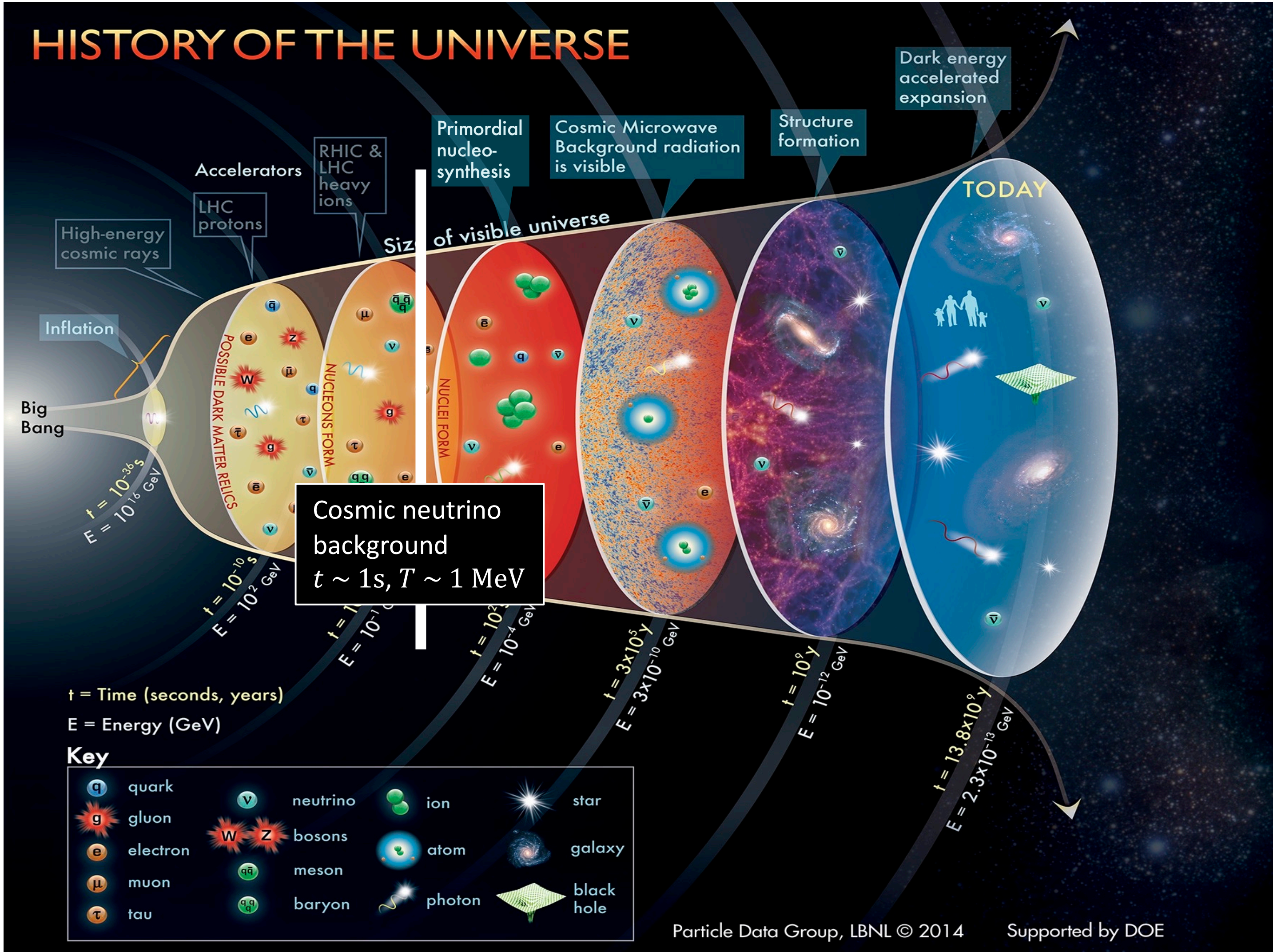
Constraining the neutrino lifetime with precision cosmology

Barenboim, Chen, Hannestad, Oldengott, Tram & Y[3]W, *JCAP* 03 (2021) 087 [arXiv:2011.01502 [astro-ph.CO]]
Chen, Oldengott, Pierobon & Y[3]W, *EPJC* 82 (2022) 7, 640 [arXiv:2203.09075 [hep-ph]]

Yvonne Y. Y. Wong, UNSW Sydney

FZU Prague, January 18, 2024


HISTORY OF THE UNIVERSE
Dark energy accelerated expansion
Structure formation
Cosmic Microwave Background radiation is visible
Primordial nucleo-synthesis
RHIC & LHC heavy ions
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POSSIBLE DARK MATTER RELICS
NUCLEONS FORM
NUCLEI FORM
Cosmic neutrino background
$t \sim 1\text{s}, T \sim 1\ \text{MeV}$
$t = 10^{-36}$ s
$E = 10^{16}$ GeV
$t = 10^{-10}$ s
$E = 10^{2}$ GeV
$t = 3\text{x}10^{5}$ y
$E = 3\text{x}10^{-10}$ GeV
$t = 10^{9}$ y
$E = 10^{-12}$ GeV
$t = 13.8\text{x}10^{9}$ y
$E = 2.3\text{x}10^{-13}$ GeV
t = Time (seconds, years)
E = Energy (GeV)
Key
quark
gluon
electron
muon
tau
neutrino
bosons
meson
baryon
ion
atom
photon
star
galaxy
black hole
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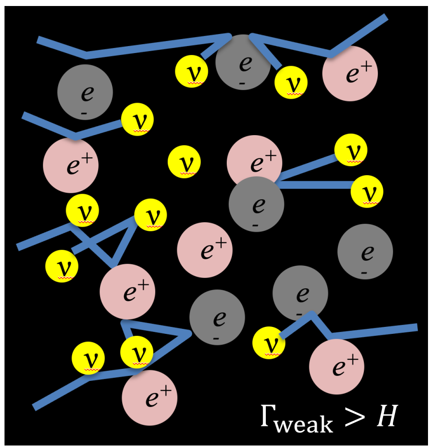
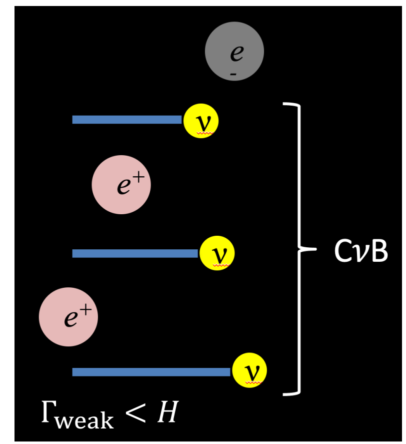
Interaction rate: $\Gamma_{\text{weak}} \sim G_F^2 T^5$

Expansion rate: $H \sim M_{\text{pl}}^{-2} T^2$

Formation of the CνB…

The CνB is formed when neutrinos decouple from the cosmic plasma.





Neutrinos "free-stream" to infinity.

Above $T \sim 1$ MeV, even the Weak Interaction occurs efficiently enough to allow neutrinos to scatter off e^+e^- and other neutrinos, and attain thermodynamic equilibrium.

Below $T \sim 1$ MeV, expansion dilutes plasma, and reduces interaction rate: the universe becomes transparent to neutrinos.

Standard-Model predictions... 1/2

Neutrino decoupling happens at $T \sim O(1)$ MeV, which is determined by the Weak Interaction.

- Given sub-eV neutrino masses, the CνB is ultra-relativistic at decoupling.
- After $e^+e^- \to \gamma\gamma$ (at $T \sim 0.5$ MeV):
  - **Fermi-Dirac distribution with temperature**: $T_{\mathrm{C\nu B}} = \left(\frac{4}{11}\right)^{1/3} T_{\mathrm{CMB}}$
  - **Energy density** per flavour: $\rho_{\mathrm{C\nu}B} = \frac{7}{8}\frac{\pi^2}{15} T_{\mathrm{C\nu B}}^4 = \frac{7}{8}\left(\frac{4}{11}\right)^{4/3} \rho_{\mathrm{CMB}}$

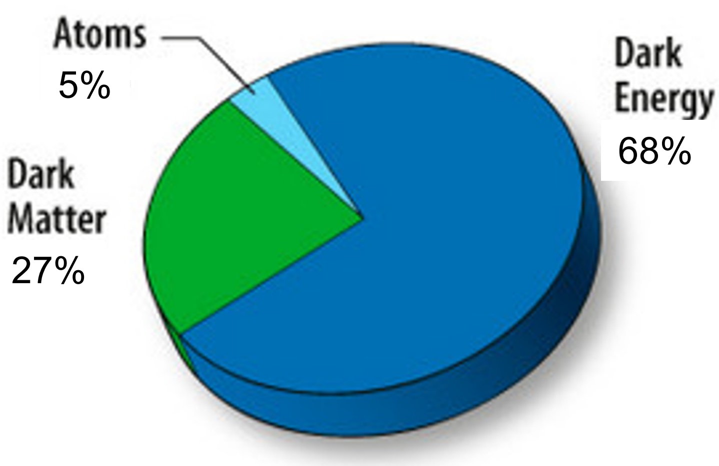
    Neutrinos + antineutrinos

High-redshift prediction
($m_\nu \lesssim T/\mathrm{MeV} \lesssim 0.5$)

