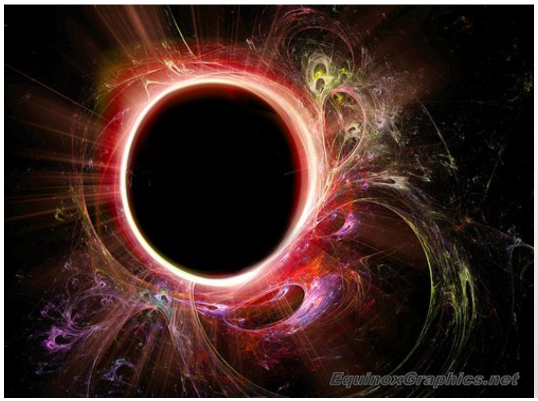
Black holes in a pencil case



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April 21st 2016

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The three roads to hep-th:

1. give-up reality

2. use hep-th as a maths tool to describe real world physics

3. use real world physics as a physics tool to test hep-th

This talk is about the third road

- Anderson SSB and Higgs

- Cosmology in the lab

- Acoustic BHs

- Unruh at CERN

- ...

Open hep-th questions can be answered by low energy experiments

Today's example: graphene and BHs

1. Black holes, doors to hep-th

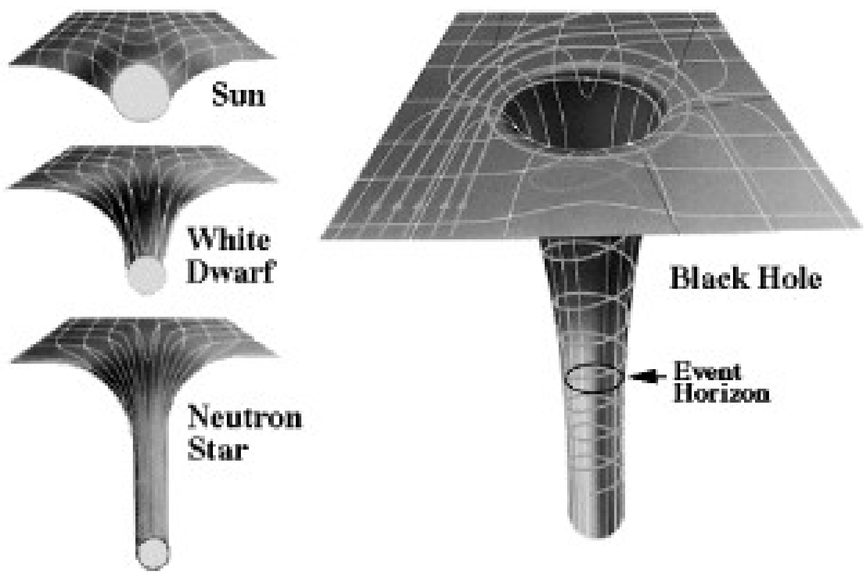
With the escape velocity argument

$$\frac{1}{2}\,\mathrm{m}\,v^2 = \mathcal{G}\frac{\mathrm{m}M}{r} \Rightarrow v_e = \sqrt{2\mathcal{G}\frac{M}{r}}$$

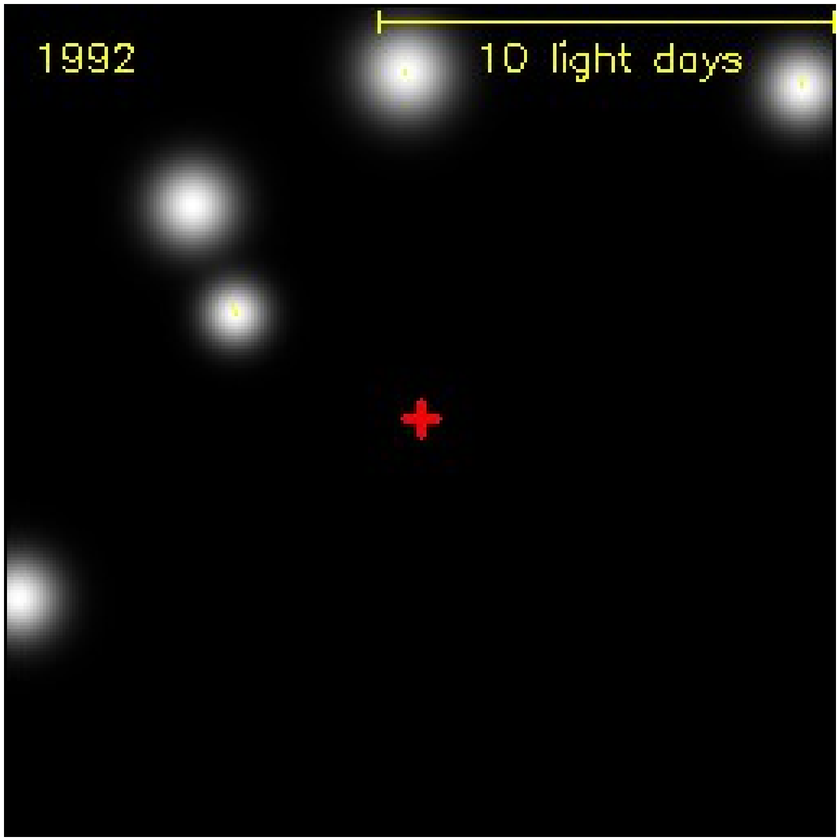
we can explain the “black”

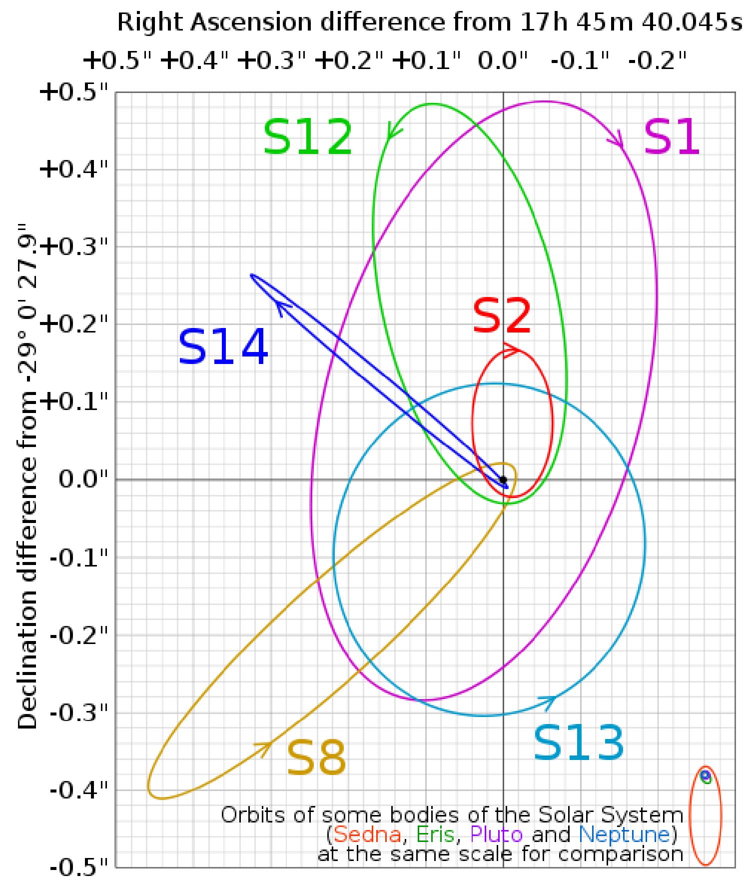
$$r_S = 2\frac{\mathcal{G}M}{c^2}$$

and, cleverly using $\mathrm{m} = \mathrm{m}$, we can explain the “hole”



BHs are no longer science fiction (only)

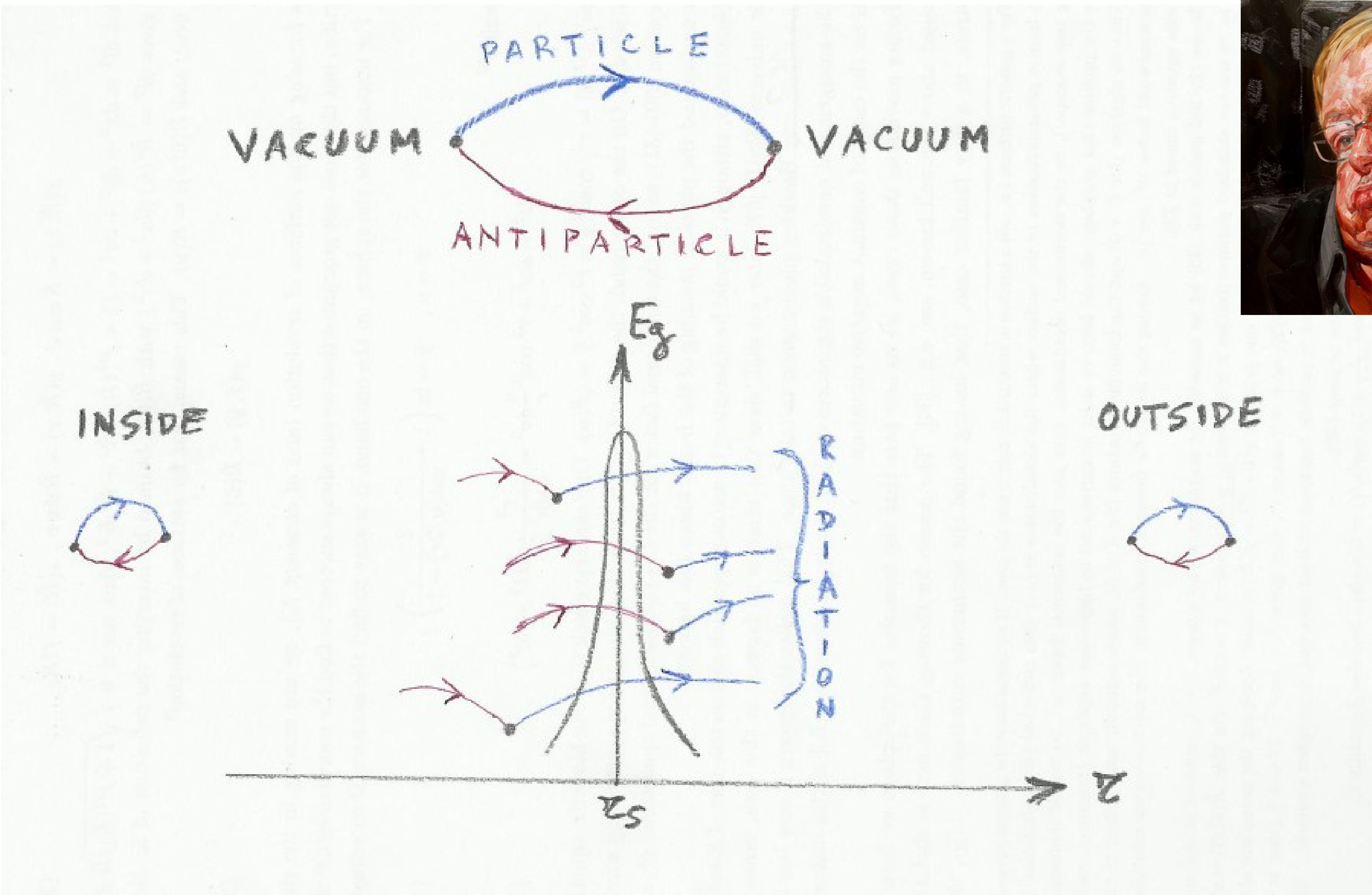




Candidate for a supermassive BH at the center of the Milky Way (~ 25000 ly away)

