

Stringy and higher spin activities @ FZU

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Cast of characters

String Field Theory	Conformal Field Theory	Higher Spins	Gravity and Holography
	Martin Schnabl		
		Joris Raeymaekers	
		Tomáš Procházka	
Theodore Erler*			
Carlo Maccaferri*			
Masaki Murata			
		Dario Francia*	
			Carlo Iazeolla
Matěj Kudrna			
Miroslav Rapčák			

Higher spins

- The consistency conditions for decoupling the longitudinal modes of free massless spin-1 or spin-2 particles are so strong, that they lead to the Maxwell (Yang-Mills) Lagrangian and Einstein-Hilbert Lagrangian unavoidably.
- The problem for higher spins has attracted many through out the history: Pauli, Fierz, Schwinger, Rarita, Fronsdal,..., Vasiliev,..., **Niederle**, Burdík,... Sagnotti, **Francia**, **Iazeolla**, ...

Higher spins

- Even at free-field level the theory is complicated

$$\mathcal{L} = \frac{1}{2} \varphi_{\mu_1 \dots \mu_{s_1}, \nu_1 \dots \nu_{s_2}, \dots} \left\{ \square \varphi^{\mu_1 \dots \mu_{s_1}, \nu_1 \dots \nu_{s_2}, \dots} - (\partial^{\mu_1} \partial_\alpha \varphi^{\alpha \mu_2 \dots \mu_{s_1}, \nu_1 \dots \nu_{s_2}, \dots} + \dots) \right. \\ \left. - (\partial^{\nu_1} \partial_\alpha \varphi^{\mu_1 \dots \mu_{s_1}, \alpha \nu_2 \dots \nu_{s_2}, \dots} + \dots) - \dots \right\},$$

Maxwell-like form for mixed symmetry tensors
(Campoleoni & Francia 2012)

- At the interacting level the theory is highly non-tractable. One attempt is to get some insight from string field theory (Francia and MS), another, better explored one is the proposal by Vasiliev.

Higher spins

- The Vasiliev equations describe not a single higher spin, but necessarily the whole tower at once:

$$\hat{F} = \frac{i}{4} dZ^A \wedge dZ_A \hat{\Phi} \star \kappa$$
$$\hat{D}\hat{\Phi} = 0$$

$$\hat{F} = \hat{d}\hat{A} + \hat{A} \star \hat{A}, \quad \hat{A} = \hat{A}(Y, Z|x)$$
$$\hat{\Phi} = \hat{\Phi}(Y, Z|x)$$

- This is perhaps a hint of something deep like string theory. They also require AdS.

Higher spins and holography

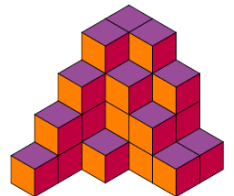
- In three dimensions the Vasiliev equations become tractable, and as a theory of gravity they are expected to have CFT dual.
- The euclidean higher spin gravity is described by an $sl(n, \mathbb{C})$ or $hs[\lambda]$ connection A :

$$F = dA + A \wedge A = 0, \quad \bar{F} = d\bar{A} + \bar{A} \wedge \bar{A} = 0$$

$$e = \frac{1}{2} (A - \bar{A}), \quad \omega = \frac{1}{2i} (A + \bar{A}),$$

Higher spins and holography

- Gaberdiel and Gopakumar (2010) proposed that the higher spin gravity is holographically dual to W_N minimal model CFT.
- Raeymaekers, Procházka and collaborators recently presented some of the most impressive checks by studying black hole like solutions on one hand and studying properties of W algebras and their representations on the other hand.



What is String Field Theory?

- Field theoretic description of all excitations of a string (open or closed) at once.
- Useful especially for physics of backgrounds: tachyon condensation or instanton physics, etc.
- Single Lagrangian field theory which around its various critical points should describe physics of diverse D-brane backgrounds, possibly also gravitational backgrounds.