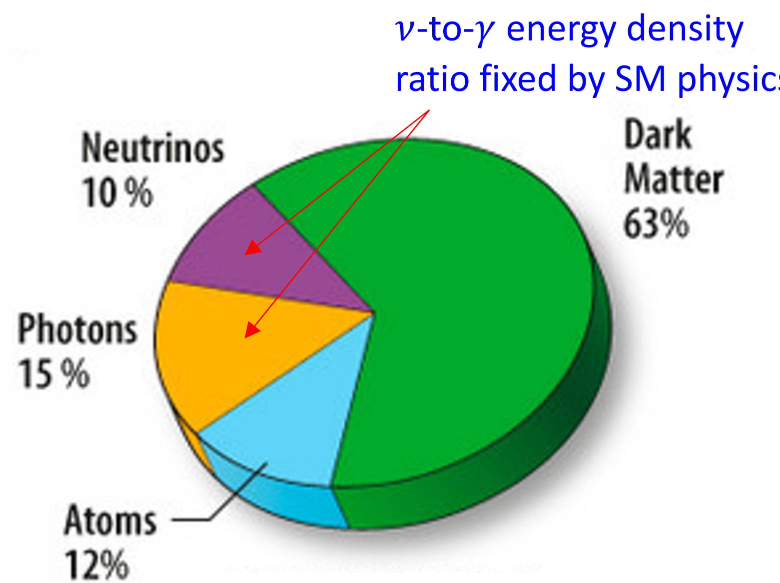
$$\frac{3\rho_\nu}{\rho_\gamma} \sim 0.68$$

The concordance flat ΛCDM model…

The simplest model capable of explaining most observational data.



Composition today



13.4 billion years ago
(at photon decoupling)

Standard-Model predictions... 2/2

As the universe expands and cools, at some point the CνB temperature will drop below the **neutrino rest mass** m_ν.

- At $T \ll m_\nu$, the CνB is non-relativistic, with **energy density** given by

$$\rho_{\mathrm{C\nu B}} \simeq m_\nu n_{\mathrm{C\nu B}}$$

Number density

Normalised to the present-day critical density

$$\Omega_\nu = \sum \frac{m_\nu}{94\ h^2\ \mathrm{eV}}$$

$h \sim 0.7$

- Expectation from **laboratory limits**:

From neutrino oscillations

$$\min \sum m_\nu = 0.06\ \mathrm{eV}$$

$$0.1\% < \Omega_\nu < 5\%$$

From β-decay end-point measurements

$$m_e \equiv \left(\sum_i |U_{ei}|^2 m_i^2\right)^{\frac{1}{2}} < 0.7\ \mathrm{eV}$$

Aker et al. [KATRIN] 2022

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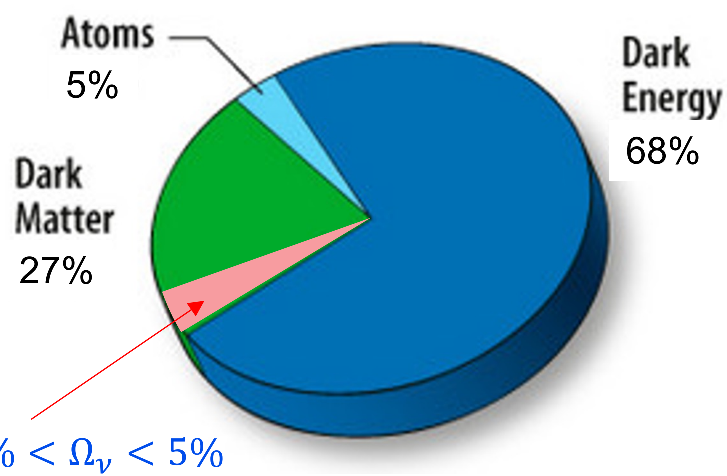
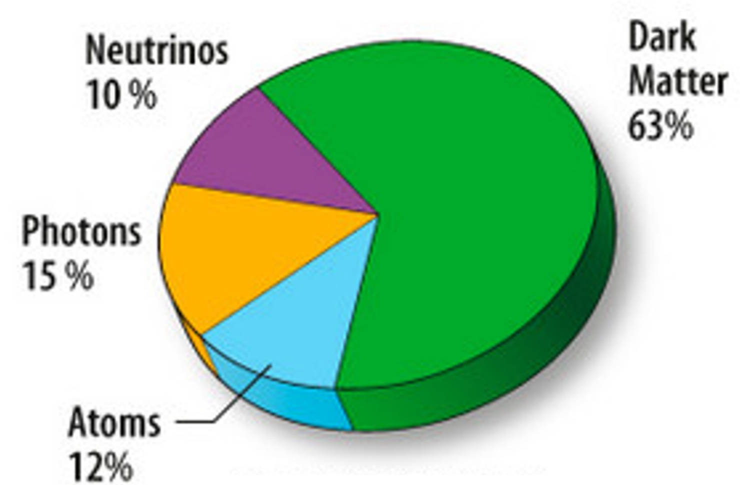


A $0.1\% < \Omega_\nu < 5\%$ subdominant dark matter component in SM neutrinos

Composition today



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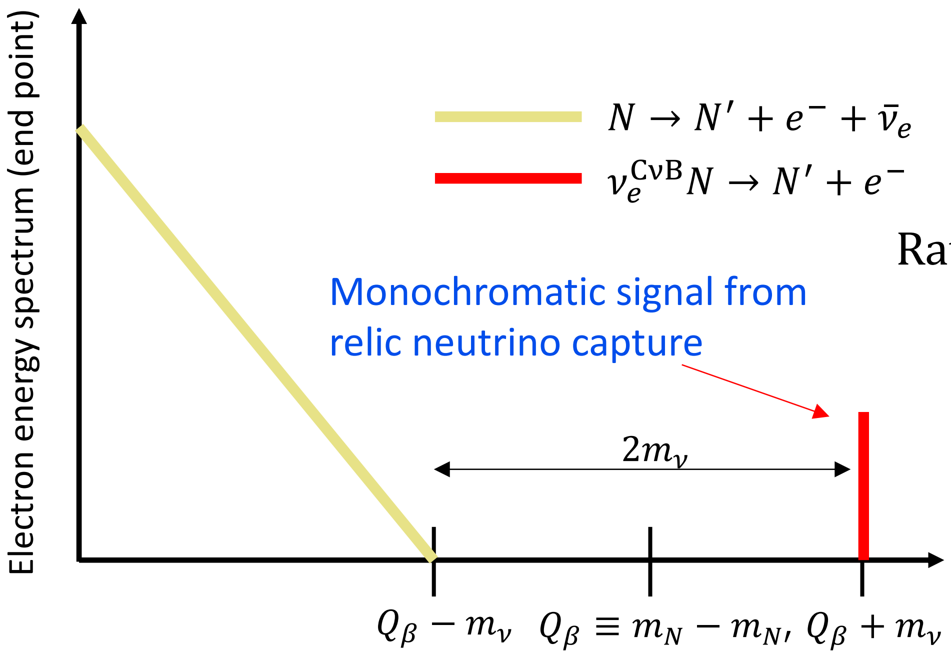
Can we detect the CνB?

Weinberg 1962
Cocco, Mangano & Messina 2007

The best idea: **neutrino capture by β-decaying nucleus**

- Feature beyond the end-point spectrum → tied to neutrino mass detection

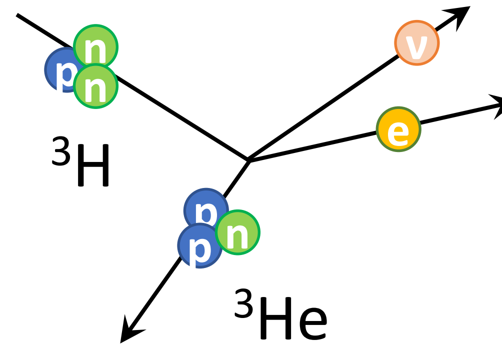


Local neutrino overdensity

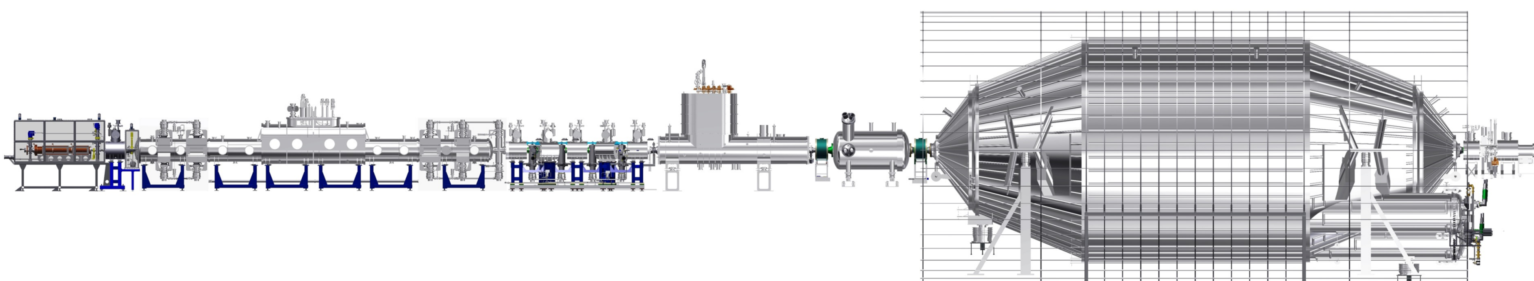
$$\mathrm{Rate} \sim 7.5 \left(\frac{n_\nu}{\bar{n}_\nu}\right) /\mathrm{year} \,/(100\ \mathrm{g\ tritium})$$

Neutrino capture with KATRIN?