M. Pössel, Einstein Online 2 (2006) 1020.

That is not the end of the story, but perhaps a beginning

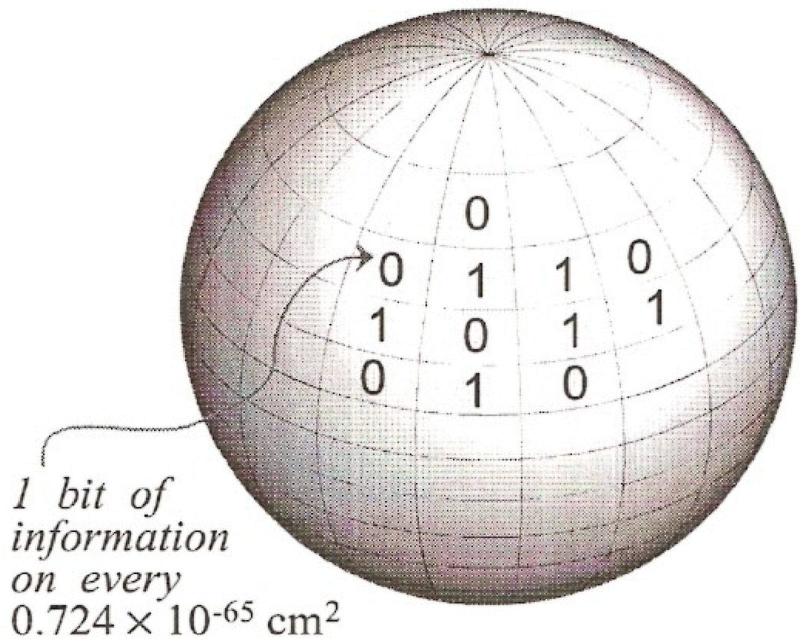


$$T_H = \frac{1}{8\pi}\,\frac{\hbar c^3}{\mathcal{G}}\,\frac{1}{k_B M} = \frac{\hbar c}{k_B}\,\frac{1}{4\pi r_S}$$

Once we have temperature we have a whole thermodynamics, in particular we have an entropy

$$S_{BH} = \frac{1}{4}\,A \;\simeq\; \frac{1}{4} k_B \frac{c^3}{\mathcal{G}\hbar} \times 4\pi r_S^2$$

$1/\ell^2_{\mathrm{Planck}}$



Quantum Gravity

Information Paradox

Discrete Spacetimes

Holographic Bound and Principle

AdS/CFT Correspondence

Emergent Gravity

SUSY

String Theory

Loop Quantum Gravity

2. Graphene for hep-th

27th Indian-Summer School of Physics

GRAPHENE

THE BRIDGE BETWEEN LOW- AND HIGH ENERGY PHYSICS

September 14 - 18, 2015, Prague, Czech Republic

Main | Program | Organizers | Venue | Presentations | Photos | Participants | Poster | History | Contact

Welcome

We cordially invite you to the 27th Indian-Summer School on Graphene - the Bridge between Low- and High-Energy Physics, to be held September 14 - 18, 2015, in Prague, Czech Republic.
The School is dedicated to the exciting area where the high-energy and condensed matter physics intersect, and where graphene is a prominent player on the scene.

The topics of the School include:

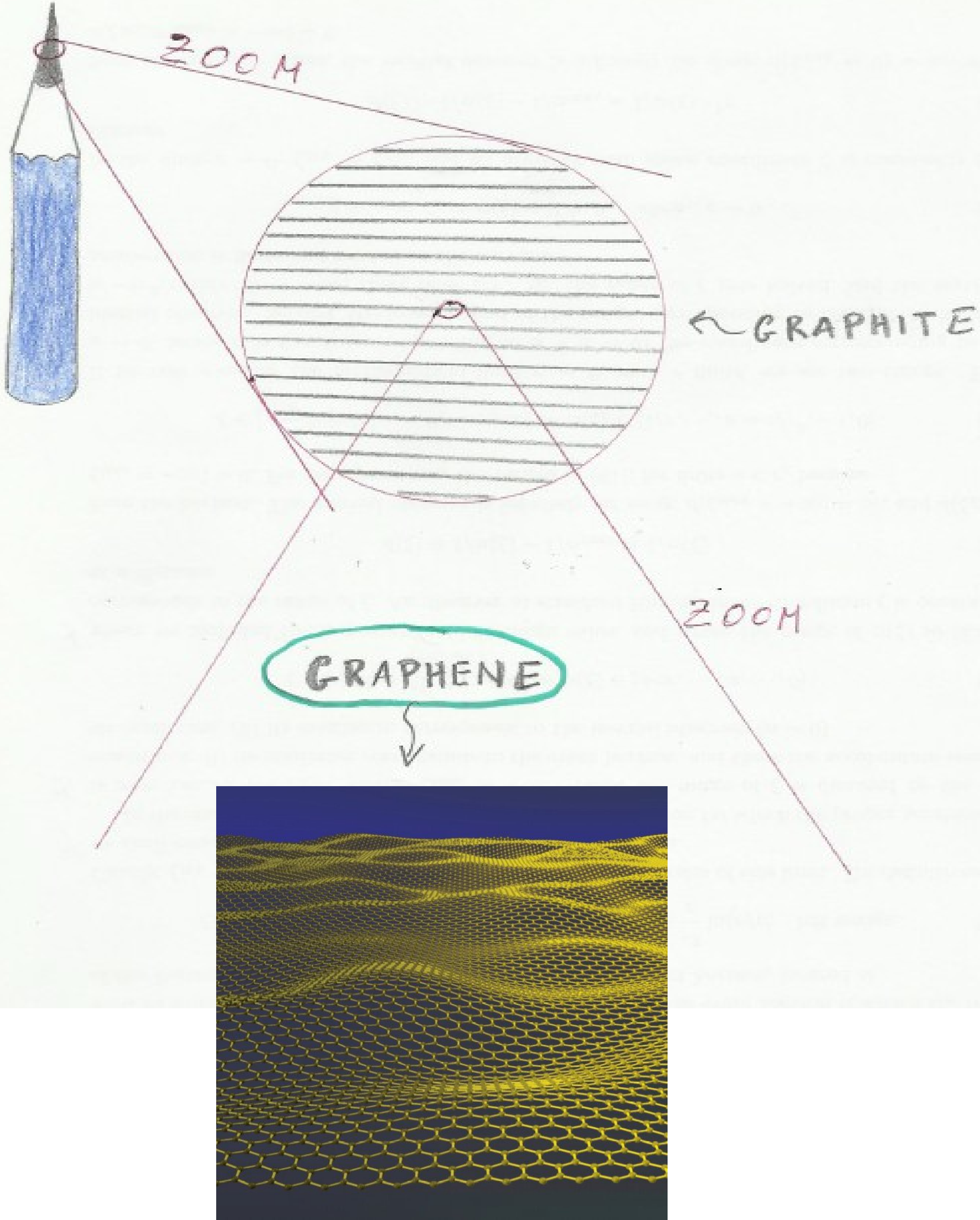
- **low-energy physics of graphene: theory, experiments, and prospects**
- **aspects of the (2+1)-dimensional quantum field theory**
- **topological quantum field theory**
- **relativistic quantum systems in condensed matter**
- **graphene and quantum cosmology**

Graphene Workshops

The Simulation Science Workshops; Modelling Graphene-like Systems took place in University College Dublin, Monday 28th April – Friday 2nd May 2014.

The Workshops coverd the following topics:

- Overview of Weyl/scale/conformal symmetry
- Introduction to Chern-Simons systems
- Unconventional supersymmetry
- Generically deformed graphene
- Topological aspects of graphene


ZOOM
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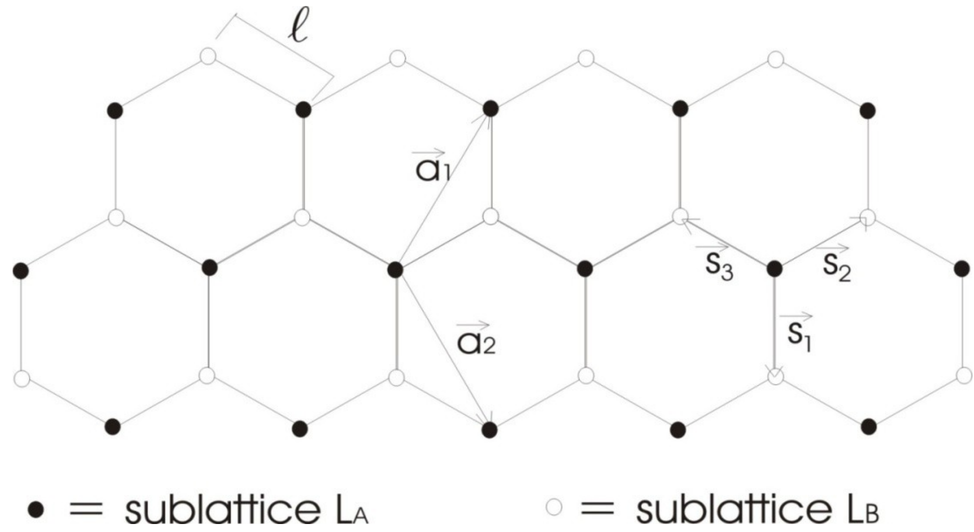
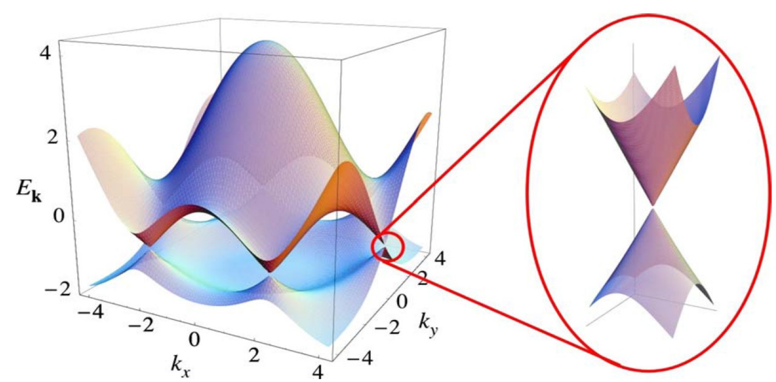
ZOOM
GRAPHENE



