D = 3$!

3D GRAVITY: TRIVIAL...

In flatland, Ligo would not detect anything!



…OR NOT SO TRIVIAL

- Negative cosmological constant $\Lambda = -\frac{1}{\ell^2}$

$$R_{\mu\nu} + \frac{2}{\ell^2} g_{\mu\nu} = 0$$

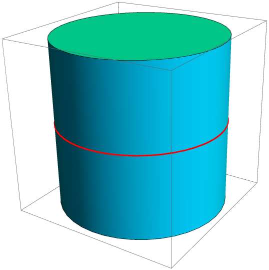
- Locally, all solutions equivalent to the Anti-de Sitter metric

$$ds^2 = -\left(1 + \frac{r^2}{\ell^2}\right) dt^2 + \frac{dr^2}{1 + \frac{r^2}{\ell^2}} + r^2 d\phi^2$$

- Subtlety: AdS_3 has a boundary ($r/\ell = \tan\theta, 0 \leq \theta \leq \pi/2$)

$$ds^2 = \frac{1}{\cos\theta^2} \left(-dt^2 + d\theta^2 + \sin^2\theta d\phi^2\right)$$

...OR NOT SO TRIVIAL



- Conserved charges like M, J are boundary integrals
- Reparametrizations which don't vanish on boundary can change M, J.
- These are global symmetries, relating physically inequivalent solutions.

The BTZ black hole

For example, there exists a Schwarzschild-like black hole solution

$$ds^2 = -\left(\frac{r^2}{\ell^2} - M\right) dt^2 + \frac{dr^2}{\frac{r^2}{\ell^2} - M} + r^2 d\phi^2$$

For $M = -1$, recover global AdS_3

Chern-Simons formulation

- AdS_3 gravity is a gauge theory with gauge group $SL(2,\mathbb{R}) \times SL(2,\mathbb{R})$

$$A = A^a_\mu T_a dx^\mu, \qquad \widetilde{A} = \widetilde{A}^a_\mu T_a dx^\mu$$

$$T_0 = \frac{1}{2}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \quad T_1 = \frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad T_2 = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

- Metric and Christoffel symbols

$$ds^2 = \ell^2 \mathrm{tr}(A - \tilde{A})^2, \qquad A + \tilde{A} \to \text{Christoffel}$$

- Einstein equations

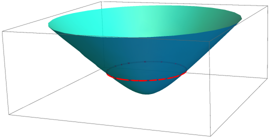
$$F \equiv dA + A \wedge A = 0, \qquad \widetilde{F} \equiv d\widetilde{A} + \widetilde{A} \wedge \widetilde{A} = 0$$

Wilson loop

- For a closed curve $\mathcal{C}$, can define Wilson loop observable

$$W[\mathcal{C}] = \mathcal{P} \exp \oint_{\mathcal{C}} A$$

- e.g. for global AdS_3



$$W[\mathcal{C}] = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

Conformal symmetry

- Near-boundary expansion ($x^{\pm} = t \pm \phi$)

$$ds^2 = d\rho^2 + e^{2\rho} dx_+ dx_- + \frac{12\pi}{c} T(x_+) dx_+^2 + \frac{12\pi}{c} \widetilde{T}(x_-) dx_-^2 + \ldots$$

- Invariant under 'conformal' tranformations

$$\begin{aligned} x^+ &\to F(x^+) + \ldots \\ x^- &\to x^- - \frac{2F''(x^+)}{F'(x^+)} e^{-2\rho} + \ldots \\ \rho &\to \rho - \frac{1}{2} \ln F'(x^+) + \ldots \end{aligned}$$

- $T(x^+)$ transforms like stress tensor in 2-dimensional conformal field theory (CFT)

$$T \to (F')^2 T - \frac{c}{24\pi} \left(\frac{F'''}{F'} - \frac{3}{2} \left(\frac{F''}{F'} \right)^2 \right)$$