Karlsruhe Tritium Neutrino Experiment



KATRIN source ~0.1 mg tritium



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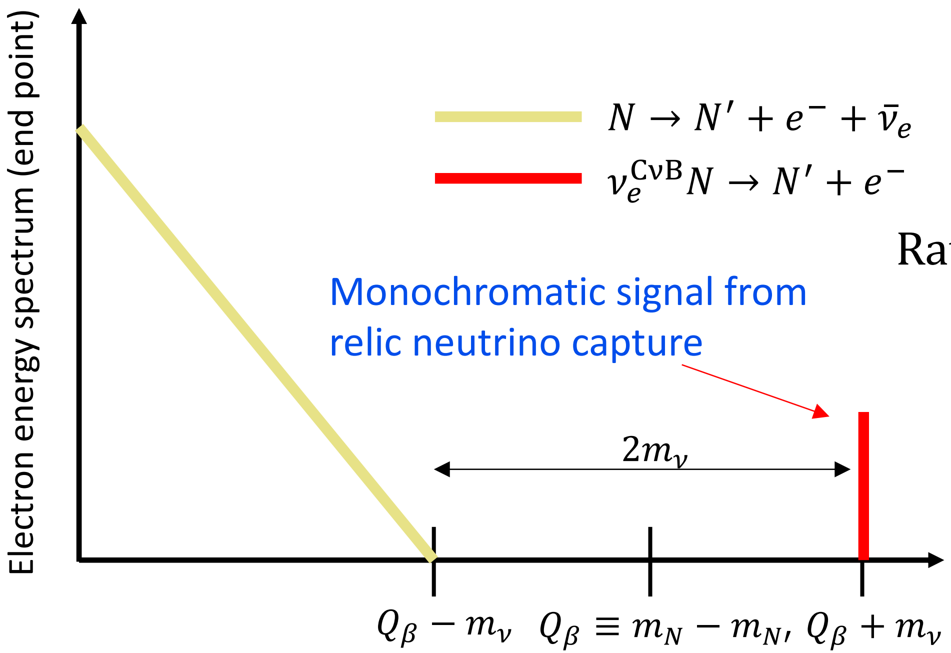

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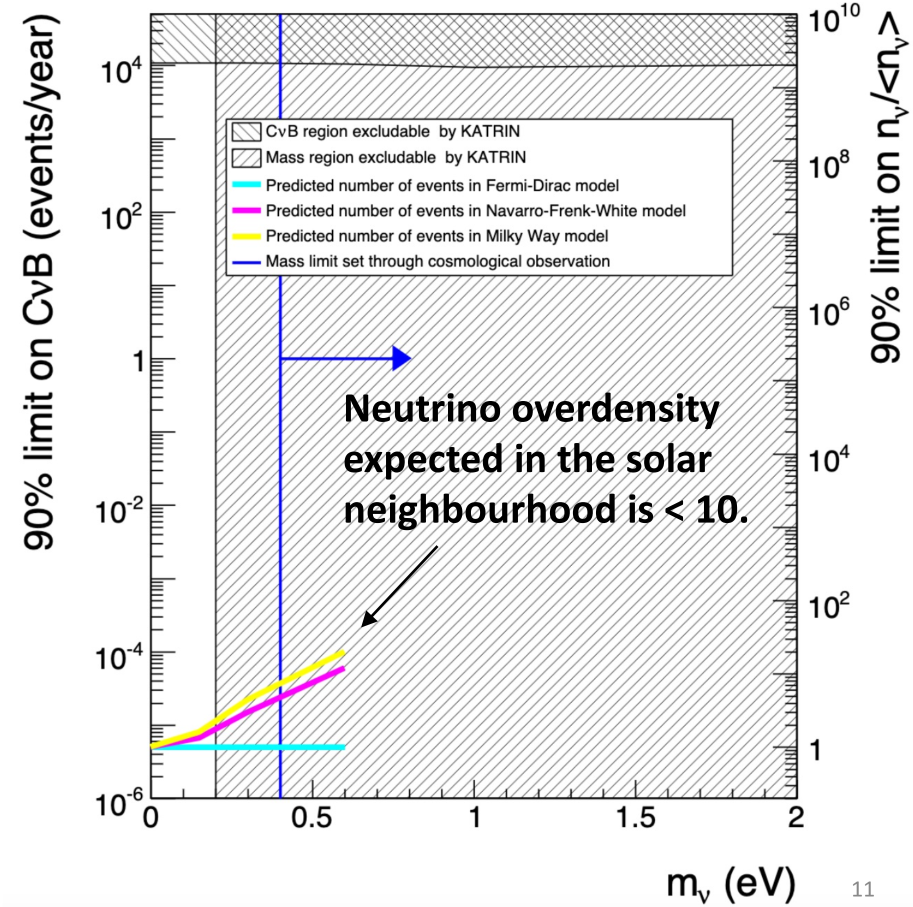
Neutrino capture with KATRIN?

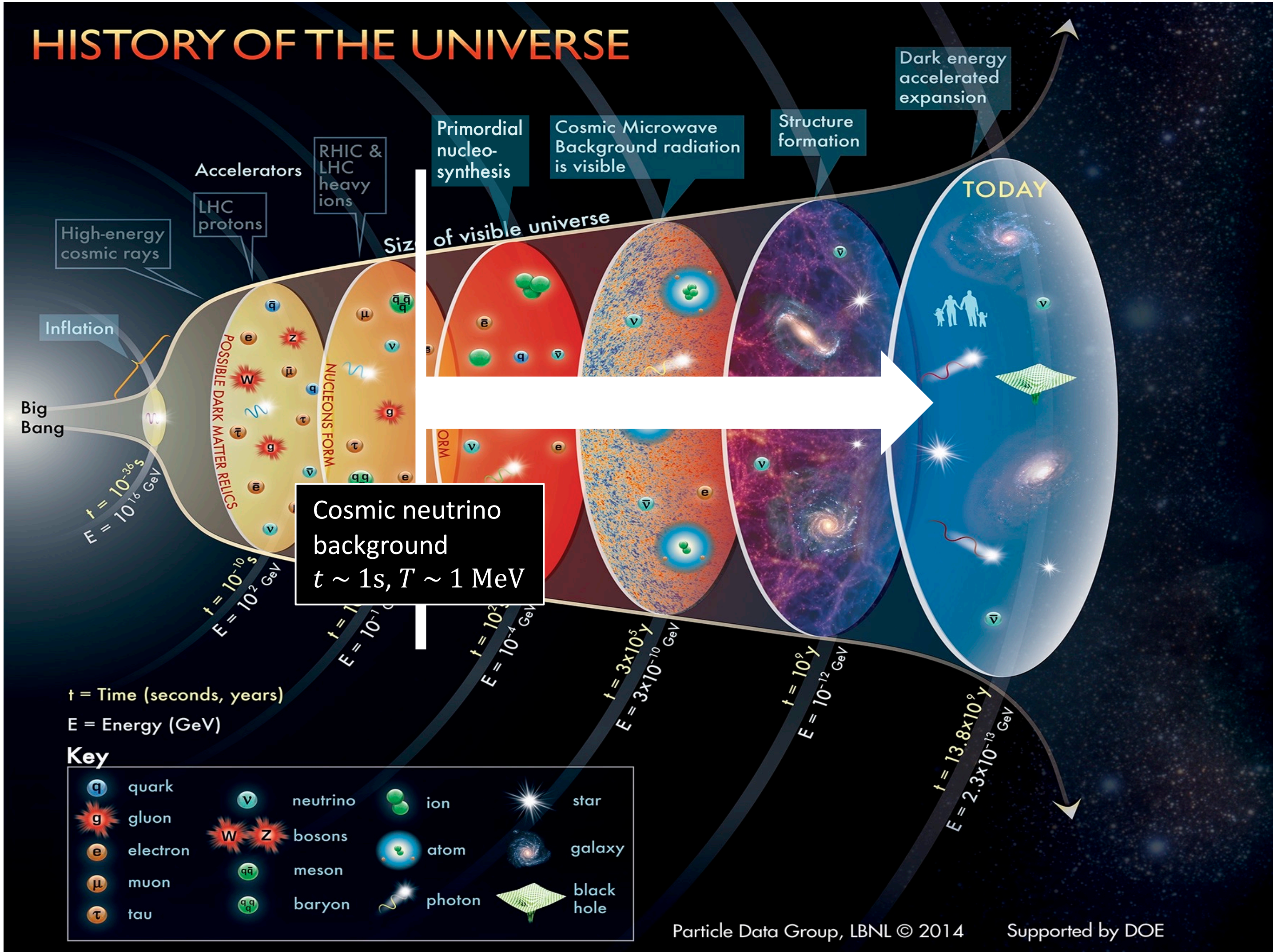
Kaboth, Formaggio & Monreal 2010
also Aker at al. [KATRIN] 2022

A 10^9 local overdensity of neutrinos is required in a 3-year run for a 90% C.L. detection of the CνB by KATRIN.