Near the Fermi points, $\vec{k}^D_\pm = (\pm\frac{4\pi}{3\sqrt{3}}, 0)$

$$H = v_F \sum_{\vec{p}} \left(\psi^\dagger_+ \vec{\sigma}\cdot\vec{p}\,\psi_+ + \psi^\dagger_- \vec{\sigma}^*\cdot\vec{p}\,\psi_-\right)$$
$$= -iv_F \int d^2x \left(\psi^\dagger_+ \vec{\sigma}\cdot\vec{\partial}\,\psi_+ + \psi^\dagger_- \vec{\sigma}^*\cdot\vec{\partial}\,\psi_-\right)$$

with $(\hbar = 1 = \ell)$, $v_F = \frac{3t}{2}$, $\vec{\sigma} \equiv (\sigma_1, \sigma_2)$, $\vec{\sigma}^* \equiv (-\sigma_1, \sigma_2)$, σ_i **the Pauli matrices and** $\psi_\pm \equiv (a_\pm, b_\pm)^{\mathrm{T}}$ **Dirac spinors.**



Picture taken from A. H. Castro Neto *et al.*, Rev. Mod. Phys. **81** (2009) 109

First scale

$$E_\ell \sim v_F/\ell \sim 4.2\,\mathrm{eV}$$

π-electrons see the graphene sheet as a continuum

$$A = v_F \int d^3x (i\psi^\dagger\, \dot{\psi} - \mathcal{H}) = i v_F \int d^3x \bar{\psi}\, \gamma^a \partial_a\, \psi$$

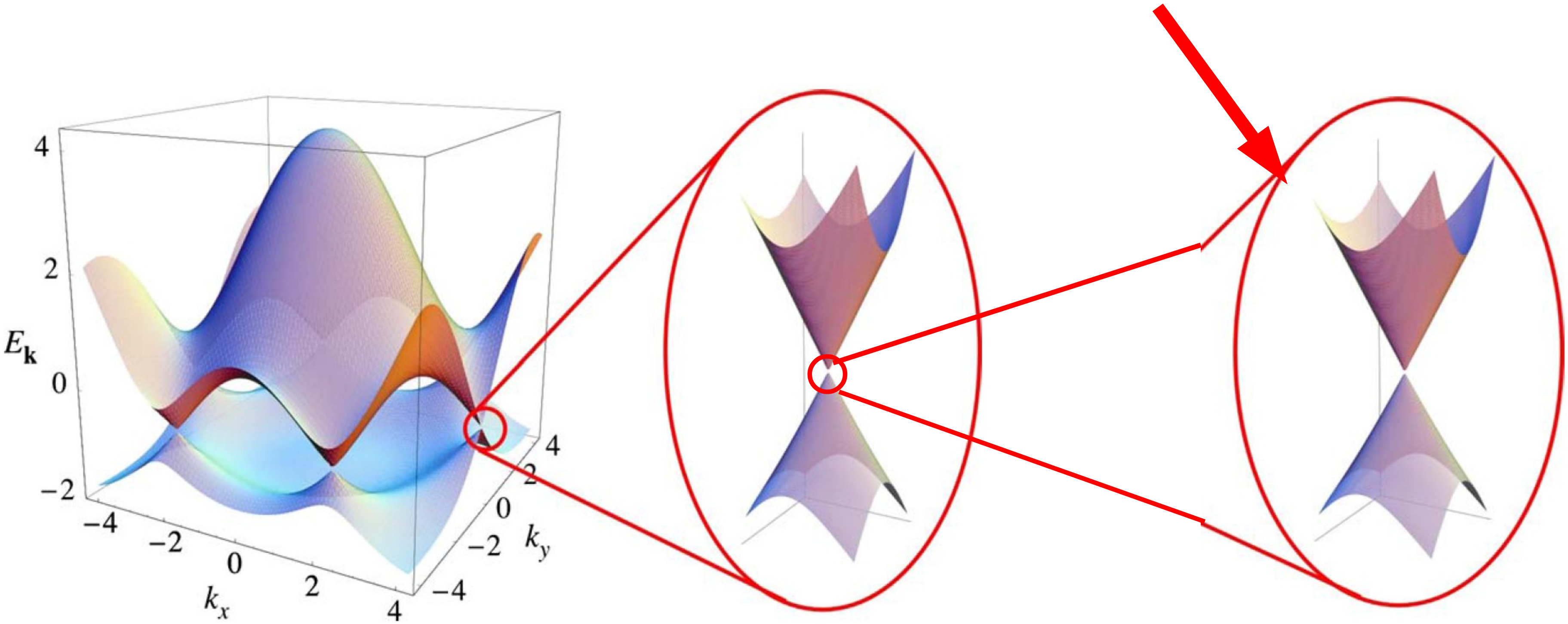
$x^0 \equiv v_F t$, $\gamma^0 = \sigma_3$, $\gamma^1 = i\sigma_2$, $\gamma^2 = -i\sigma_1$, **with** $[\gamma^a, \gamma^b]_+ = 2\eta^{ab}$.
No mixing of Fermi points (conf. flat)
Second scale, r radius of curvature,

$$E_r \sim v_F/r < E_\ell$$

No impurities, vacancies, wrinkles, Coulomb scatterers, resonant scatterers

deformation type	$\Omega_\mu (= \omega_\mu^a J_a)$	$E_\mu (= e_\mu^a P_a)$	scale
intrinsic/inelastic	curvature	torsion	$\leq E_r$
extrinsic/elastic ℓ	$\sim \partial u$	$\sim u$	$\leq E_\ell$

The tree regimes



small wavelength	medium wavelength	large wavelength
E > 4 eV	**E ~ 1 eV**	**E ~ 10 meV**
elas; dis; n-lin	elas; cont/dis; lin	inelas; cont; lin

The simplest setting is to go to regimes where only intrinsic curvature counts

$$R^{ij}{}_{kl} = \epsilon^{ij}\epsilon_{kl}\epsilon^{mn}\partial_m\omega_n = \epsilon^{ij}\epsilon_{lk}2\mathcal{K}$$

where $\partial_i\omega \equiv \omega_i$.

We include time, in the most gentle way

$$g^{\rm graphene}_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & & \\ 0 & g_{ij} & \end{pmatrix}$$

$\partial_t g_{ij} = 0$.

SO(2)-valued disclination field $\to$ SO(1,2)-valued disclination field, so $R^i{}_{jkl} \to R^\lambda{}_{\mu\nu\rho}$, etc.

The very long wavelength/very small energy transport properties of graphene, are well described by the following action

$$\mathcal{A} = i v_F \int d^3x \sqrt{g}\; \bar\psi \gamma^\mu (\partial_\mu + \Omega_\mu)\psi$$

3. Hawking on graphene

This *exotic* situation, on the graphene side, is *very meagre*,
on the hep-th side
Local Weyl symmetry

$$g_{\mu\nu}(x) \to \phi^2(x) g_{\mu\nu}(x) \quad \text{and} \quad \psi(x) \to \phi^{-1}(x)\psi(x)$$

and

$$\mathcal{A} \to \mathcal{A}$$

This is a huge and powerful symmetry:
Classical physics in $g_{\mu\nu}$ = classical physics in $\phi^2 g_{\mu\nu}$

Important cases are *conformally flat spacetimes*

$$g_{\mu\nu} = \phi^2 \eta_{\mu\nu}$$

Do not be misled by the words here: in (2+1)d conformally flat is even a (BTZ) BH!

For CF spacetimes the effects of curvature are null on the *classical physics* of a massless Dirac field

But, *quantum mechanically*, Weyl is implemented through

$$\psi'(x) = U\psi(x)U^{-1} = \phi^{-1}(x)\psi(x)$$

and

$$|0\rangle' = U|0\rangle$$

with

$$U = \exp\left\{\int d^3y \ln\phi(y)\psi^\dagger(y)\psi(y)\right\}$$

the vacuum is a condensate, hence, e.g.,

$$S^{CF}(x_1, x_2) = \phi^{-1}(x_1)\phi^{-1}(x_2)S^{flat}(x_1, x_2)$$

where

$$S(x_1, x_2) \equiv \langle 0|\psi(x_1)\bar{\psi}(x_2)|0\rangle$$

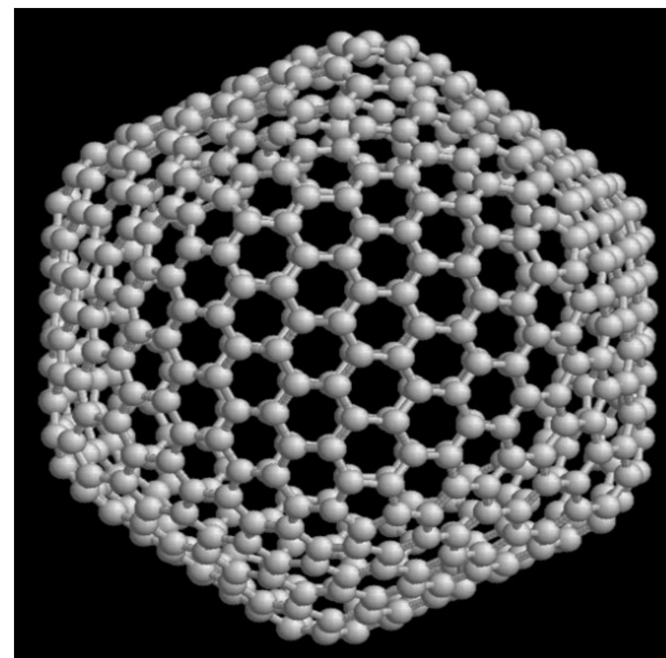
How can we make CF *spacetimes* with

$$g^{2+1}_{\mu\nu}(x,y) = \begin{pmatrix} 1 & 0 \\ 0 & g^{(2)}_{\alpha\beta}(x,y) \end{pmatrix}$$

The condition is

$$C_{\mu\nu} = \epsilon_{\mu\lambda\kappa}\nabla^{\lambda}R^{(3)}{}^{\kappa}_{\nu} + \epsilon_{\nu\lambda\kappa}\nabla^{\lambda}R^{(3)}{}^{\kappa}_{\mu} = 0$$

All surfaces of constant Gaussian curvature $\mathcal{K}$, give a CF spacetime!