HOLOGRAPHY

- Fourier modes

$$T(x^+) = \sum_n L_n e^{inx^+}$$

- Poisson brackets give Virasoro algebra

$$-i\{L_m, L_n\}_{PB} = (m-n)L_{m+n} + \frac{c}{12}m^3\delta_{m,-n}$$

with

$$c = \frac{3\ell}{2G_N}$$

- Holography: semi-classical 3D AdS gravity is a CFT with large central charge, which lives at the boundary

VIRASORO REPRESENTATIONS

- Primary:

$$L_0|h\rangle = h|h\rangle, \qquad L_n|h\rangle = 0, n > 0$$

- Vacuum state $|(1,1)\rangle$ is annihilated by one raising operator

$$L_{-1}|(1,1)\rangle = 0$$

- Other 'short' representations: Kac table

$$|(r,s)\rangle, \qquad r, s, \in \mathbb{N}_0$$

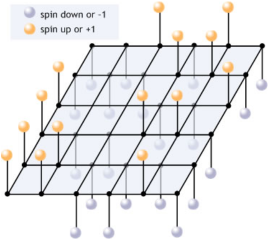
$$h_{(r,s)} = -\frac{1}{24} - \frac{rs}{2} + \frac{1}{48}(13-c)\left(r^2+s^2\right) + \frac{1}{48}\sqrt{(1-c)(25-c)}\left(s^2-r^2\right)$$

- E.g.

$$\left(L_{-2} - bL_{-1}^2\right)|(2,1)\rangle = 0 \qquad \left(c = 13 - 6\left(b + \frac{1}{b}\right)\right)$$

$$\left(L_{-3} - \frac{b}{2}(L_{-2}L_{-1} + L_{-1}L_{-2}) + \frac{b^2}{4}L_{-1}^3\right)|(3,1)\rangle = 0 \qquad (1)$$

- 2D Ising model near critical temperature; CFT with $c = 1/2$



- Described by 3 operators from Kac table

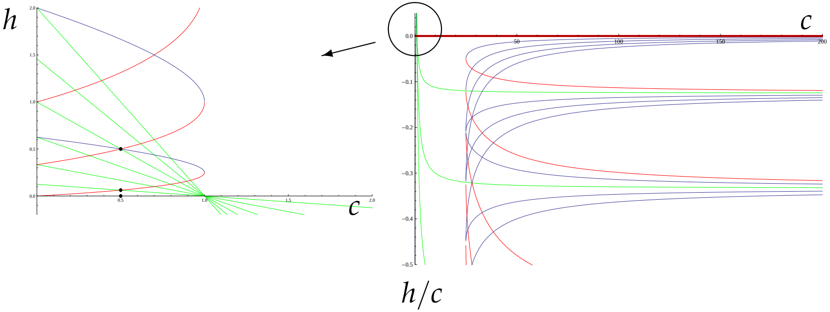
$$\begin{aligned} \mathbf{1} &= \mathcal{O}_{(1,1)} & \text{identity} \\ \epsilon &= \mathcal{O}_{(2,1)} & \text{energy density} \\ \sigma &= \mathcal{O}_{(1,2)} & \text{spin} \end{aligned}$$

Holography for Kac states?

- What is the 3D gravity version of the $|(r,s)\rangle$ Kac states?
- Large c behaviour

$$h_{(r,s)} = -\frac{cr^2}{24} + \ldots$$
$$(L_{-r} + \ldots)\,|(r,1)\rangle = 0$$

LARGE c LIMIT



in red: $|r,1\rangle$ primaries, in green: $|r,r\rangle$ primaries, in blue: other $|r,s\rangle$ primaries

- Holographic meaning of BTZ-like metrics

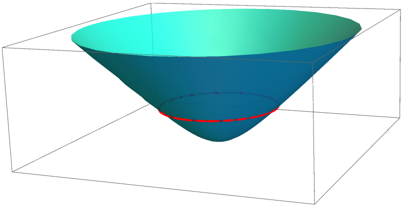
$$ds^2 = -\left(\frac{r^2}{\ell^2} - M\right) dt^2 + \frac{dr^2}{\frac{r^2}{\ell^2} - M} + r^2 d\phi^2$$