First look at bosonic OSFT

Open string field theory uses the following data

$$\mathcal{H}_{BCFT}, \quad *, \quad Q_B, \quad \langle \cdot \rangle.$$

Let all the string degrees of freedom be assembled in

$$|\Psi\rangle = \sum_i \int d^{p+1}k \phi_i(k) |i, k\rangle,$$

Witten (1986) proposed the following action

$$S = -\frac{1}{g_o^2} \left[\frac{1}{2} \langle \Psi * Q_B \Psi \rangle + \frac{1}{3} \langle \Psi * \Psi * \Psi \rangle \right],$$

First look at OSFT

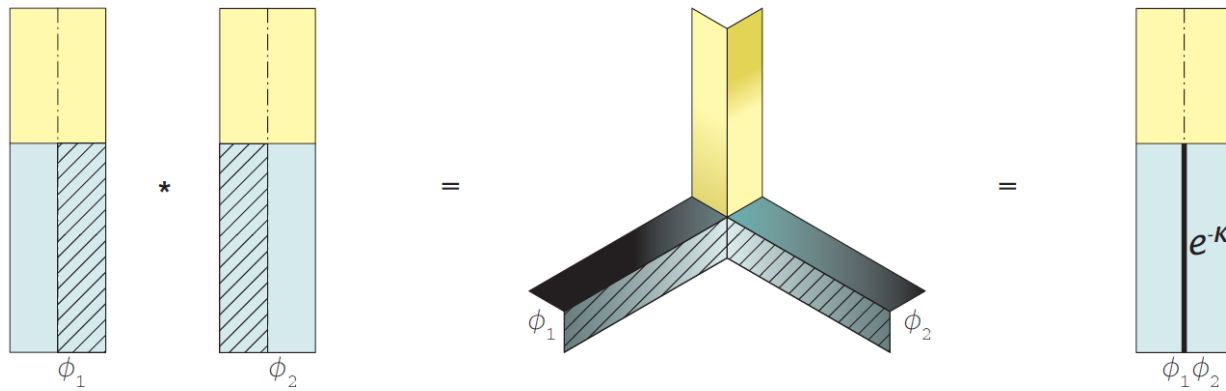
This action has a huge gauge symmetry

$$\delta\Psi = Q_B\Lambda + \Psi * \Lambda - \Lambda * \Psi,$$

provided that the star product is associative, Q_B acts as a graded derivation and $\langle . \rangle$ has properties of integration.

Note that there is a gauge symmetry for gauge symmetry so one expects infinite tower of ghosts – indeed they can be naturally incorporated by lifting the ghost number restriction on the string field. Solving the master equation in the superstring case is still an open problem. See our work with Berkovits, Kroyter, Okawa, Torii and Zwiebach.

Witten's star product



- The elements of string field star algebra are states in the BCFT, they can be identified with a piece of a worldsheet.
- By performing the path integral on the glued surface in two steps, one sees that in fact:

$$|\phi_1\rangle * |\phi_2\rangle = |\phi_1 e^{-K} \phi_2\rangle.$$

Witten's star product as operator multiplication

We have just seen that the star product obeys

$$|\phi_1\rangle * |\phi_2\rangle = |\phi_1 e^{-K} \phi_2\rangle.$$

And therefore states $\hat{\phi} = e^{K/2} \phi e^{K/2}$ obey

$$|\hat{\phi}_1\rangle * |\hat{\phi}_2\rangle = |\widehat{\phi_1 \phi_2}\rangle$$

The star product and operator multiplication are thus isomorphic!

Simple subsector of the star algebra

- The star algebra is formed by vertex operators and the operator K . The simplest subalgebra relevant for tachyon condensation is therefore spanned by K and c . Let us be more generous and add an operator B such that $QB=K$.

- The building elements thus obey

$$c^2 = 0, \quad B^2 = 0, \quad \{c, B\} = 1$$

$$[K, B] = 0, \quad [K, c] = \partial c$$

- The derivative Q acts as $Q_B K = 0, \quad Q_B B = K, \quad Q_B c = cKc$.

Classical solutions

This new understanding lets us construct solutions to OSFT equations of motion $Q_B\Psi + \Psi * \Psi = 0$ easily.

It does not take much trying to find the simplest solution is $\Psi = \alpha c - cK$

$$Q\Psi = \alpha(cKc) - (cKc)K$$

$$\Psi * \Psi = \cancel{\alpha^2 c^2} - \cancel{\alpha c^2 K} - \alpha c K c + (cK)(cK)$$

More general solutions are of the form

$$\Psi = Fc \frac{KB}{1 - F^2} cF,$$

Here $F=F(K)$ is arbitrary

M.S. 2005, Okawa, Erler 2006

Classical solutions

- What do these solutions correspond to?
- In 2011 with Murata we succeeded in computing their energy

$$E = \frac{1}{2\pi^2} \oint_C \frac{dz}{2\pi i} \frac{G'(z)}{G(z)}$$

in terms of the function $G(z) = 1 - F^2(z)$

- For simple choices of G , one can get perturbative vacuum, **tachyon vacuum**, or exotic **multibrane solution**. At the moment the multibrane solutions appear to be a bit singular. (see also follow-up work by Hata and Kojita)