One immediately thinks of the sphere

$$\mathcal{K} = \frac{1}{r^2}$$

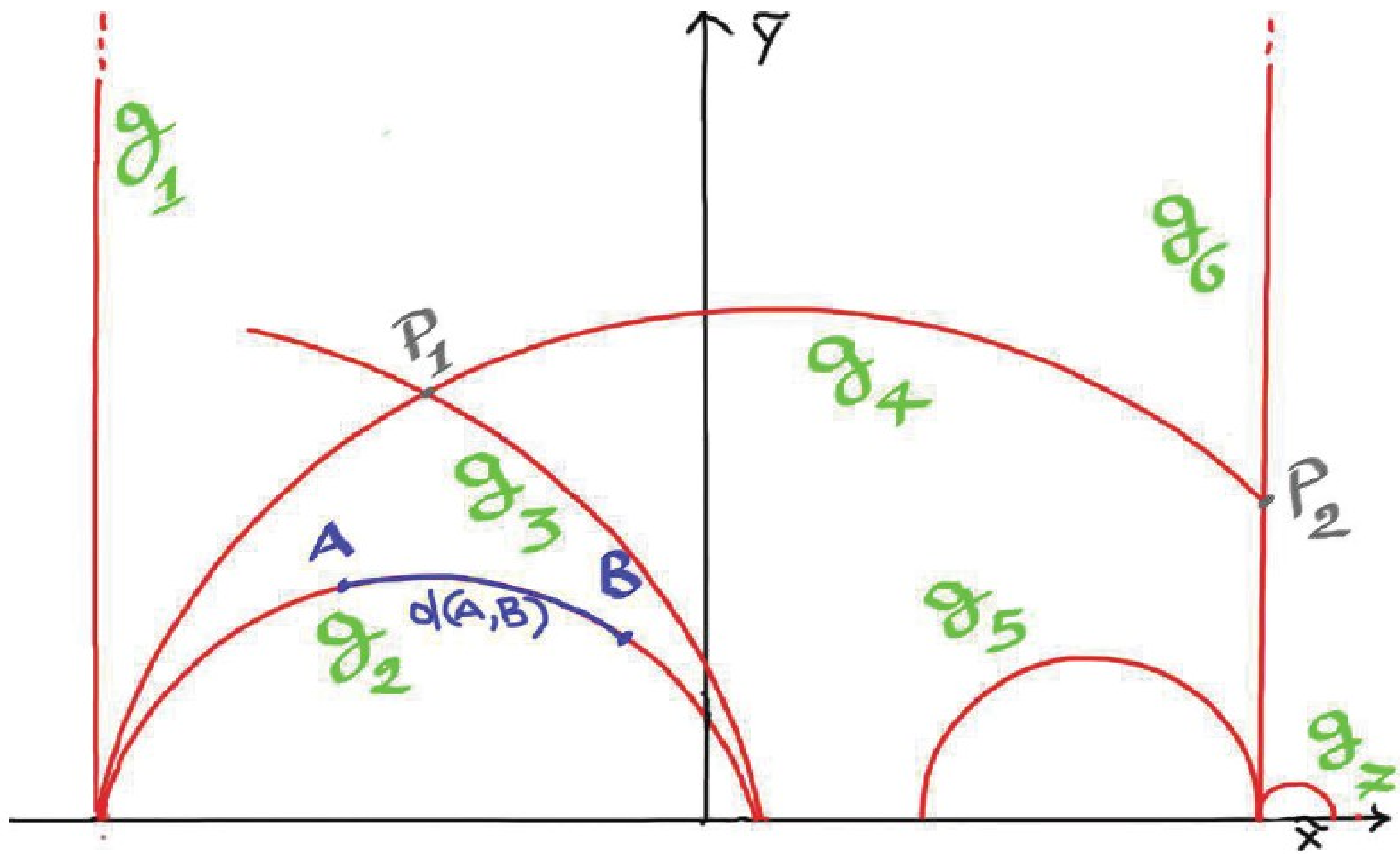
Interesting, but no horizons in sight...

There is another case, that is

$$\mathcal{K} = -\frac{1}{r^2}$$

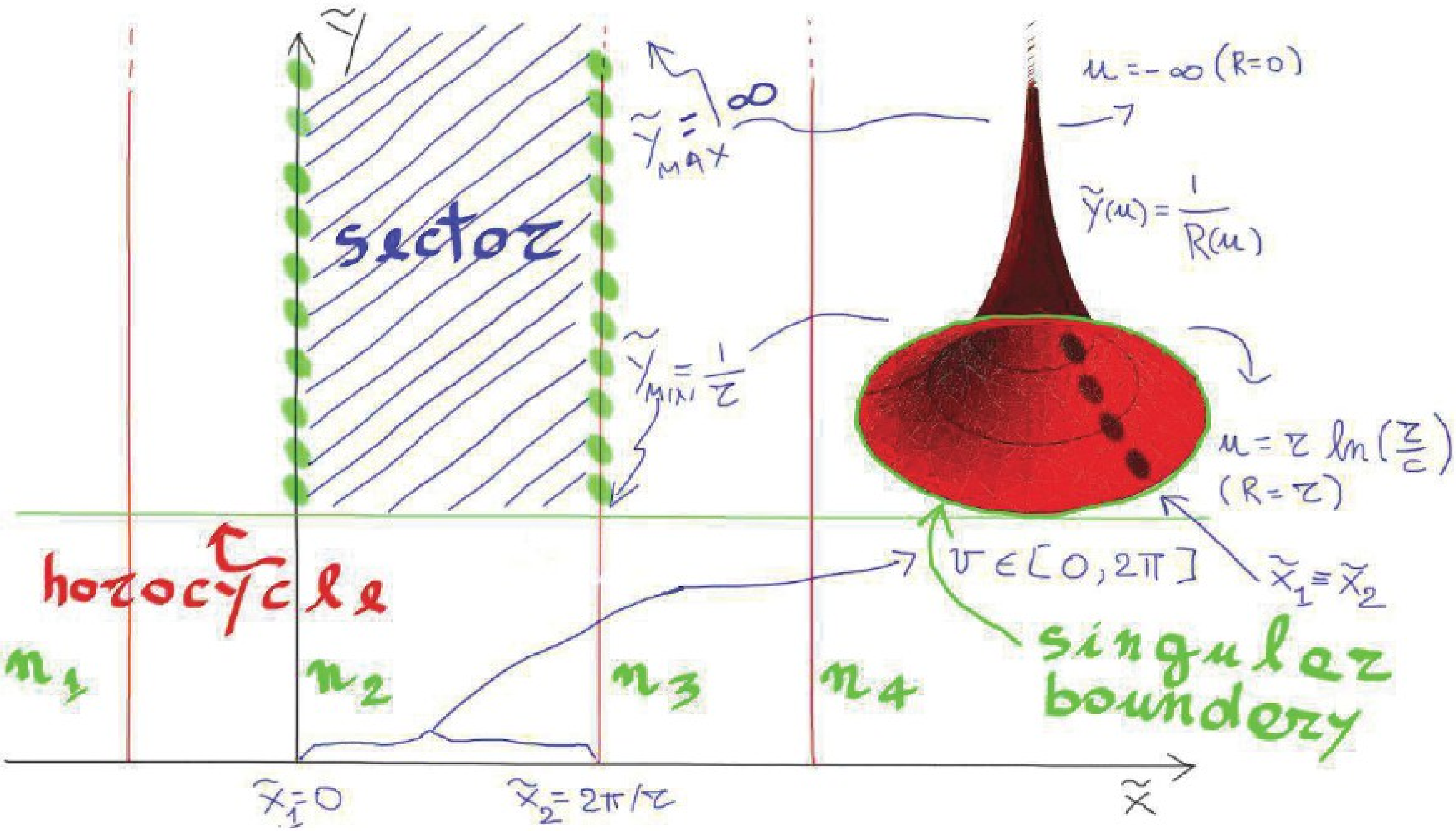
which brings us into Lobachevsky geometry

$$ds^2_{\text{graphene}} = dt^2 - \frac{r^2}{\tilde{y}^2}(d\tilde{x}^2 + d\tilde{y}^2)$$



$$ds^2_{\text{graphene}} = \frac{r^2}{\tilde{y}^2}\left[\frac{\tilde{y}^2}{r^2}dt^2 - d\tilde{y}^2 - d\tilde{x}^2\right]$$

$\tilde{y} = 0$ **event horizon**



$$\tilde{x} = \frac{v}{r} \quad \tilde{y} = \frac{e^{-u/r}}{c} \qquad v \in [0, 2\pi] \quad u \in [-\infty, r\ln(r/c)]$$

Hilbert horizon

The line element of this surface of revolution is

$$dl^2 = du^2 + R^2(u)dv^2$$

where

$$R(u) = c\, e^{u/r}$$

That is a Beltrami pseudosphere

The Hilbert horizon is the maximal circle

$$R(u_{Hh}) = r$$

The parameter c sets:

(a) the origin of u ($R(u = 0) = c$)

(b) its pace (the bigger r/c, the farther is the boundary)

For graphene

$$c \equiv \ell$$

Altogether

$$ds_B^2 = \frac{\ell^2}{r^2}\, e^{2u/r} \left[\frac{r^2}{\ell^2}\, e^{-2u/r} \left(dt^2 - du^2\right) - r^2 dv^2\right] = \varphi^2(u)\, ds_R^2$$

ds_R^2 **is the line element of the Rindler spacetime, e.g. for the right wedge**

$$\eta \equiv \frac{r}{\ell} t \qquad\qquad \xi \equiv -\frac{r}{\ell} u$$

The two horizons merge in a limit

$$\xi_{Eh} = -\infty \quad \text{vs} \quad \xi_{Hh} = -\frac{r^2}{\ell} \ln \frac{r}{\ell}$$