→ Direct detection of the CνB in the laboratory is not going to happen any time soon…

But there are other ways to infer the presence of the CνB and to constrain its properties.

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gluon
electron
muon
tau
neutrino
bosons
meson
baryon
ion
atom
photon
star
galaxy
black hole
Particle Data Group, LBNL © 2014
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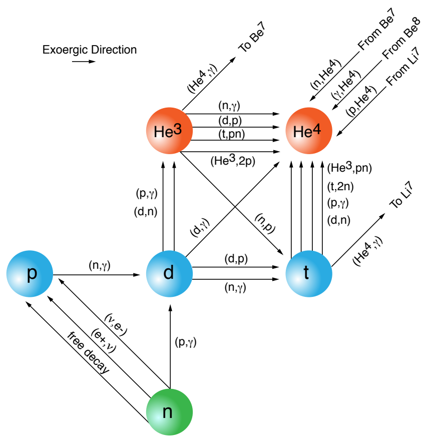
Indirect evidence for the CνB...

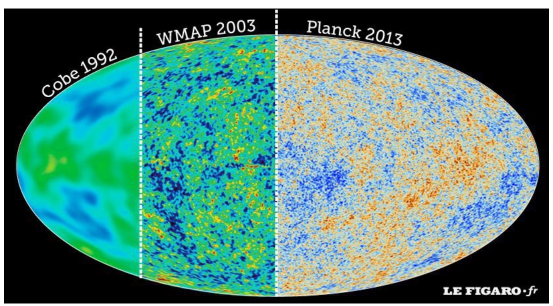
We can look at the imprints of the CνB on cosmological observables:

- To see if they are consistent with expectations
- And if so, to constrain (non-standard) neutrino properties

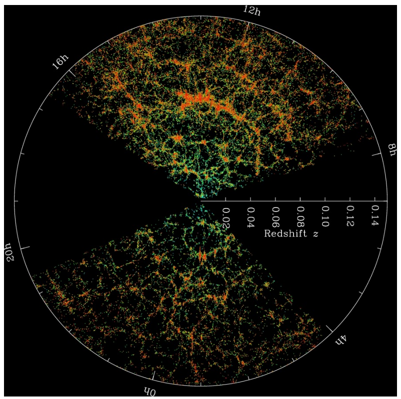
Light element abundances



CMB anisotropies



Large-scale matter distribution

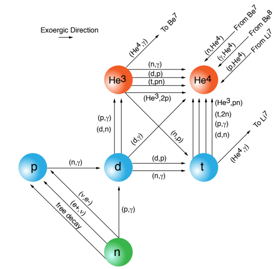


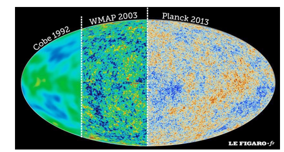
Looking for the CνB in precision cosmological observables…

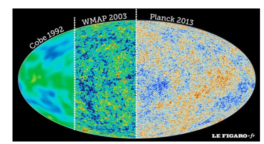
What can cosmological observables tell us?

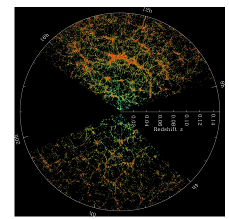
They may look different, but ultimately what each observable can tell us about the universe are:

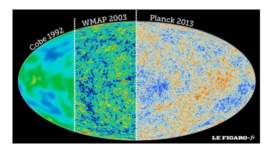
- Universal expansion rate at different times
  - How much matter, radiation, "in-between" (e.g., neutrinos), vacuum energy, etc.
- Growth of fluctuations under gravity
  - Kinematic properties and interactions of the various types of stuff in the universe; good for neutrino physics
- Distance measurements
  - Spatial geometry, dark energy; not directly relevant for neutrino physics but has indirect effects on inference

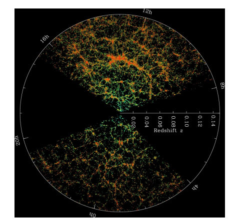






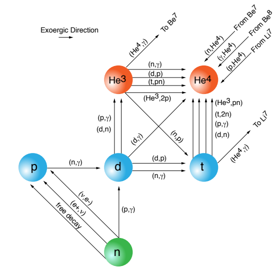


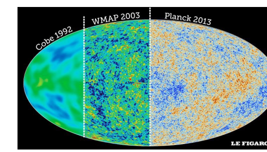


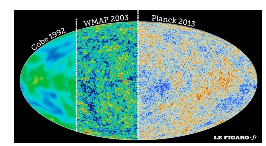


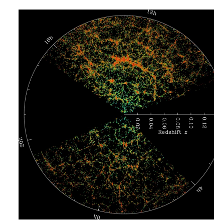
Testing CνB prediction against observations…

- Universal expansion rate at different times
  - Testing the radiation energy density at the nucleosynthesis and CMB epochs.
- Growth of fluctuations under gravity
  - Testing the "free-streaming" nature (or lack thereof) of the non-photon radiation content at the CMB epoch.









Testing the radiation energy density via the expansion rate…

CνB & the expansion rate...

The Hubble expansion rate depends on the energy content of the universe:

$$H^2(a(t)) = H_0^2(\Omega_m a^{-3} + \Omega_r a^{-4} + \Omega_\Lambda + \Omega_k a^{-2} + \cdots)$$

Scale factor

Matter

Radiation

Cosmological constant

Spatial curvature

Neutrinos = radiation at early times
= matter at late times

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Standard cosmology at pre-CMB times

Do current cosmological observations prefer $N_{\rm eff}^{\rm SM}$?

$$\rho_{\rm CMB} + \sum \rho_{\rm C\nu B} = \left[1 + N_{\rm eff} \times \frac{7}{8}\left(\frac{4}{11}\right)^{4/3}\right] \rho_{\rm CMB}$$

$$N_{\rm eff}^{\rm SM} = 3.0440 \pm 0.0002$$

Bennett et al, 2020, 2021;
Froustey, Pitrou & Volpe, 2020

For 3 SM families, includes m_e/T corrections, non-instantaneous decoupling, finite-temperature QED, and neutrino oscillations.

Nucleosynthesis & N_{eff}...

Constraining N_{eff} with the primordial elemental abundances has a long history.

Volume 66B, number 2 PHYSICS LETTERS 17 January 1977

COSMOLOGICAL LIMITS TO THE NUMBER OF MASSIVE LEPTONS

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National Radio Astronomy Observatory[1] *and Yale University*[2], *USA*

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University of Chicago, Enrico Fermi Institute (LASR), 933 E 56th, Chicago, Ill. 60637, USA

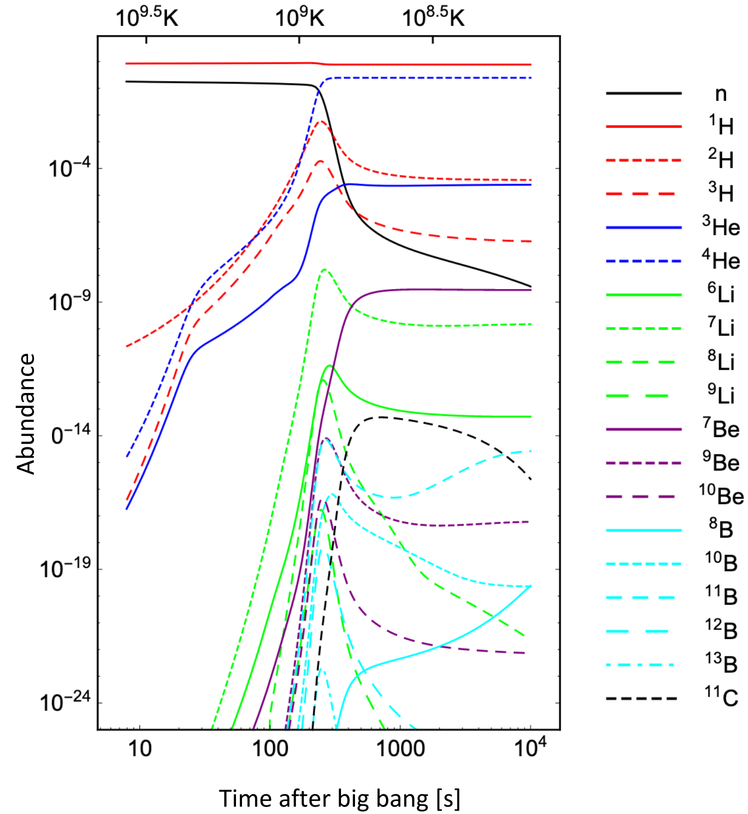
James E. GUNN
University of Chicago and California Institute of Technology[2], *USA*

Received 29 November 1976

If massive leptons exist, their associated neutrinos would have been copiously produced in the early stages of the hot, big bang cosmology. These neutrinos would have contributed to the total energy density and would have had the effect of speeding up the expansion of the universe. The effect of the speed-up on primordial nucleosynthesis is to produce a higher abundance of ^{4}He. It is shown that observational limits to the primordial abundance of ^{4}He lead to the constraint that the total number of types of heavy lepton must be less than or equal to 5.

$N_{\mathrm{eff}} < 5$

Pitrou, Coc, Uzan & Vangioni 2018



How much of these elements is produced depends on how fast the universe expands.