Classical solutions in super OSFT

- Recently there was a significant progress by **Erler**, who found the tachyon vacuum for Berkovits super-OSFT

$$S = \frac{1}{2g^2} \left\langle \left\langle (e^{-\Phi} Q_B e^{\Phi})(e^{-\Phi} \eta_0 e^{\Phi}) - \int_0^1 dt (e^{-t\Phi} \partial_t e^{t\Phi}) \{ (e^{-t\Phi} Q_B e^{t\Phi}), (e^{-t\Phi} \eta_0 e^{t\Phi}) \} \right\rangle \right\rangle$$

where Φ is picture-number and ghost-number zero string field. This action can be also written as

$$S = - \int_0^1 dt \text{Tr} [(\eta \Psi_t) \Psi_Q] \quad \Psi_Q \equiv g(t)^{-1} Q g(t), \quad \Psi_\eta \equiv g(t)^{-1} \eta g(t), \quad \Psi_t \equiv g(t)^{-1} \partial_t g(t)$$

The equation of motion takes the form $\eta_0 (e^{-\Phi} Q_B e^{\Phi}) = 0$

Classical solutions in super OSFT

- On a non-BPS D-brane the solution can be conveniently looked for in the basis K, B, c, γ and γ^{-1} suggested by Berkovits and M.S.

- Erler looked for solutions such that

$$\Psi = c \frac{1}{1+K} - Q \left(c \frac{B}{1+K} \right)$$

Corresponding g can be constructed by $g = Q_0 \Psi \beta$

With a clever choice of β he found:

$$g = (1 + \zeta) \left(1 + Q\zeta \frac{B}{1+K} \right) \quad g^{-1} = \left(1 - Q\zeta \frac{B}{1+K+V} \right) (1 - \zeta)$$

$$\zeta \equiv \gamma^{-1} c$$

$$V \equiv \frac{1}{2} \gamma^{-1} \partial c$$

for which he could have computed the energy etc.

OSFT = physics of backgrounds

- So far all the discussion concerned background independent solutions and aspects of OSFT.
- The new theme of the past year or two, is that OSFT can be very efficient in describing BCFT backgrounds and their interrelation.
- Traditionally, this has been studied using the boundary states. General construction not known!

Boundary states

- Describe possible boundary conditions from the closed string channel point of view.
- Conformal boundary states obey:
 - 1) the gluing condition $(L_n - \bar{L}_{-n})|B\rangle = 0$
 - 2) Cardy condition (modular invariance)
 - 3) sewing relations (factorization constraints)

See e.g. reviews by Gaberdiel or by Cardy

Boundary states

- The gluing condition is easy to solve:
For any spin-less primary $|V_\alpha\rangle$ we can define

$$||V_\alpha\rangle\rangle = \sum_{IJ} M^{IJ}(h_\alpha) L_{-I} \bar{L}_{-J} |V_\alpha\rangle$$

where M^{IJ} is the inverse of the real symmetric Gram matrix

$$M_{IJ}(h_\alpha) = \langle V^\alpha | L_I L_{-J} | V_\alpha \rangle$$

where $L_{-X} \equiv L_{-n_k} \dots L_{-n_1}$
(with possible null states projected out).

Boundary states – Cardy's solution

- By demanding that

$$\langle\langle \alpha \| q^{\frac{1}{2}(L_0 + \bar{L}_0 - \frac{c}{12})} \| \beta \rangle\rangle = \text{Tr}_{\mathcal{H}_{\alpha\beta}^{\text{open}}} \left(\tilde{q}^{L_0 - \frac{c}{24}} \right)$$

and noting that RHS can be expressed as

$$\sum_i n_{\alpha\beta}^i \chi_i(\tilde{q})$$

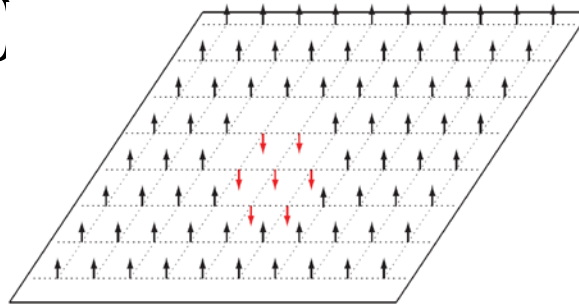
Cardy derived integrality constraints on the boundary states. Surprisingly, for certain class of rational CFT's he found an elegant solution (relying on Verlinde formula)

$$\|B_i\rangle\rangle = \sum_j \frac{S_i^j}{\sqrt{S_0^j}} |j\rangle\rangle$$

where S_i^j is the modular matrix.