hence

$$\xi_{Hh} \to \xi_{Eh} \quad \text{for} \quad \frac{\ell}{r} \to 0$$

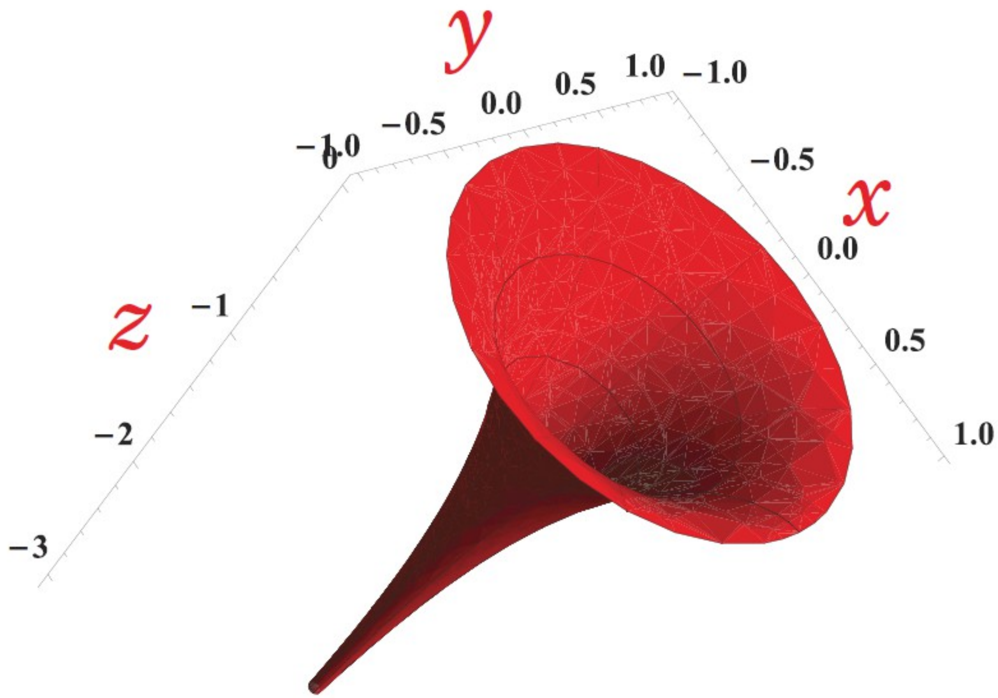
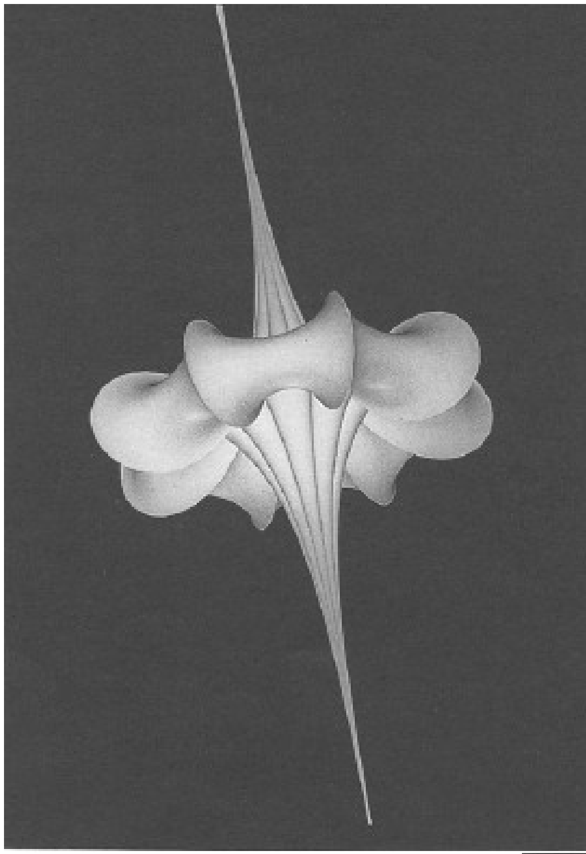
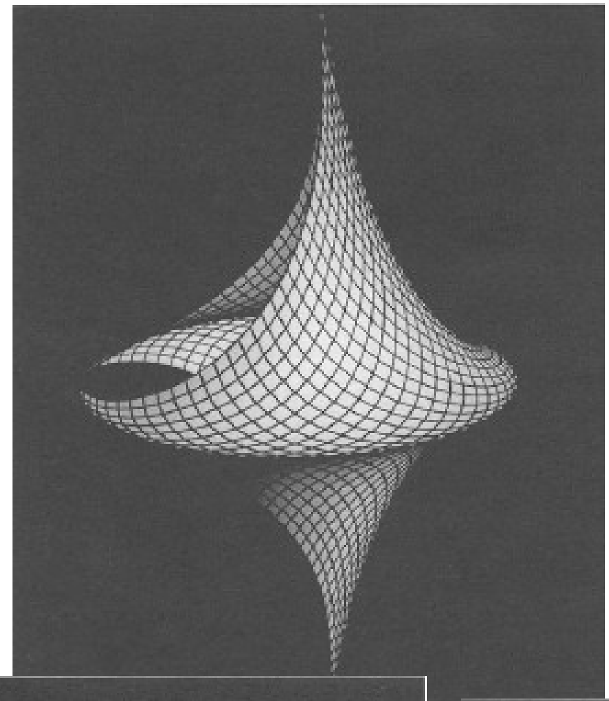


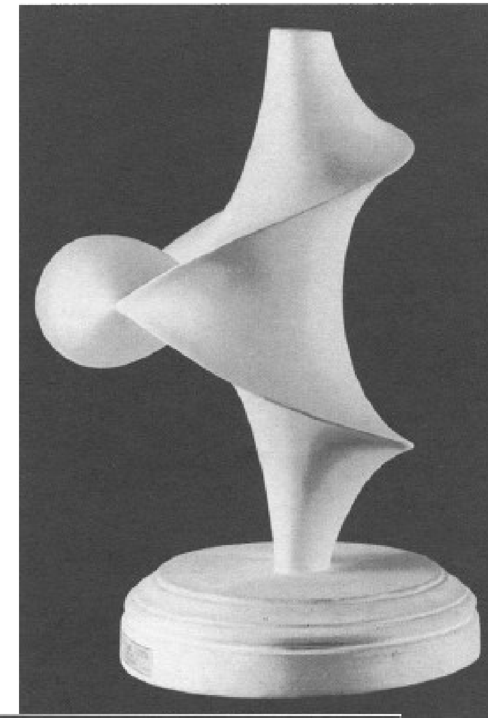
Table 1: Quantification of how good is to approximate the Hilbert horizon of the Beltrami spacetime, $R = r$, with a Rindler event horizon. The closer $\zeta_B \equiv -(\ell/r)^2/\ln(r/\ell)$ is to zero, the better is the approximation. In the table we indicate three values of r, the corresponding values of ζ_B, and we also explicitly indicate the corresponding values of ℓ/r (recall that $\ell \simeq 2$Å.). This latter parameter is also a measure of how close to zero is $\tilde{y}_{Hh} = 1/r$, in units of the lattice spacing ℓ: $1/(r/\ell)$. The values are all approximate.

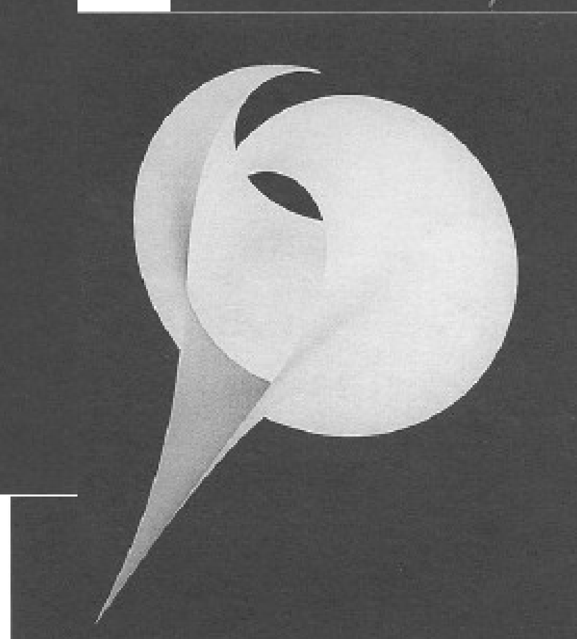
r	ζ_B	ℓ/r
20Å	-4×10^{-3}	0.1
1 μm	-5×10^{-9}	2×10^{-4}
1 mm	-3×10^{-15}	2×10^{-7}

There is more!









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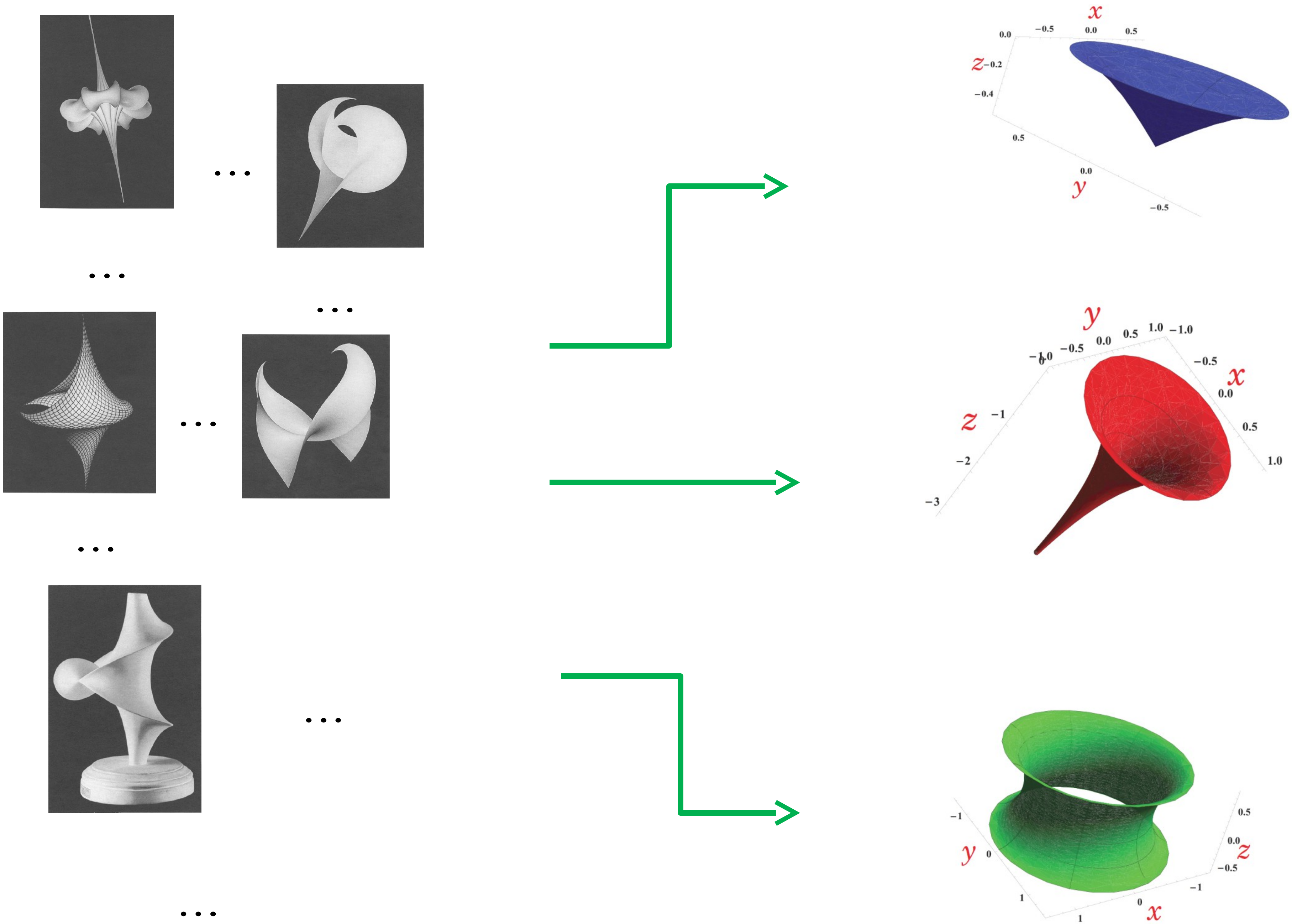
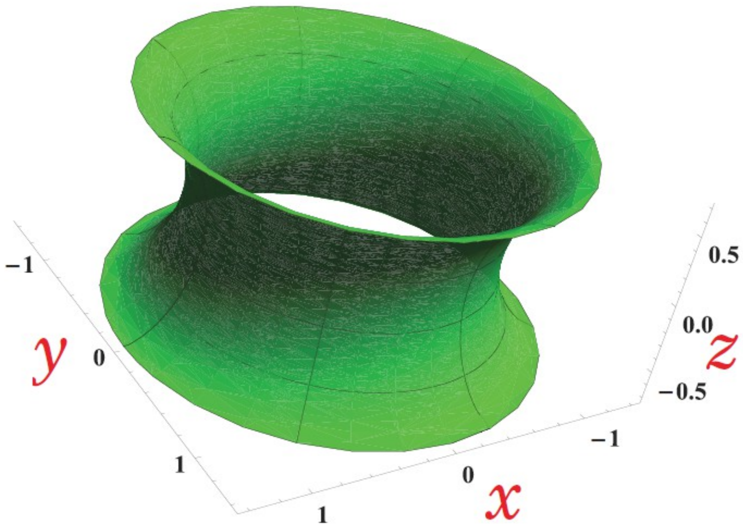
R. McLachlan, Maths. Int. **16**, 31 (1994).