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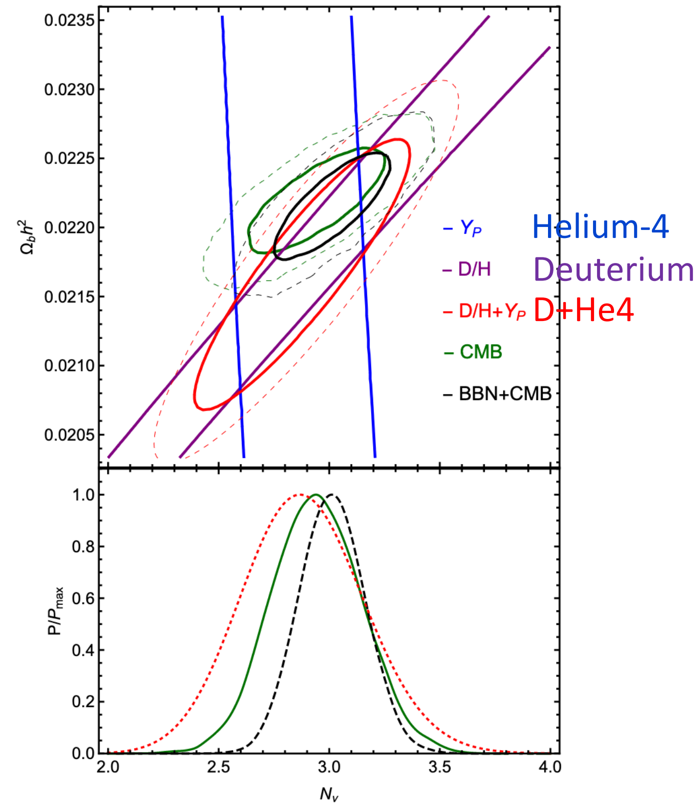
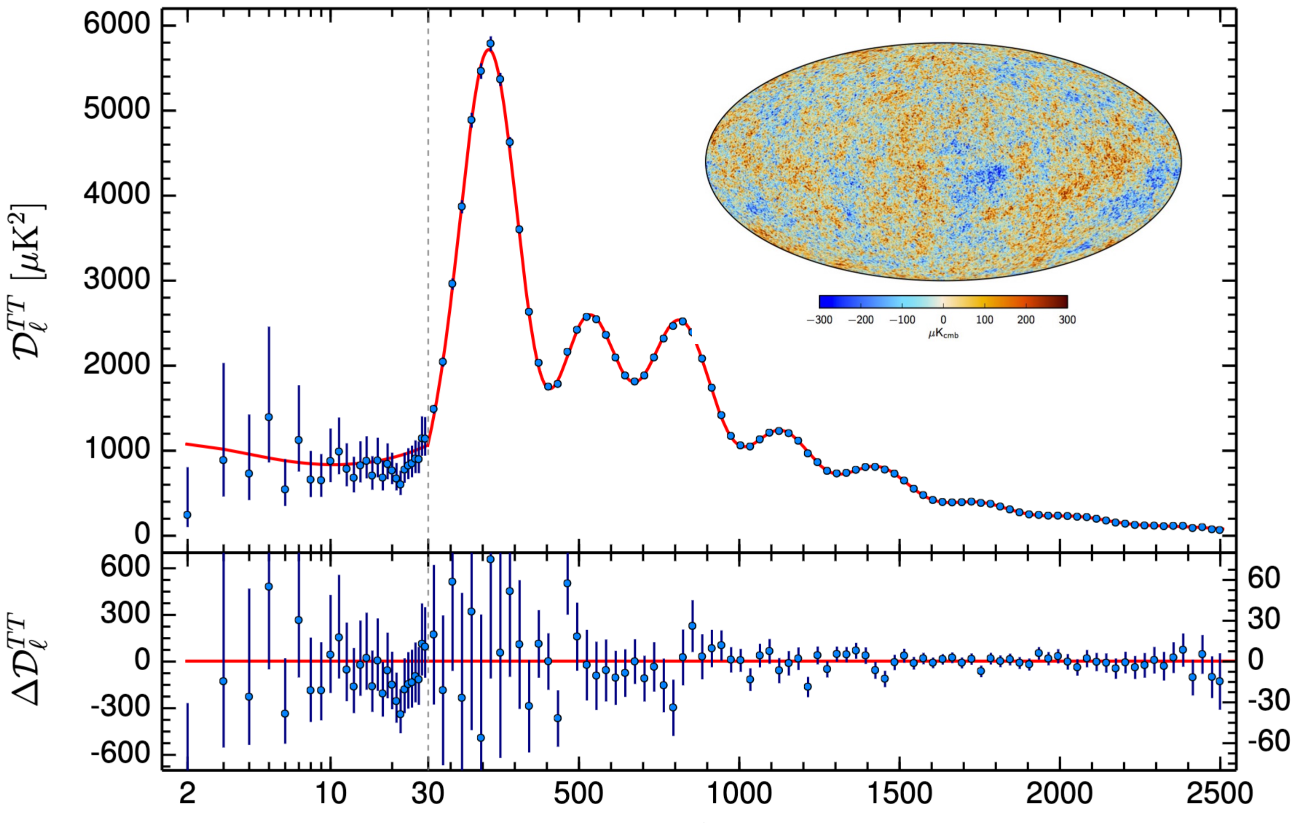


$N_{\text{eff}} = 2.88 \pm 0.27$ (68% CL)

Neutrino energy density is consistent with SM prediction $N_{\text{eff}} = 3$; it's definitely not $N_{\text{eff}} = 0$.

CMB anisotropies & N_{eff}…

Temperature auto-correlation
(TT power spectrum)



CMB anisotropies & N_{eff}...

Varying N_{eff} changes the universal expansion rate at photon decoupling.

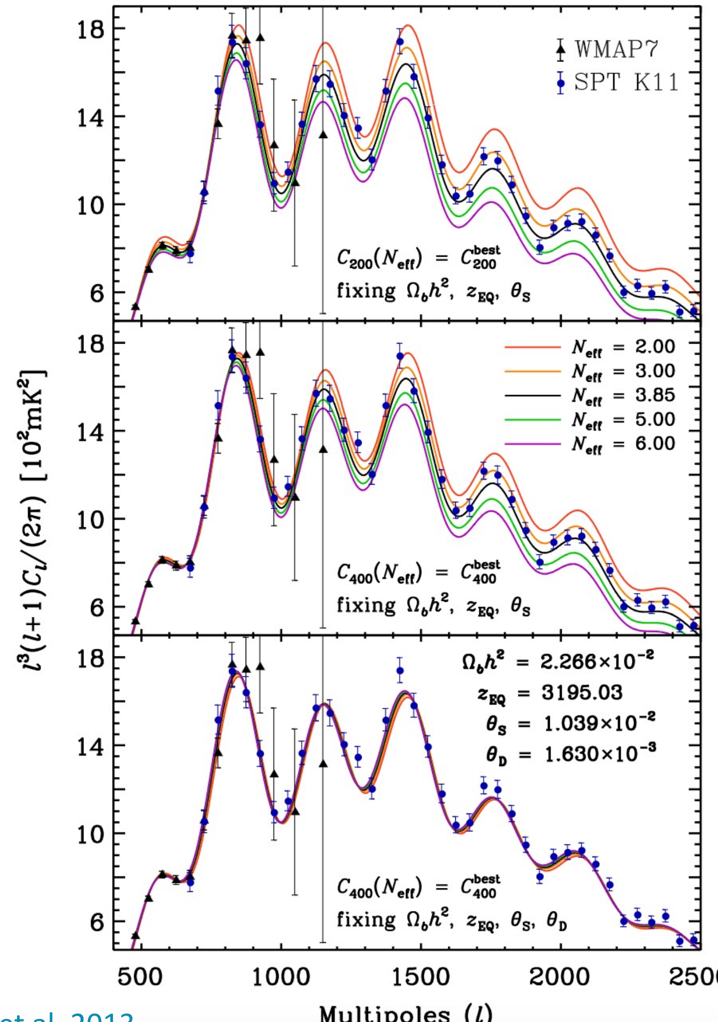
- Irreducible signature in the damping tail of the TT power spectrum
- Current constraint from Planck:

$$N_{\text{eff}} = 2.99 \pm 0.34 \ (95\% \ \text{CL})$$

TTTEEE+lowE+lensing+BAO;
7-parameters

Aghanim et al. [Planck] 2021

Inferred neutrino energy density consistent with SM prediction of $N_{\text{eff}} = 3.044$ to 10%.