for arbitrary values of M

- Correspond to CFT states with

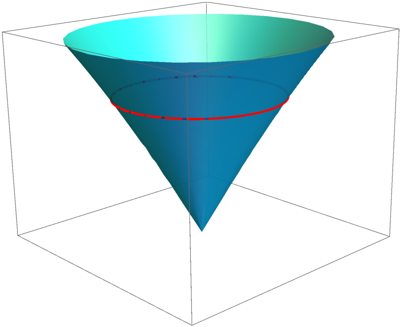
$$h = \frac{c}{24} M$$

$M = -1$: GLOBAL AdS_3



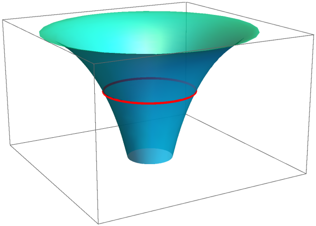
- Invariant under L_{-1}, $h = -\frac{c}{24}$
- Dual to vacuum state $|(1,1)\rangle$
- Wilson loop trivial, $W[\mathcal{C}] = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$

$-1 < M < 0$: CONICAL DEFECTS



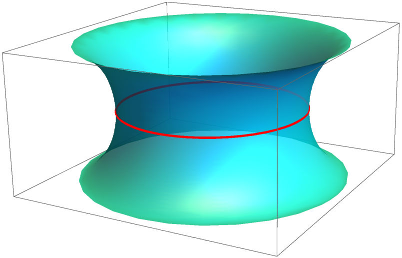
- Conical defect with deficit angle $\delta = 2\pi(1 - \sqrt{-M})$
- Dual to primary $|h = \frac{cM}{24}\rangle$
- Wilson loop $W[\mathcal{C}] = \begin{pmatrix} \cos 2\pi\sqrt{-M} & \sin 2\pi\sqrt{-M} \\ -\sin 2\pi\sqrt{-M} & \cos 2\pi\sqrt{-M} \end{pmatrix}$

$M = 0$: EXTREMAL BLACK HOLE



- Extremal $M = J = 0$ black hole
- Dual to $|h = 0\rangle$
- Wilson loop $W[\mathcal{C}] = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$

$M > 0$: BTZ BLACK HOLE



- Dual to $|h = \frac{cM}{24}\rangle$ primary
- Bekenstein-Hawking entropy = Cardy degeneracy in unitary CFT

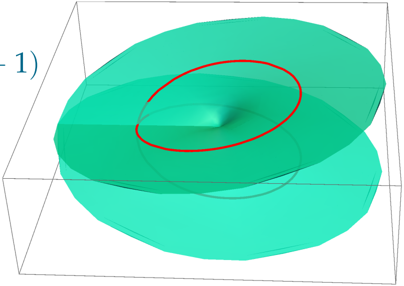
$$S_{BH} = \frac{2\pi\sqrt{M}l}{4G_N} = 4\pi\sqrt{\frac{ch}{6}} = S_{Cardy}$$

- Wilson loop $W[\mathcal{C}] = \begin{pmatrix} \cosh 2\pi\sqrt{M} & \sinh 2\pi\sqrt{M} \\ -\sinh 2\pi\sqrt{M} & \cosh 2\pi\sqrt{M} \end{pmatrix}$

$M = -n^2$: CONICAL SURPLUSES

$\delta = -2\pi(n-1)$

$n = 2$:



Wilson loop trivial! $W[\mathcal{C}] = (-1)^n \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