x
y
z

$$ds^2_{Hyp} = \left(\frac{\mathcal{R}^2}{c^2} - M\right)^{-1} ds^2_{BTZ}$$



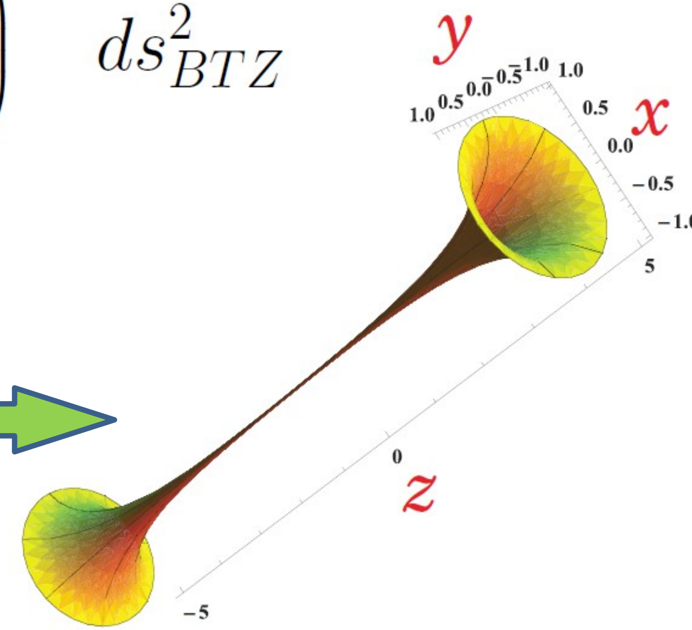


Table 3: Quantification of how good is to approximate the Hilbert horizon of the hyperbolic pseudosphere spacetime with a BTZ black hole event horizon. The closer $\zeta_{Hyp} \equiv (\mathcal{R}_{Hh} - \mathcal{R}_{Eh})/\mathcal{R}_{Eh}$ is to zero, the better is the approximation. In the table we indicate three values of ℓ/r comparable to those used in Tables 1 and 2, the corresponding values of ζ_{Hyp}, then how close the Hilbert horizon of this spacetime ($R = r\sqrt{1 + (\ell^2/r^2)}$) is to the Hilbert horizon of the Beltrami spacetime ($R = r$) (that is also a measure of how well the hyperbolic pseudosphere spacetime can be identified with the Beltrami spacetime). In the last column are the values of the BTZ mass M in terms of graphene parameters. All the values are approximate.

ℓ/r	ζ_{Hyp}	$(R_{Hh} - r)/r$	M
0.1	5×10^{-3}	5×10^{-2}	10^{-2}
10^{-4}	5×10^{-9}	5×10^{-5}	10^{-8}
10^{-7}	5×10^{-15}	5×10^{-8}	10^{-14}

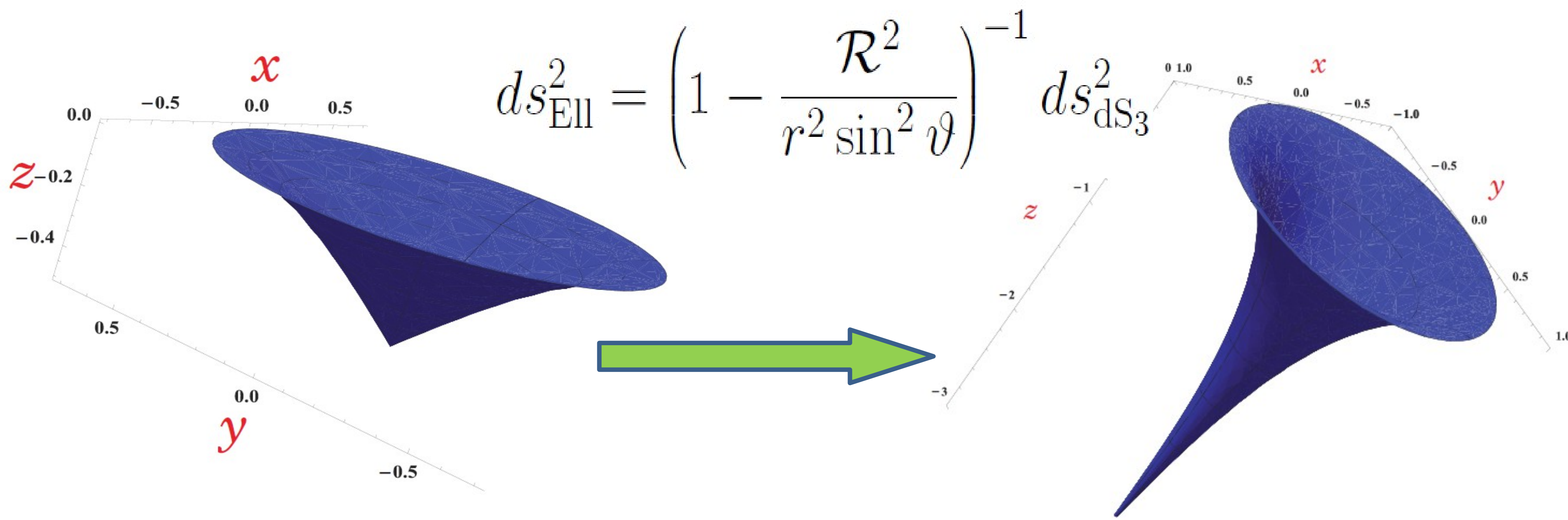


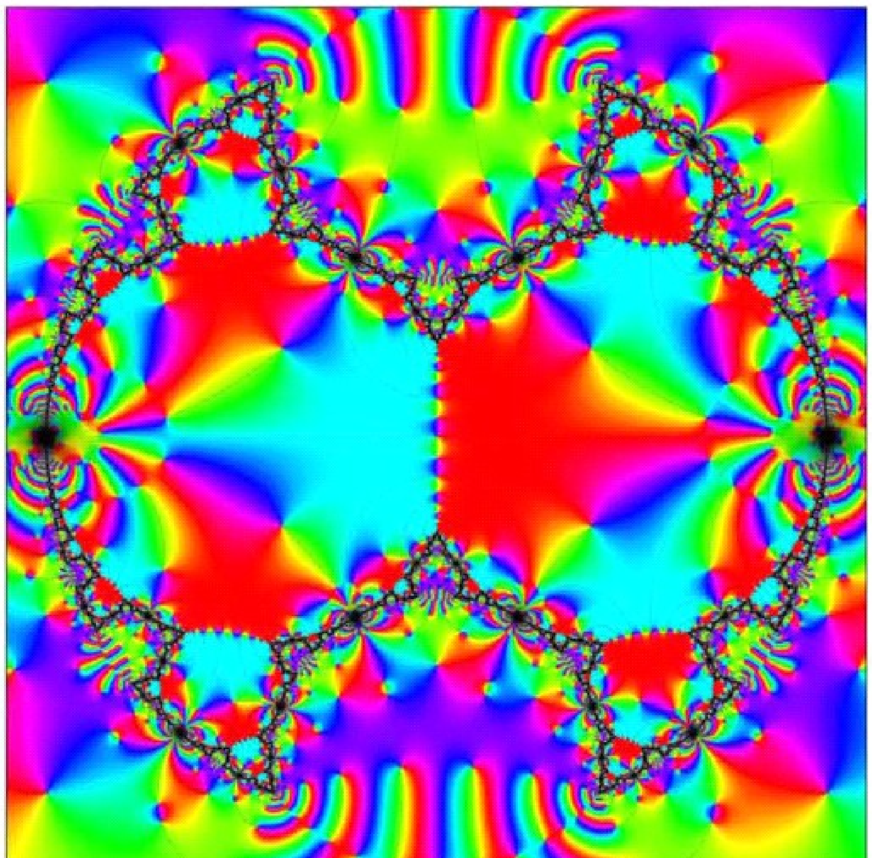
Table 2: Quantification of how good is to approximate the Hilbert horizon of the elliptic pseudosphere spacetime with a cosmological event horizon. The closer $\zeta_{Ell} \equiv (\mathcal{R}_{Eh} - \mathcal{R}_{Hh})/r$ is to zero, the better is the approximation. In the table we indicate three values of ℓ/r comparable to those used in Table 1, the corresponding values of ζ_{Ell}, and of how close the Hilbert horizon of this spacetime ($R = r\cos\vartheta$) is to the Hilbert horizon of the Beltrami spacetime ($R = r$). The latter column, then, is also a measure of how well the elliptic pseudosphere spacetime can be identified with the Beltrami spacetime. The values are all approximate.