Hou et al. 2013

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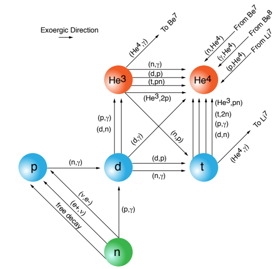
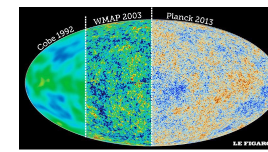
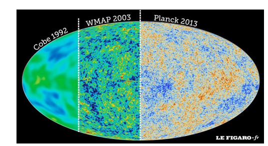
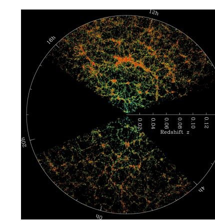








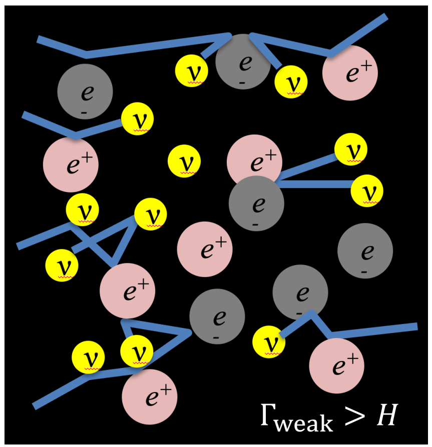
Testing free-streaming…

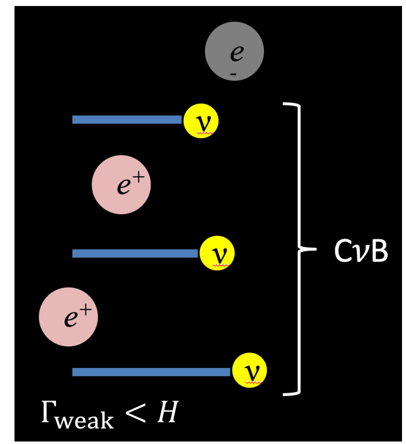
Interaction rate: $\Gamma_{\text{weak}} \sim G_F^2 T^5$

Expansion rate: $H \sim M_{\text{pl}}^{-2} T^2$

Formation of the CνB...

The CνB is formed when neutrinos decouple from the cosmic plasma.





Neutrinos "free-stream" to infinity.

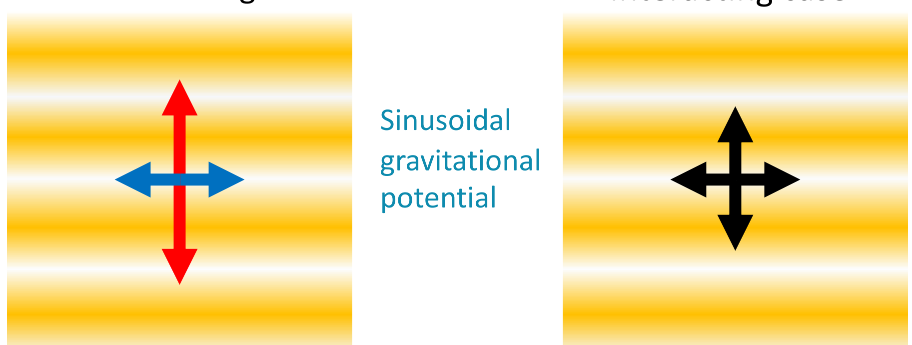
Above $T \sim 1$ MeV, even the Weak Interaction occurs efficiently enough to allow neutrinos to scatter off e^+e^- and other neutrinos, and attain thermodynamic equilibrium.

Below $T \sim 1$ MeV, expansion dilutes plasma, and reduces interaction rate: the universe becomes transparent to neutrinos.

Free-streaming in inhomogeneities…

Standard Model neutrinos free-stream after decoupling.

- Free-streaming in a spatially inhomogeneous background induces shear stress (or momentum anisotropy).
- Conversely, interactions transfer momentum and, if sufficiently efficient, can wipe to out shear stress.



Why is this interesting for the CMB?

Neutrino shear stress (or lack thereof) leaves distinct imprints on the spacetime metric perturbations at CMB formation times.

Conformal Newtonian gauge

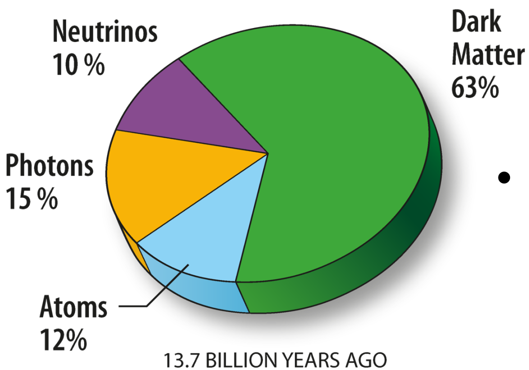
Scale factor

$$ds^2 = a^2(\tau)[-(1+2\psi)d\tau^2 + (1-2\phi)dx^i dx_i]$$

where $$k^2(\phi - \psi) = 12\pi G a^2(\bar{\rho} + \bar{P})\sigma$$

Shear stress
At CMB times, mainly from ultra-relativistic neutrinos and photons.

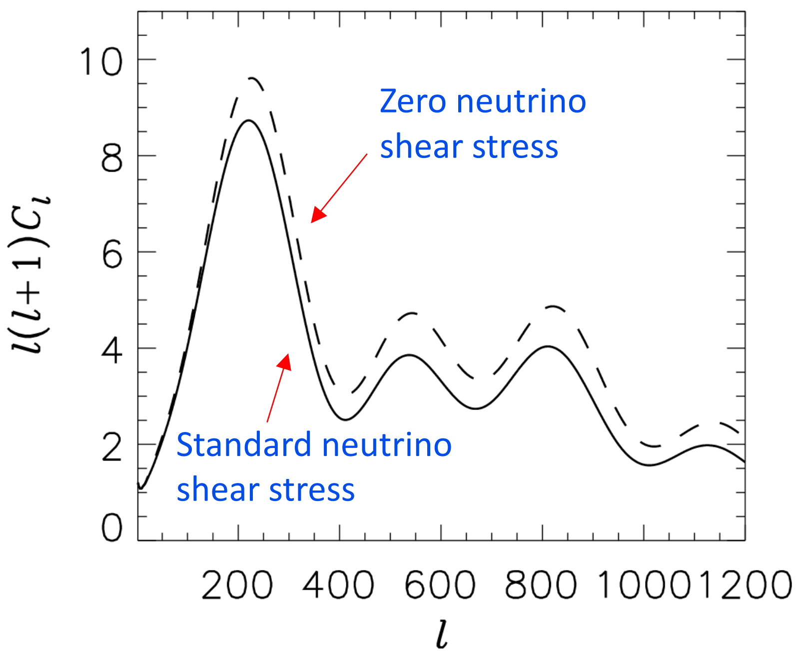
Mean energy density & pressure



- The **CMB temperature fluctuations** respond to changes in $(\phi - \psi)$

  → Observable effects in the **CMB TT power spectrum**

Neutrino shear & the CMB TT spectrum…

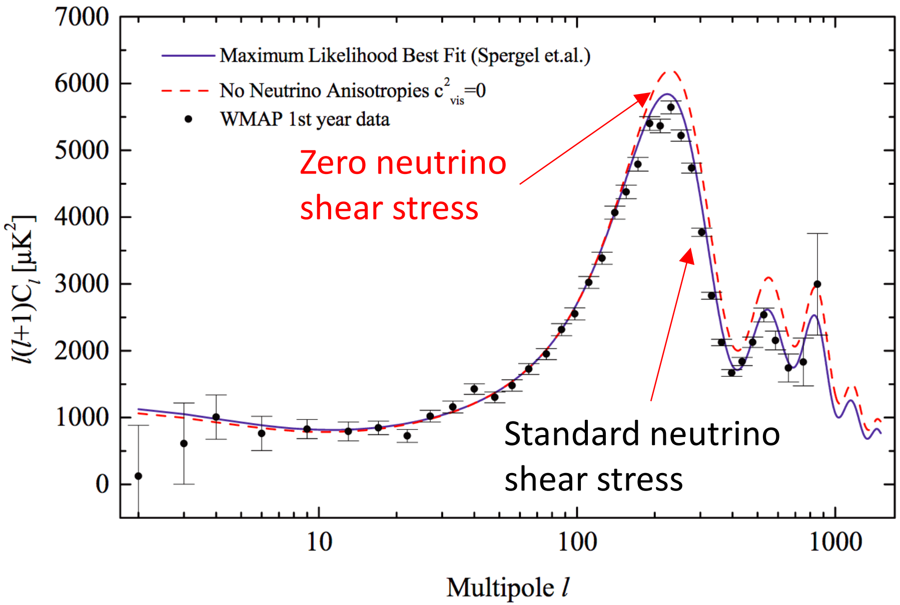


Hannestad 2005

Removing neutrino shear stress enhances power at multipoles $\ell \gtrsim 200$.

- Effect is mildly degenerate with the primordial fluctuation amplitude and spectral tilt.
- But even with **WMAP-1st year data**, it was already possible to exclude zero neutrino shear stress at $\gtrsim 2\sigma$.

Neutrino shear & the CMB TT spectrum…



Melchiorri & Trotta 2005

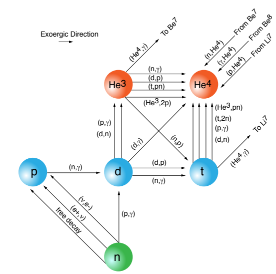
Removing neutrino shear stress enhances power at multipoles $\ell \gtrsim 200$.

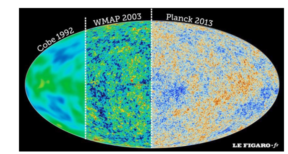
- Effect is mildly degenerate with the primordial fluctuation amplitude and spectral tilt.
- But even with **WMAP-1st year data**, it was already possible to exclude zero neutrino shear stress at $\gtrsim 2\sigma$.