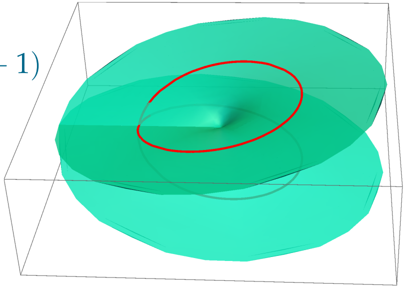
$$h = -\frac{cn^2}{24}$$

Annihilated by L_{-n}

$M = -n^2$: CONICAL SURPLUSES

$\delta = -2\pi(n-1)$

$n = 2:$



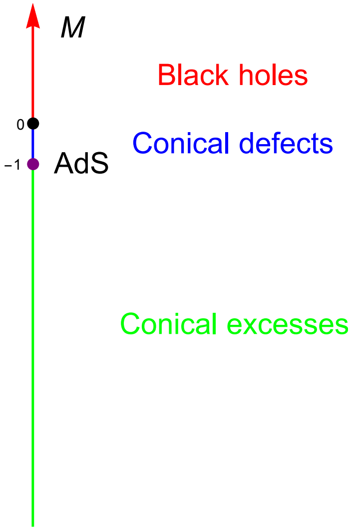
Wilson loop trivial! $W[\mathcal{C}] = (-1)^n \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$$h = -\frac{cn^2}{24} + \frac{1}{24}(n-1)(1+13n) + \ldots$$

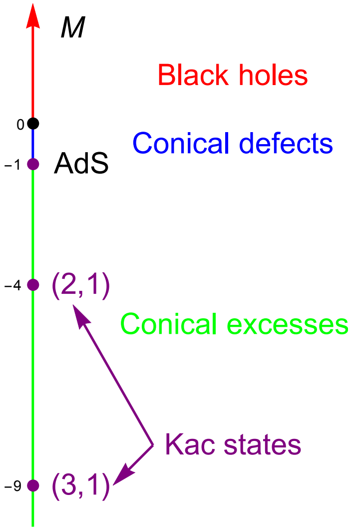
Annihilated by $L_{-n} - \frac{6}{c}\sum_{m=1}^{n-1} \frac{L_{-m}L_{m-s}}{m(m-s)} + \ldots$

Dual to $|(n,1)\rangle$ in Kac table!

Summary so far



Summary so far



Other $|r, s\rangle$ Kac states?

- Add matter: 2×2 matrices $C, \widetilde{C}$

$$
\begin{aligned}
dC + AC - C\widetilde{A} &= 0 \\
d\widetilde{C} - A\widetilde{C} + \widetilde{C}\widetilde{A} &= 0
\end{aligned}
$$

- Single-particle excitations describe $|r, 2\rangle$ Kac states
- Multi-particle excitations describe general $|r, s\rangle$ Kac states

- Massless higher spins in D dimensions: physical polarizations sit in traceless symmetric tensor

$$h_{i_1 \ldots i_s}, \qquad h_{jji_3 \ldots i_s} = 0 \qquad i_r = 1, \ldots, D-2$$

- No physical polarizations in $D = 3$!

Chern-Simons description

- Chern-Simons gauge theory with gauge group $SL(N,\mathbb{R}) \times SL(N,\mathbb{R})$:

$$F \equiv dA + A \wedge A = 0, \qquad \widetilde{F} \equiv d\widetilde{A} + \widetilde{A} \wedge \widetilde{A} = 0$$

- Describes spins $2, 3, \ldots, N$: under $SL(N,\mathbb{R}) \supset SL(2,\mathbb{R})$

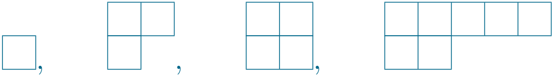
$$\mathrm{adj} = \mathbf{3} \oplus \mathbf{5} \oplus \ldots \oplus \mathbf{N}$$

- Symmetry algebra extends from Virasoro to $\mathcal{W}_N$

Kac states versus classical solutions

- Kac-like short representations labelled by two $sl(N)$ Young tableaux

$$|\Lambda_1, \Lambda_2\rangle$$

e.g. $N = 3 : \Lambda =$  , . . .

- Solutions with trivial Wilson loop are classfied by single Young diagram Λ
- Correspond to states $|\Lambda, 0\rangle$ in CFT. Evidence from:
  - Higher spin charges at large c Campoleoni et. al. 2013
  - Symmetries
  - Correlation functions Hijano et. al. 2013
- Adding matter gives general $|\Lambda_1, \Lambda_2\rangle$ states

The light state enigma

- There exists a Lie algebra $sl(\lambda)$, for any $\lambda \in \mathbb{R}$:

$$\begin{array}{rl} \text{generated by} & J_0, J_1, J_2 \\ \text{relations} & [J_a, J_b] = \epsilon_{ab}{}^c J_c \\ & -J_0^2 + J_1^2 + J_2^2 = \frac{1}{4}(\lambda^2 - 1) \end{array}$$