Ising model CFT

- Originally Ising model was intended as a toy model for ferro- to paramagnetic phase transition (Lenz 1920, Ising 1925)



$$Z = \sum_{\sigma_i = \pm 1} e^{\beta \sum_{(i,j)} \sigma_i \sigma_j}$$

- At long distances and critical temperature the Ising model is described by the simplest possible unitary CFT with $c = 1/2$ and with 3 primary operators:

1	(0,0)	identity
ε	($1/2, 1/2$)	local energy operator
σ	($1/16, 1/16$)	local spin operator

Boundary states – Cardy's solution

- The modular S-matrix takes the form

$$S = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \end{pmatrix}$$

- And thus the Ising model conformal boundary

states are $\|\tilde{0}\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|\varepsilon\rangle + \frac{1}{\sqrt[4]{2}}|\sigma\rangle$

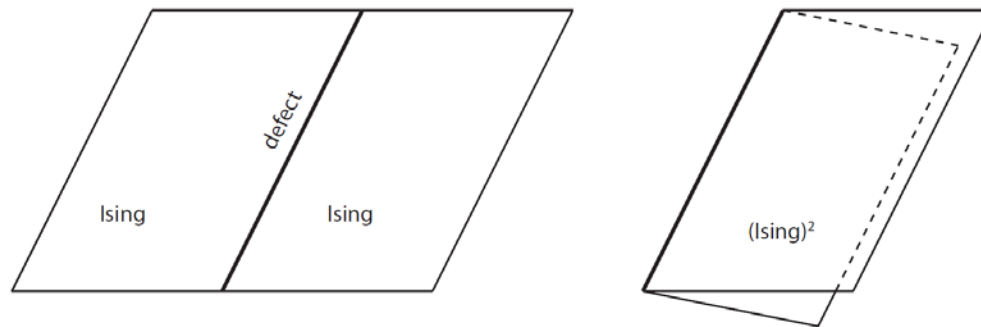
$$\|\tilde{\varepsilon}\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|\varepsilon\rangle - \frac{1}{\sqrt[4]{2}}|\sigma\rangle$$

$$\|\tilde{\sigma}\rangle = |0\rangle - |\varepsilon\rangle$$

- The first two boundary states describe fixed (+/-) boundary condition, the last one free boundary condition

(Ising)²

- This model naturally arises when one considers Ising model on a plane with a defect line and employs the folding trick.

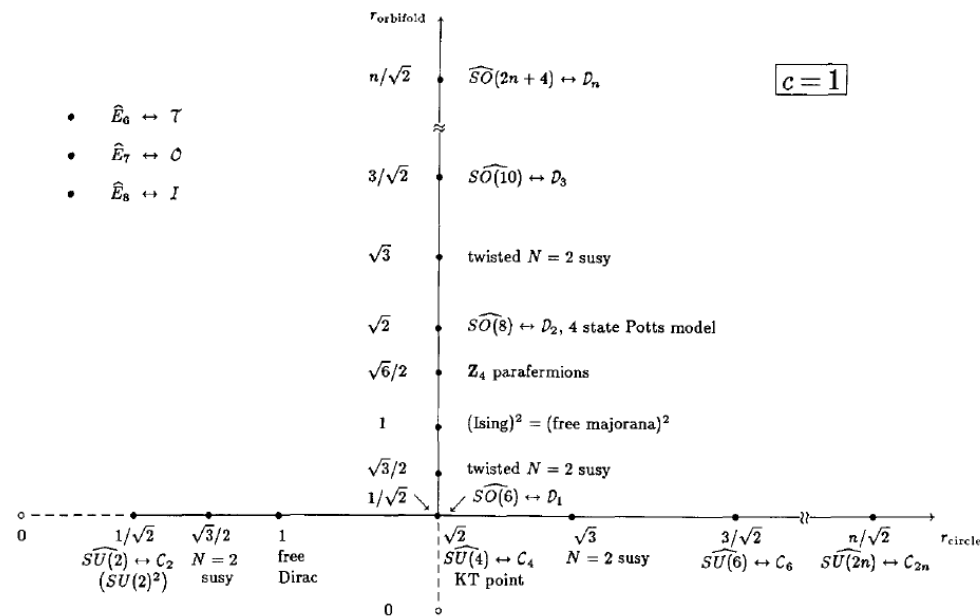


(Ising)²

- (Ising)² model is well known point on the orbifold branch of the moduli space of $c=1$ models