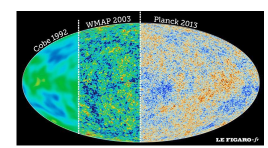
Half-time conclusions…

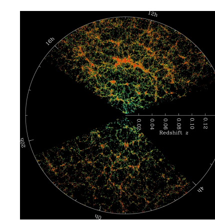
Precision cosmological observations are **consistent with the existence of a standard-model neutrino background** up to ~400,000 year post big bang.

- Universal expansion rate at different times
  - Testing the radiation energy density at the nucleosynthesis and CMB epochs.
- Growth of fluctuations under gravity
  - Testing the “free-streaming” nature (or lack thereof) of the non-photon radiation content at the CMB epoch.









It sounds boring. But the fact that everything is so consistent also means that we can use this fact to place constraints on non-standard neutrino interactions, including neutrino decay.

Constraining invisible neutrino decay and the neutrino lifetime…

Invisible neutrino decay…

Invisible here means the decay products do **not** include a photon.

- **SM 1 → 3 decay:** $\nu_j \rightarrow \nu_i \nu_k \bar{\nu}_k$, but the rate is suppressed by $m_\nu^6 E$.
  → For sub-eV neutrino masses, the neutrino lifetime would be $> 10^{10}$ longer than the present age of the universe, i.e., not very interesting. Bahcall, Cabibbo & Yahil 1972

- **Beyond SM**: generically one could consider

$$\nu_H \rightarrow \nu_l + \phi$$

SM neutrinos (sub-eV masses)

Some almost massless boson (scalar, pseudo-scalar, vector)

  - More freedom with the coupling strength and hence lifetime.
  - Predicted by a many extensions to the SM (mostly linked to neutrino mass generation or dark matter). Gelmini & Roncadelli 1981; Chikashige, Mohapatra & Peccei 1981; Schechter & Valle 1982; Dror 2020; Ekhterachian, Hook, Kumar & Tsai 2021; etc.

Isotropisation timescale...

Given the decay process, to use free-streaming requirements to constrain invisible neutrino decay we need to determine the rate at which neutrino shear stress is lost due to the interaction.

→ What is the **isotropisation timescale** given a specific interaction?

Tracking neutrino perturbations...

The standard approach is to use the **relativistic Boltzmann equation** to describe the neutrino phase space distribution $f_i(x^\mu, P^i)$.

Liouville operator

$$P^\mu \frac{\partial f_i}{\partial x^\mu} - \Gamma^\nu_{\rho\sigma} P^\rho P^\sigma \frac{\partial f_i}{\partial P^\nu} = 0$$

Gravitational effects

- **Split** into $f_i(x^\mu, P^i) = \bar{f}_i(x^0, |P^i|) + F_i(x^\mu, P^i)$
- **Linearise** and go to Fourier space $x^i \leftrightarrow k^i$
- **Decompose** $F_i(x^o, k^i, P^i)$ into a Legendre series in $k \cdot P$.

Integrate in momentum:
$\ell = 0$ → density and pressure perturbations
$\ell = 1$ → velocity perturbations
$\ell \geq 2$ → anisotropies

Ma & Bertschinger 1995

Adding a short-range particle interaction...

To describe a **short-range interaction**, add a collision integral to the RHS of the relativistic Boltzmann equation for $f_i(x^\mu, P^i)$.

Liouville operator

$$P^\mu \frac{\partial f_i}{\partial x^\mu} - \Gamma^\nu_{\rho\sigma} P^\rho P^\sigma \frac{\partial f_i}{\partial P^\nu} = m_i \left(\frac{\mathrm{d} f_i}{\mathrm{d}\sigma}\right)_C$$

Collision integral

Gravitational effects

- **Split** into $f_i(x^\mu, P^i) = \bar{f}_i(x^0, |P^i|) + F_i(x^\mu, P^i)$
- **Linearise** and go to Fourier space $x^i \leftrightarrow k^i$
- **Decompose** $F_i(x^o, k^i, P^i)$ into a Legendre series in $k \cdot P$.

Integrate in momentum:
$\ell = 0 \rightarrow$ density and pressure perturbations
$\ell = 1 \rightarrow$ velocity perturbations
$\ell \geq 2 \rightarrow$ anisotropies

Ma & Bertschinger 1995

Collision integral and the isotropisation rate…

Given an interaction Lagrangian, the collision integral for $f_i(x^\mu, P^i)$ is

$$m_i \left(\frac{\mathrm{d}f_i}{\mathrm{d}\sigma}\right)_C = \frac{1}{2}\left(\prod_j^N \int g_j \frac{\mathrm{d}^3\mathbf{n}_j}{(2\pi)^3 2E_j(\mathbf{n}_j)}\right)\left(\prod_k^M \int g_k \frac{\mathrm{d}^3\mathbf{n}_k}{(2\pi)^3 2E_k(\mathbf{n}_k)}\right)$$
$$\times (2\pi)^4 \delta_D^{(4)}\left(p + \sum_j^N n_j - \sum_k^M n'_k\right) |\mathcal{M}_{i+j_1+\cdots+j_N \leftrightarrow k_1+\cdots+k_M}|^2$$
$$\times [f_{k_1}\cdots f_{k_N}(1\pm f_i)(1\pm f_{j_1})\cdots(1\pm f_{j_N}) - f_i f_{j_1}\cdots f_{j_N}(1\pm f_{k_1})\cdots(1\pm f_{k_M})]$$

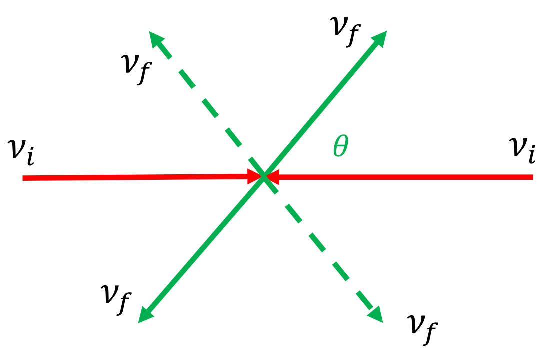
- **To compute the isotropisation rate**, follow the previous procedure of linearisation and decomposition into a Legendre series.

→ The damping rate of the quadrupole ($\ell = 2$) moment represents the lowest-order isotropisation rate of the neutrino ensemble.

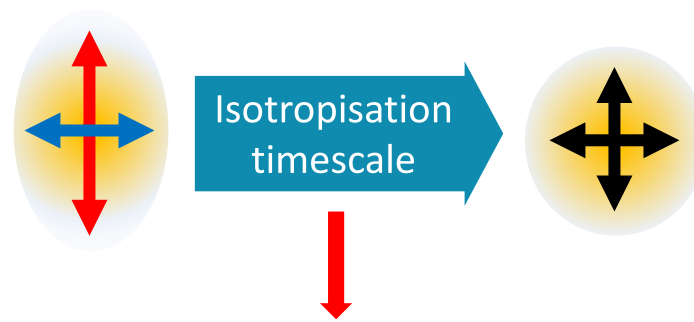
Tedious stuff, but this is really the only correct way to calculate these things, else you can get it very wrong…
However, the result can usually be understood in simple terms. → **Next slide**

Warm-up: Isotropisation from self-interaction...

Consider a 2 → 2 scattering event $\nu_i + \nu_i \rightarrow \nu_f + \nu_f$.



→ Particles in two head-on ν_i beams need only scatter once to transfer their momenta equally in all directions.



$T_{\text{isotropise}} \sim 1/\Gamma_{\text{scattering}}$

Scattering rate

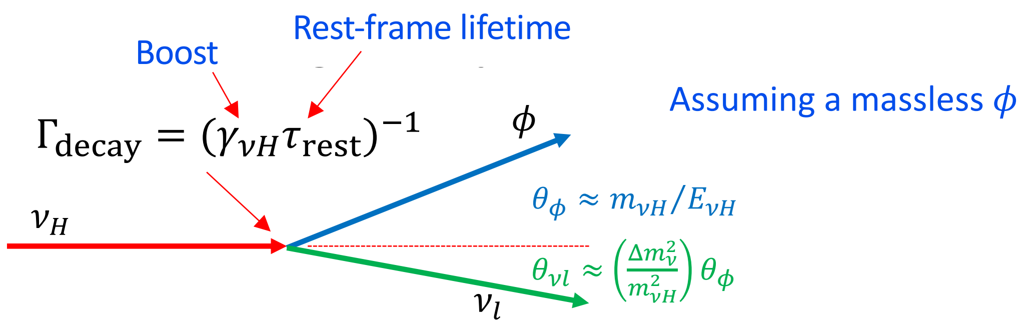
- The probability of ν_f emitted at any angle θ is the same for all $\theta \in [0, \pi]$.

That was easy…. Now let’s try relativistic $1 \to 2$ decay.

Isotropisation from relativistic $1 \rightarrow 2$ decay…

How long does it take $\nu_H \rightarrow \nu_l + \phi$ and its inverse process to wipe out momentum anisotropies? (Hint: it's not the lifetime of ν_H.)