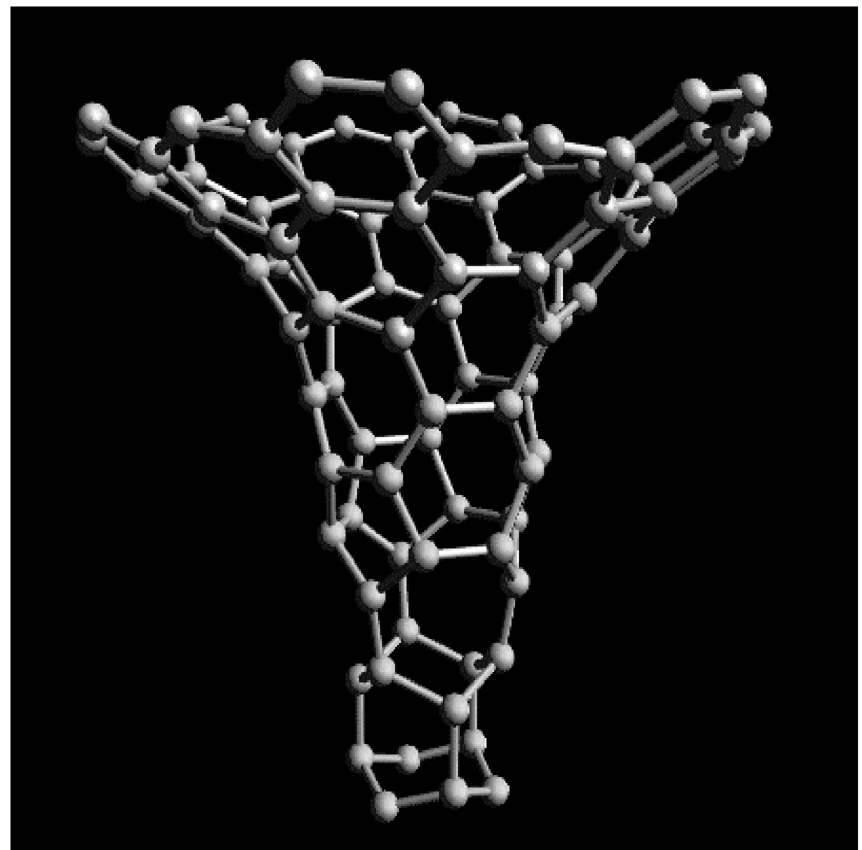
$\vartheta \sim \ell/r$	ζ_{Ell}	$(R_{Hh} - r)/r$
0.1	5×10^{-4}	5×10^{-3}
10^{-4}	5×10^{-13}	5×10^{-9}
10^{-7}	5×10^{-22}	5×10^{-15}

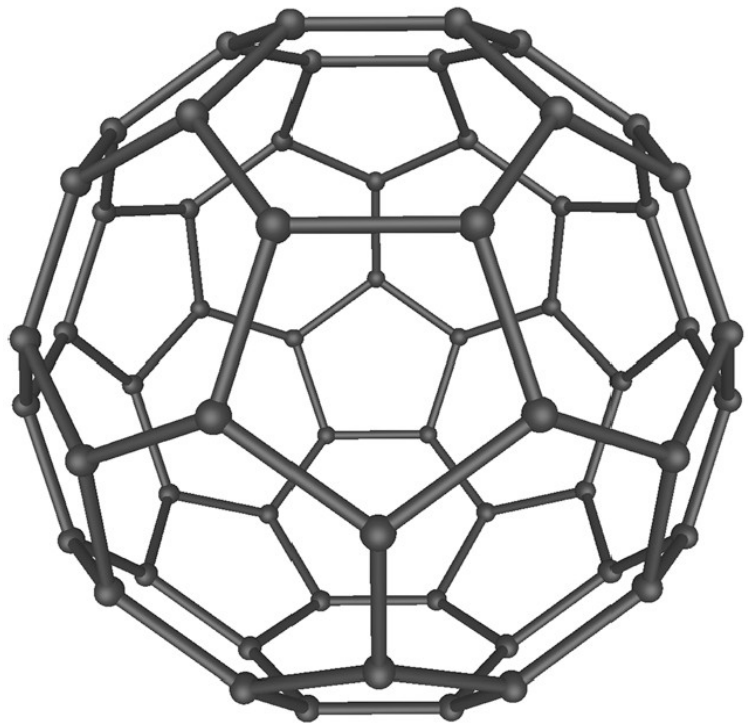
4. In the lab

There is a LONG list of things to do

First, we built the 'graphenic Beltrami' on computer





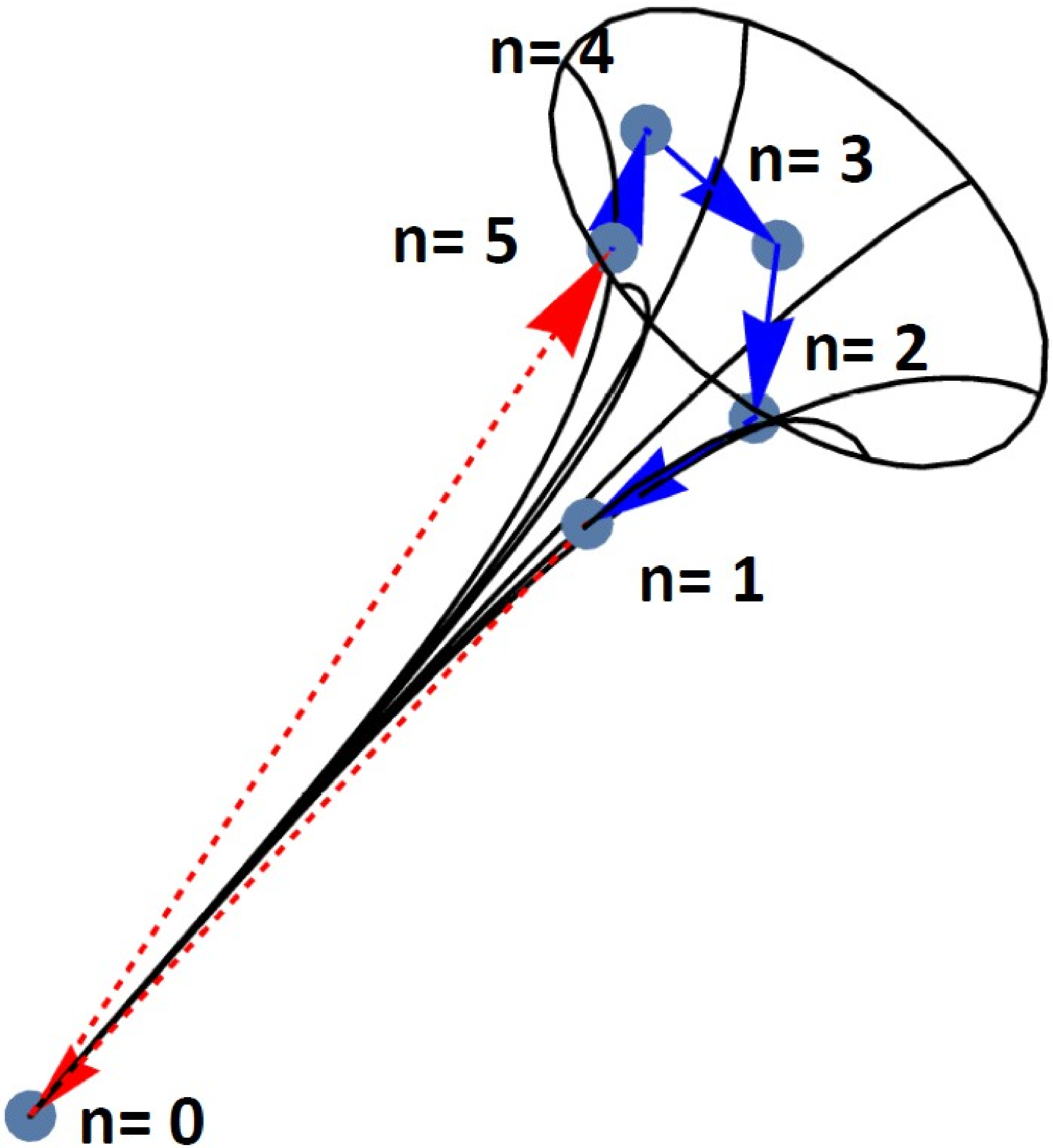


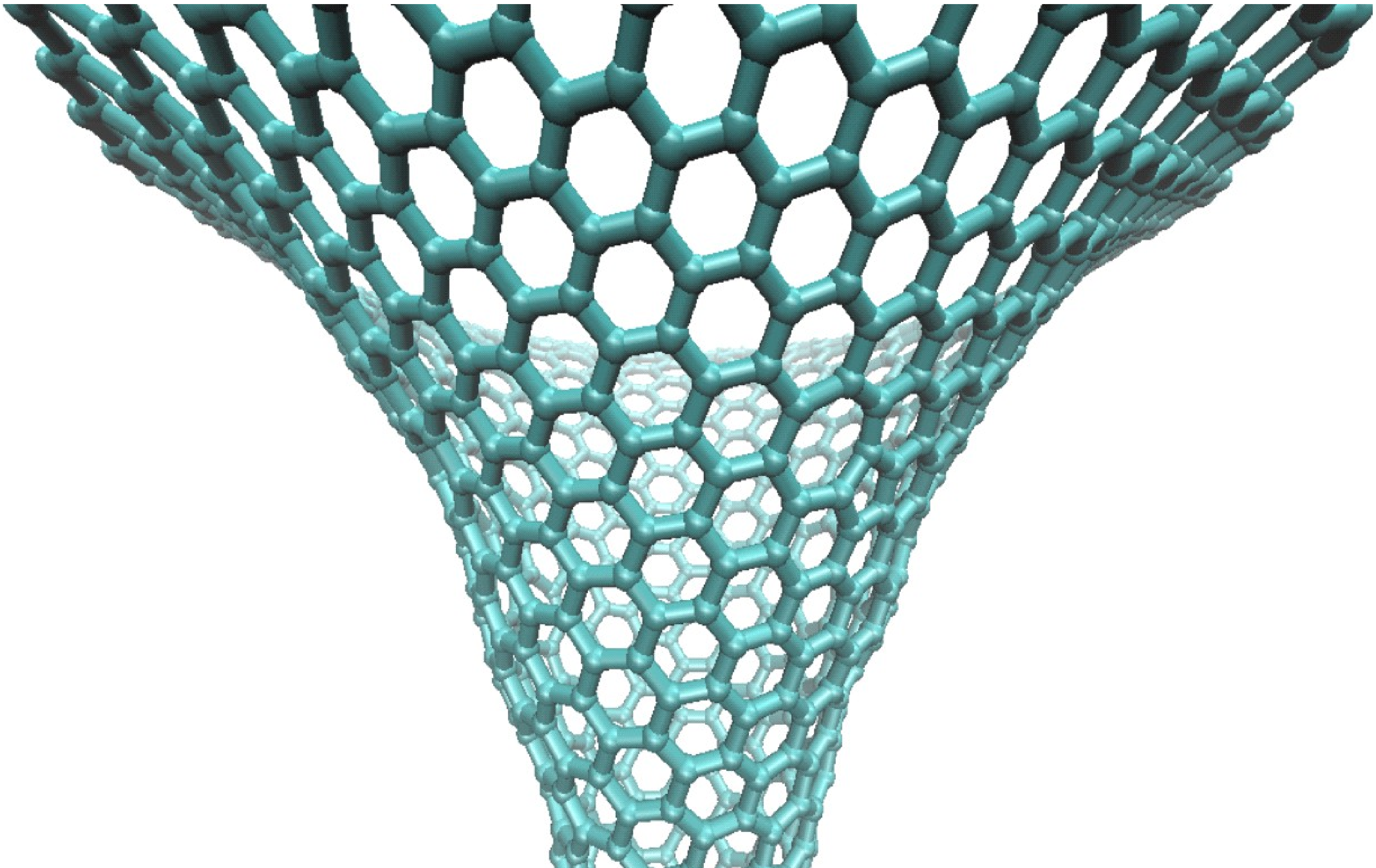
$$F_Y^B \equiv \{\exp\{u + iv\} | v = 2\pi n/5, u = r \ln(|n|/6)\}$$

where $n \in \mathbf{Z}_5$. The sign of n is related to the spiral's chirality (ssb of the parity in Y).