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P. Ginsparg / Curiosities at $c=1$



- $\widehat{E}_6 \leftrightarrow \tau$
- $\widehat{E}_7 \leftrightarrow \mathcal{O}$
- $\widehat{E}_8 \leftrightarrow I$

(Ising)²

- Even though Ising model itself has only 3 bulk primaries, (Ising)² has infinite number of them (Yang 1987)

$\Delta = \bar{\Delta}$	Multiplicity	(Ising) ² Examples	Orbifold Examples
$n^2 = 0, 1, 4, \dots$	1	$1 \otimes 1, \varepsilon \otimes \varepsilon$	$1, \partial X \bar{\partial} X$
$\frac{(n+1)^2}{2} = \frac{1}{2}, 2, \frac{9}{2}, \dots$	2	$1 \otimes \varepsilon, \varepsilon \otimes 1$	$\cos(\sqrt{2}X), \cos(\sqrt{2}\tilde{X})$
$\frac{(2n+1)^2}{8} = \frac{1}{8}, \frac{9}{8}, \frac{25}{8}, \dots$	1	$\sigma \otimes \sigma$	$\sqrt{2} \cos(\frac{X}{\sqrt{2}})$
$\frac{(2n+1)^2}{16} = \frac{1}{16}, \frac{9}{16}, \frac{25}{16}, \dots$	2	$1 \otimes \sigma, \sigma \otimes 1, \varepsilon \otimes \sigma, \sigma \otimes \varepsilon$	twist fields, excited twist fields

- It is precisely equivalent to a free boson on an orbifold S^1/Z_2 with radius $R_{orb} = \sqrt{2}$ (in our units $\alpha' = 1$.)
- While some boundary states are readily available, the complete list can be expected to be quite rich

Boundary states in (Ising)²

- Here is the list found by Affleck and Oshikawa (1996)

(Ising) ² D-brane	Interpretation	Energy = $\langle 1 \rangle$	$\frac{\langle \partial X \bar{\partial} X \rangle}{\langle 1 \rangle}$	Position
$1 \otimes \varepsilon$	fractional D0	$\frac{1}{2}$	+1	πR
$\varepsilon \otimes 1$	fractional D0	$\frac{1}{2}$	+1	πR
$1 \otimes 1$	fractional D0	$\frac{1}{2}$	+1	0
$\varepsilon \otimes \varepsilon$	fractional D0	$\frac{1}{2}$	+1	0
$1 \otimes \sigma$	fractional D1	$\frac{1}{\sqrt{2}}$	-1	-
$\sigma \otimes 1$	fractional D1	$\frac{1}{\sqrt{2}}$	-1	-
$\varepsilon \otimes \sigma$	fractional D1	$\frac{1}{\sqrt{2}}$	-1	-
$\sigma \otimes \varepsilon$	fractional D1	$\frac{1}{\sqrt{2}}$	-1	-
$\sigma \otimes \sigma$	centered bulk D0	1	+1	$\frac{\pi R}{2}$
$\sum_i a_i(\phi) AT, i\rangle\rangle$	generic bulk D0	1	+1	ϕR
$\sum_i b_i(\tilde{\phi}) AT, i\rangle\rangle$	generic bulk D1	$\sqrt{2}$	-1	-

Now we would like to find all this from OSFT !?!

Numerical solutions in OSFT

- To construct new D-branes in a given BCFT with central charge c using OSFT, we consider strings ‘propagating’ in a background $\text{BCFT}_c \otimes \text{BCFT}_{26-c}$ and look for solutions which do not excite any primaries in BCFT_{26-c} .
- In the case of Ising the boundary spectrum is particularly simple:

D-brane	Energy	Boundary spectrum
$ \tilde{1}\rangle$	$\frac{1}{\sqrt{2}}$	1
$ \tilde{\varepsilon}\rangle$	$\frac{1}{\sqrt{2}}$	1
$ \tilde{\sigma}\rangle$	1	1, ε