- There exists $sl(\lambda) \times sl(\lambda)$ higher spin theory (spins $2, 3, \ldots$) coupled to matter (spin 0) Prokushkin, Vasiliev 1998
- For $0 \leq \lambda \leq 1$, dual to unitary CFT Gaberdiel, Gopakumar 2010

$$\frac{SU(N)_k \times SU(N)_1}{SU(N)_{k+1}}, \qquad N, k \to \infty, \lambda = \frac{N}{N+k} \text{ fixed}$$

Symmetry algebra is $\mathcal{W}_\infty[\lambda]$, see T. Procházka's work.

- Contains mysterious light states $|\Lambda, \Lambda\rangle$. Our work $\to$ possibly present in higher spin theory as a kind of nonperturbative states.

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String Theory as a Higher Spin Theory

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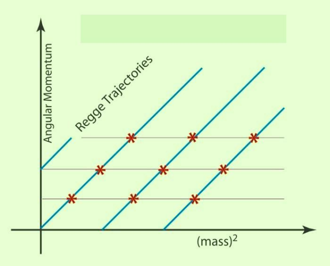
ABSTRACT: The symmetries of string theory on $AdS_3 \times S^3 \times \mathbb{T}^4$ at the dual of the symmetric product orbifold point are described by a so-called Higher Spin Square (HSS). We show that the massive string spectrum in this background organises itself in terms of representations of this HSS, just as the matter in a conventional higher spin theory does so in terms of representations of the higher spin algebra. In particular, the entire untwisted sector of the orbifold can be viewed as the Fock space built out of the multiparticle states of a *single* representation of the HSS, the so-called 'minimal' representation. The states in the twisted sector can be described in terms of tensor products of a novel family of representations that are somewhat larger than the minimal one.

- String theory is a massive higher spin theory



- Higher spin fields become massless in tensionless limit
- Hints of hidden massless higher spin gauge symmetry, spontaneously broken in Minkowski vacuum

String theory in its most symmetric phase

- Vasiliev's work: higher spin theories live in AdS background
- AdS_3 is special: symmetries are enlarged $\rightarrow \mathcal{W}$-symmetry
- Gaberdiel-Gopakumar's concrete candidate: tensionless limit of strings on

$$AdS_3 \times S^3 \times T^4$$

- can study through dual CFT, the 'symmetric orbifold of T^4'

$$\frac{(T^4)^N}{S^N}$$

in large N limit

The gauge algebra of string theory

- Harmonic oscillators from 4 real bosonic and 4 real fermionic fields: $i = 1, \ldots 4, m \in \mathbb{N}_0, r \in \mathbb{N} + \frac{1}{2}$

$$\left[a^i_m, \left(a^j_n\right)^\dagger\right] = \delta^{ij}\delta_{mn} \qquad \left\{\psi^i_r, \left(\psi^j_s\right)^\dagger\right\} = \delta^{ij}\delta_{rs}$$

- Lie algebra of operators which annihilate both the in- and out-Fock vacuum
- Basis: normal-ordered monomials with at least one creation- and annihilation operator
  E.g. $(a^1_1)^\dagger a^3_5, \qquad (a^2_3)^\dagger(\psi^1_1)^\dagger\psi^3_4, \ldots$
- Called 'higher spin square' or *HSS*

- Chern-Simons gauge fields $A, \widetilde{A}$ valued in HSS

$$F \equiv dA + A \wedge A = 0, \qquad \widetilde{F} \equiv d\widetilde{A} + \widetilde{A} \wedge \widetilde{A} = 0$$

- Matter: scalar fields $C, \widetilde{C}$ valued in HSS

$$\begin{aligned} dC + AC - C\widetilde{A} &= 0 \\ d\widetilde{C} - A\widetilde{C} + \widetilde{C}\widetilde{A} &= 0 \end{aligned}$$

- Checked conjecture that spectrum accounts for full 'untwisted sector' of CFT

ONGOING EFFORTS

- Systematic quantization of ‘conical’ solutions
- Multi-centered solutions
- Adding interactions to the higher spin square theory

Thank you!