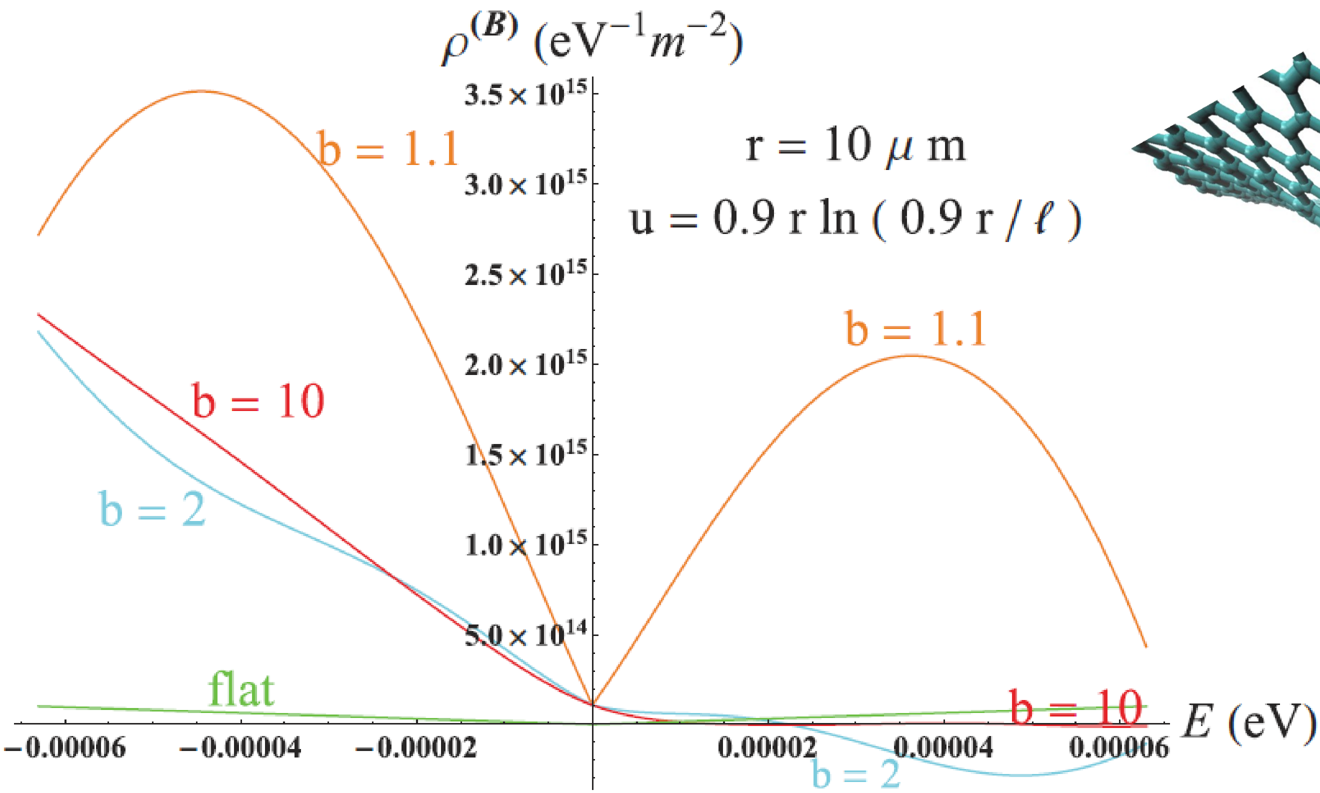
The 6 discrete values of $u = r \ln(R/r)$ correspond to the point at infinity, $R/r = 0$, and to the 5 tangent cones with apertures $\alpha_n = \arcsin(n/6)$

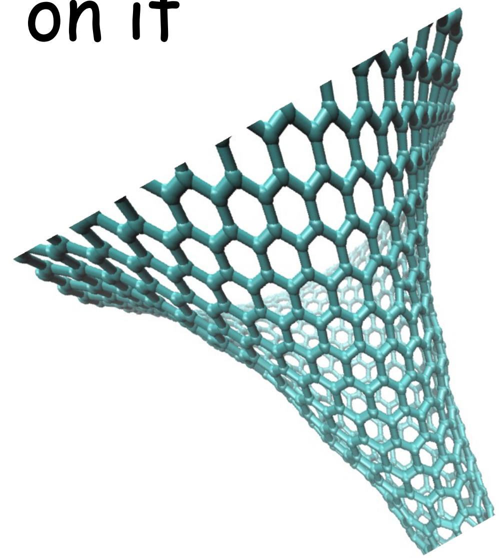
F_Y^B is a cyclic loxodromic subgroup of $SL(2, \mathbf{Z})$ of order 5, hence the NEC group of Beltrami


n= 4
n= 3
n= 5
n= 2
n= 1
n= 0



Then, we shall measure Hawking on it

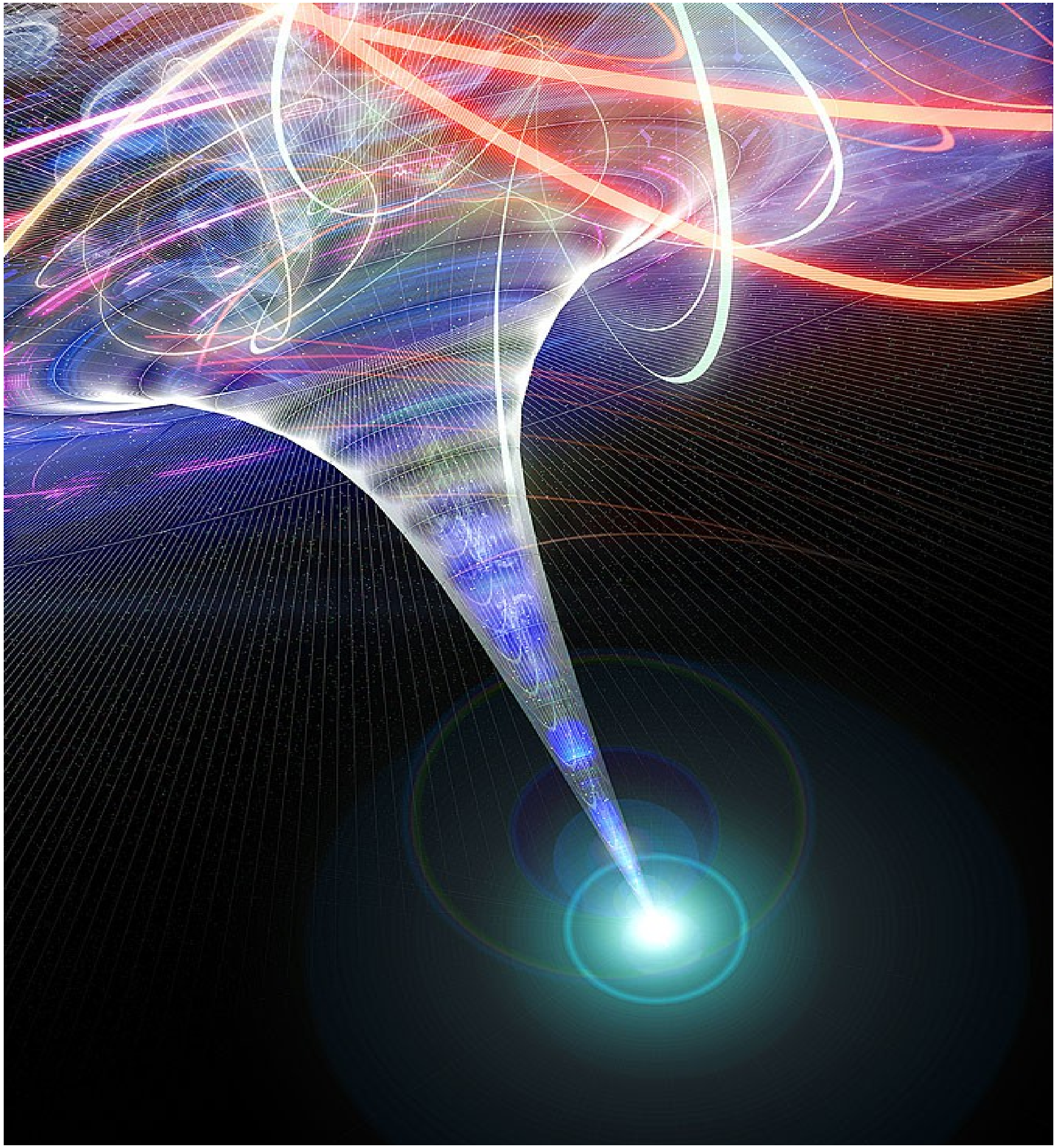




$$\rho^{(B)}_{thermal}(E,u,r) = \frac{4}{\pi}\frac{1}{(\hbar v_F)^2}\frac{r^2}{\ell^2}e^{-2u/r}\frac{E}{\exp\left[E/(k_B\mathcal{T}(u,r))\right]-1}$$

$$\mathcal{T}(r\ln(r/\ell)) = \frac{\hbar v_F}{k_B}\,\frac{1}{2\pi r}$$

Then, all the rest...



5. Selected refs

AI, P Pais, Phys Rev D 92 (2015) 125005 (fresh theory)

S Taioli, R Gabbrielli, S Simonucci, NM Pugno, AI, J Phys: Cond Mat 28 (2016) 13LT01 (fresh computer sim.)

AI, G Lambiase, Phys Lett B 716 (2012) 334; Phys Rev D 90 (2014) 025006 (the Hawking phenomenon)

AI, Ann Phys 326 (2011) 1334; Int J Mod Phys D 24 (2015) 1530013 (seminal work AND detailed review)