Boundary state from Ellwood invariants

- The coefficients of the boundary state

$$|B_\Psi\rangle = \sum_{\alpha} n_{\Psi}^{\alpha} ||V_{\alpha}\rangle\rangle$$

can be computed from OSFT solution via

$$n_{\Psi}^{\alpha} = 2\pi i \langle I | \mathcal{V}^{\alpha}(i) | \Psi - \Psi_{\text{TV}} \rangle^{\text{BCFT}_0 \otimes \text{BCFT}_{\text{aux}}}$$

$$\mathcal{V}^{\alpha} = c\bar{c}V^{\alpha} e^{2i\sqrt{1-h_{\alpha}}Y} w$$

See: Kudrna, Maccaferri, M.S. (2012)

Alternative attempt:

Kiermaier, Okawa, Zwiebach (2008)

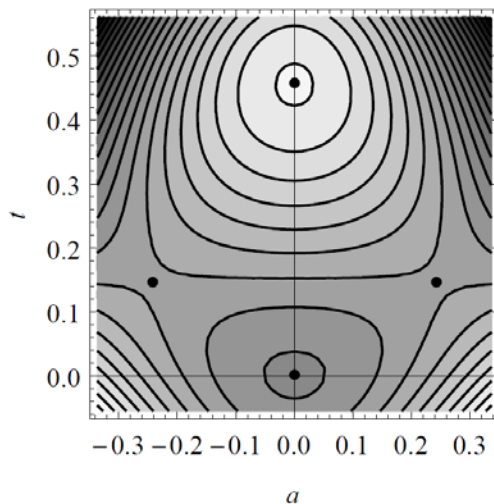
Tachyon condensation on the σ -brane

- Truncating the string field e.g. to level 2:

$$|\psi\rangle = tc_1|0\rangle + ac_1|\epsilon\rangle + uc_{-1}|0\rangle + vc_1L_{-2}^I|0\rangle + wc_1L_{-2}^R|0\rangle$$

we are interested in the stationary points of the potential

$$\begin{aligned} \mathcal{V}(t, a, u, v, w) = & -\frac{1}{2}t^2 - \frac{1}{4}a^2 - \frac{1}{2}u^2 + \frac{1}{8}v^2 + \frac{51}{8}w^2 + \frac{27}{64}va^2 - \frac{255}{64}wa^2 + \frac{11}{16}ua^2 + \\ & - \frac{165\sqrt{3}}{3456}tuv - \frac{8415\sqrt{3}}{3456}tuw + \frac{1049\sqrt{3}}{9216}tv^2 + \frac{256423563\sqrt{3}}{746496}tw^2 + \frac{66\sqrt{3}}{128}ut^2 - \\ & - \frac{15\sqrt{3}}{256}vt^2 - \frac{765\sqrt{3}}{256}wt^2 + \frac{19\sqrt{3}}{192}tu^2 + \frac{27\sqrt{3}}{64}t^3 + \frac{425\sqrt{3}}{1536}tvw + \frac{27}{16}ta^2. \end{aligned}$$



Picture shows only t and a dependence.
Taken from M. Rapčák's thesis.

Tachyon condensation on the σ -brane

- Going to higher levels, we should properly take care of the Ising model **null-states**
- Null states form an ideal, e.g.

$$(L_{-2} - \frac{3}{4}L_{-1}^2)|\varepsilon\rangle * |\varepsilon\rangle = \frac{2 \times 3^3}{7^2 \times 11} K^{-9} \left(L_{-6} + \frac{22}{9}L_{-4}L_{-2} - \frac{31}{36}L_{-3}^2 - \frac{16}{27}L_{-2}^3 \right) |0\rangle + \dots$$

So we can consistently set them to zero and reduce the complexity.

Tachyon condensation on the σ -brane

- Already in the lowest truncation levels we see two solutions corresponding to 1- and ϵ -branes

Level	0.5	2.0	2.5
$2\pi^2\mathcal{V}(\psi)$	-0.16971	-0.24579	-0.26454
Percentage	57.9 %	83.9 %	90.3 %
$c_1 0\rangle$	0.14815	0.20553	0.21454
$c_1 \epsilon\rangle$	± 0.24348	± 0.27818	± 0.29230
$c_{-1} 0\rangle$		0.07382	0.07305
$c_1L_{-2}^I 0\rangle$		-0.09006	-0.10418
$c_1L_{-2}^R 0\rangle$		0.02750	0.02643
$c_{-1} \epsilon\rangle$			± 0.02764
$c_1L_{-2}^I \epsilon\rangle$			± 0.02178
$c_1L_{-2}^R \epsilon\rangle$			± 0.00915

Level	$2\pi^2\mathcal{V}(\Psi)$	n_{Ψ}^1	n_{Ψ}^{ϵ}	n_{Ψ}^{σ}
2	0.74917	0.73370	0.89339	± 0.73942
4	0.72656	0.72213	0.48762	± 0.77824
6	0.71933	0.71585	0.72112	± 0.80182
8	0.71596	0.71401	0.62984	± 0.81011
10	0.71404	0.71216	0.70480	± 0.81679
12	0.71280	0.71154	0.66492	± 0.82018
14	0.71193	0.71065	0.70130	± 0.82331
16	0.71129	0.71035	0.67919	± 0.82517
18	0.71080	0.70983	0.70060	± 0.82699
20	0.71043	0.70978	0.69080	± 0.82815
∞	0.70663	0.70688	0.70433	± 0.83935
Expected	0.70711	0.70711	0.70711	± 0.84090

Solution first found by M. Rapčák for his bachelor thesis.

Positive energy solutions on the 1-brane

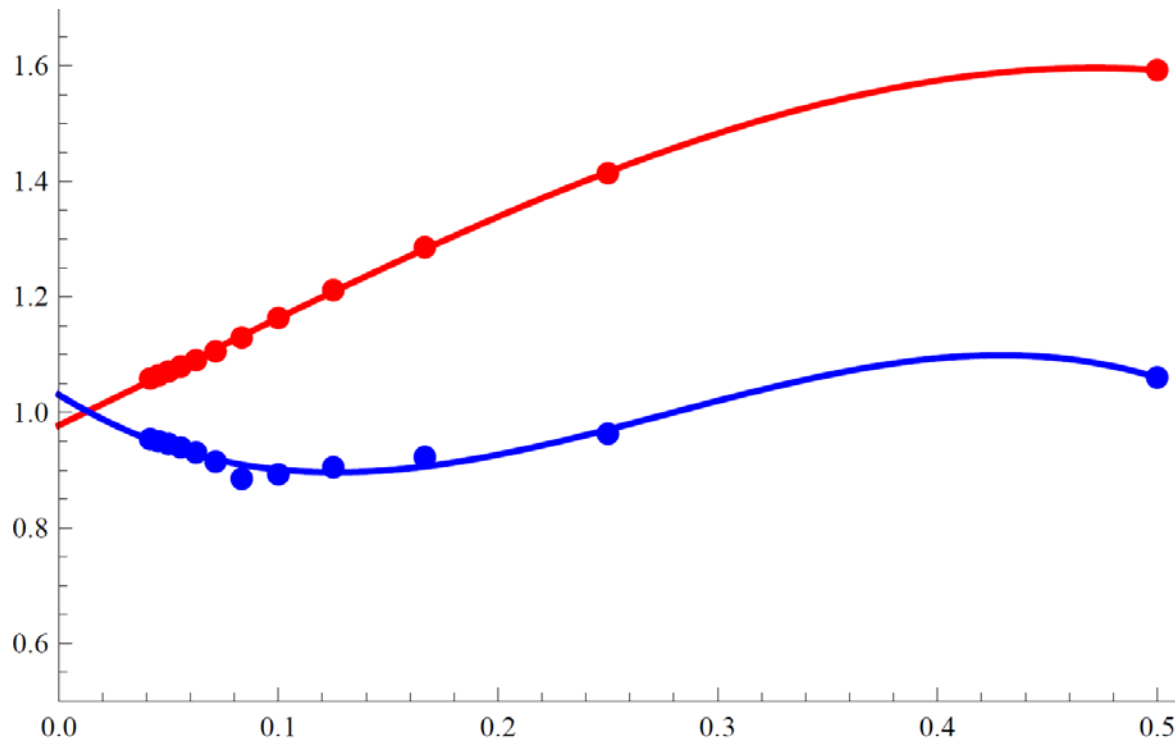
- A real surprise awaited us on the Ising 1-brane. Starting with a complex solution at level 2 we found a real solution at level 14 and higher! And with positive energy !

L	Energy	n_{ψ}^{\parallel}	n_{ψ}^{ϵ}	n_{ψ}^{σ}	Im/Re
2	$1.59267 + 0.726878i$	$1.06048 - 0.184547i$	$-9.73471 - 5.23904i$	$-0.343579 - 0.970819i$	0.788398
4	$1.41414 + 0.201521i$	$0.962899 - 0.142672i$	$-0.66854 + 1.99191i$	$-0.369755 - 0.564227i$	0.438384
6	$1.28579 + 0.0766818i$	$0.922618 - 0.113783i$	$-3.86207 - 0.373757i$	$-0.389334 - 0.394362i$	0.307463
8	$1.2116 + 0.0305389i$	$0.904803 - 0.0868545i$	$-0.575138 + 0.822662i$	$-0.372165 - 0.281935i$	0.221002
10	$1.16345 + 0.0100715i$	$0.892563 - 0.0617353i$	$-2.48552 + 0.0026107i$	$-0.376292 - 0.192323i$	0.152223
12	$1.12943 + 0.0012257i$	$0.885097 - 0.0310941i$	$-0.569512 + 0.245609i$	$-0.368913 - 0.093986i$	0.0748655
14	1.10568	0.914693	-1.93951	-0.266065	0
16	1.09045	0.930444	-0.950873	-0.206326	0
18	1.07936	0.939178	-1.69824	-0.174965	0
20	1.07084	0.945384	-1.04849	-0.15003	0
22	1.06405	0.949943	-1.55407	-0.13398	0
24	1.0585	0.953671	-1.08669	-0.119182	0
Exp.	1	1	-1	0	

Solution first found by M. Kudrna for his Ph.D. thesis.

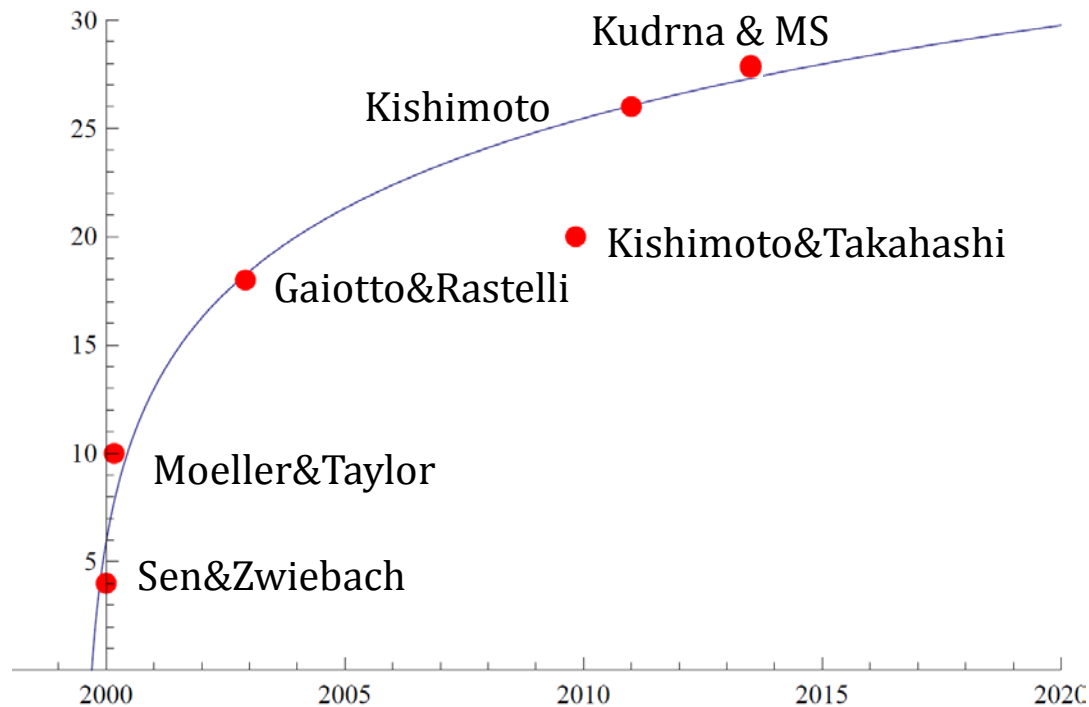
Positive energy solutions on the 1-brane

- Cubic extrapolations of **energy** and **Ellwood invariant** (boundary entropy) to infinite level



Future of level truncation

- Hopefully soon level 30 should be reached



Required tools:

- universal basis
- conservation laws
- C++
- SU(1,1) singlet basis
- parallelism
- we use **metacentrum**
- ???

N.B.: Level 30 is interesting, as we should see the oscillation for the tachyon vacuum energy predicted by Gaiotto and Rastelli.

OSFT Conclusions

- High level numerical computations in OSFT have the potential to discover new boundary states (e.g. they could have predicted existence of fractional D-branes) .
- The key tool for physical identification are the boundary states obtained from the **generalized Ellwood invariants**.
- First well behaved **positive energy solution** discovered ! (describing σ -brane on a 1-brane, or perhaps double branes)
- We are coming to an era of possible computer exploration of the OSFT landscape – stay tuned!