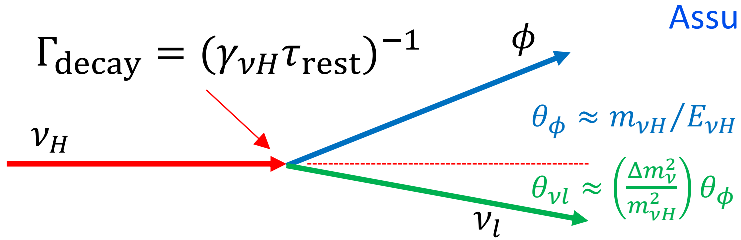
- In relativistic decay, the decay products are **beamed**.



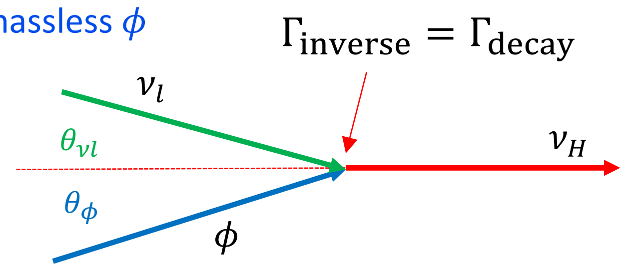
Isotropisation from relativistic $1 \rightarrow 2$ decay…

How long does it take $\nu_H \rightarrow \nu_l + \phi$ and its inverse process to wipe out momentum anisotropies? (Hint: it's not the lifetime of ν_H.)

- In relativistic decay, the decay products are **beamed**.
- Inverse decay also only happens when the daughter particles meet **strict momentum/angular requirements**.



Assuming a massless ϕ



→ Isotropisation is going to take a loooong time compared with the ν_H lifetime.

How long? Part 1

Two works in the 2000s that considered how long it would take relativistic $1 \rightarrow 2$ decay and inverse decay to isotropise a neutrino ensemble.

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CMB signals of neutrino mass generation

Z. Chacko, Lawrence J. Hall, Takemichi Okui, and Steven J. Oliver
Phys. Rev. D **70**, 085008 – Published 12 October 2004

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Constraining invisible neutrino decays with the cosmic microwave background

Steen Hannestad and Georg G. Raffelt
Phys. Rev. D **72**, 103514 – Published 14 November 2005

- **Neither** work actually calculated it… But this is the isotropisation timescale they used:

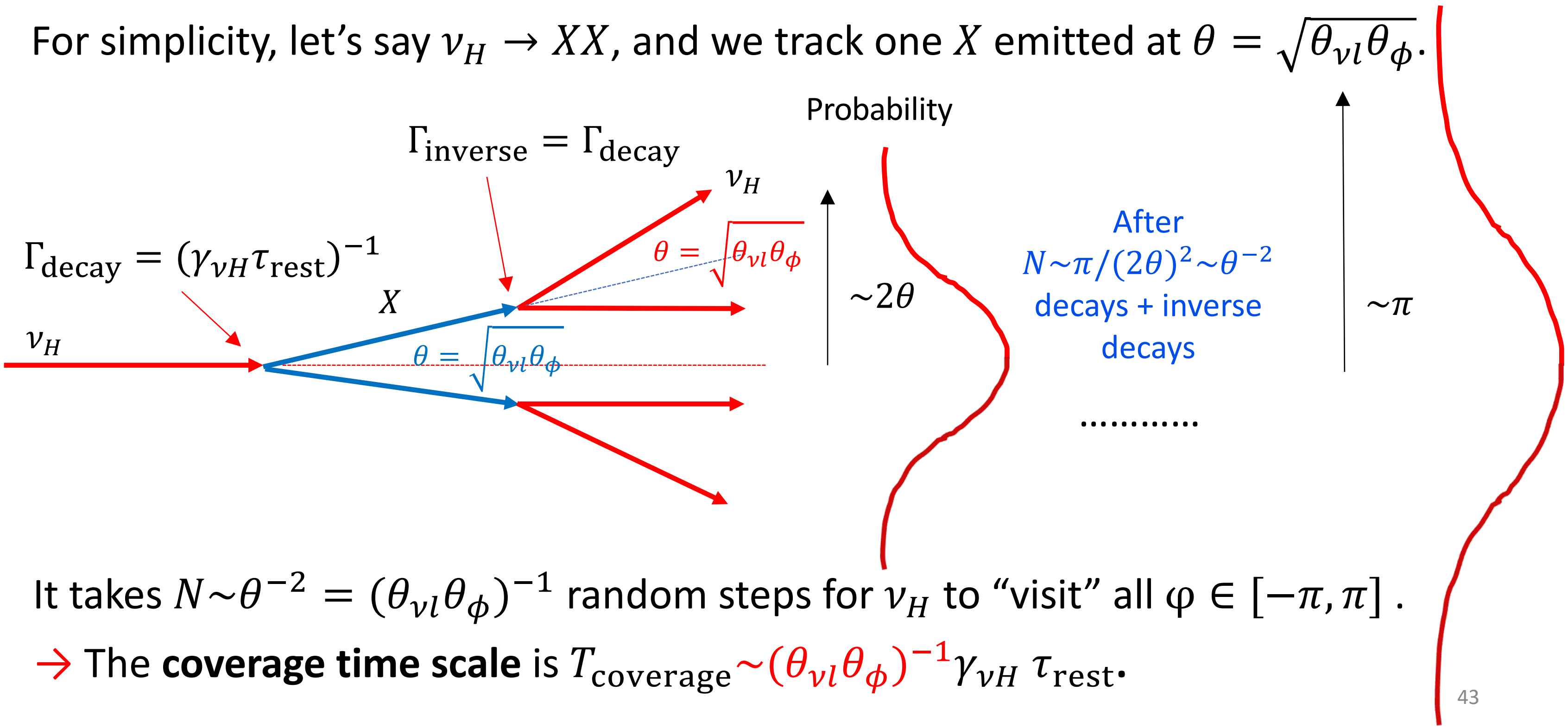
$$T \sim (\theta_{\nu l} \theta_{\phi})^{-1} \gamma_{\nu H}\, \tau_{\text{rest}}$$

- Their argument is as follows.

How long? Part 1

Let's look at what happens to ν_H after one decay and inverse decay.

- For simplicity, let's say $\nu_H \to XX$, and we track one X emitted at $\theta = \sqrt{\theta_{\nu l}\theta_\phi}$.



- It takes $N \sim \theta^{-2} = (\theta_{\nu l}\theta_\phi)^{-1}$ random steps for ν_H to "visit" all $\varphi \in [-\pi, \pi]$.

  → The **coverage time scale** is $T_{\text{coverage}} \sim (\theta_{\nu l}\theta_\phi)^{-1}\gamma_{\nu H}\,\tau_{\text{rest}}$.

How long? Part 1

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- Taking T_{coverage} to be the isotropisation timescale and assuming massless decay products, the free-streaming bound on the $\boldsymbol{\nu_H}$ **rest-frame lifetime** was found to be:

$$\tau_{\text{rest}} \gtrsim 10^9 \left(\frac{m_{\nu H}}{0.05\ \text{eV}}\right)^3 \text{s}$$

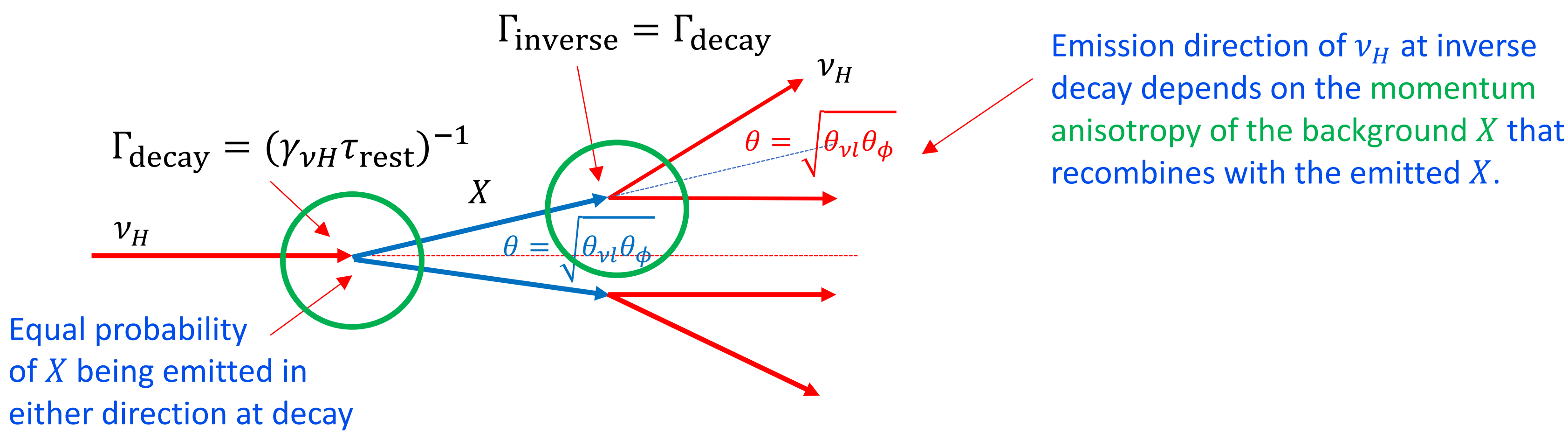
Many updates to the number since (e.g., WMAP to Planck), but no one really questioned the modelling behind this bound in the next 15 years…

Is T_{coverage} the isotropisation time scale?

Barenboim, Chen, Hannestad, Oldengott, Tram & Y[3]W 2021
Chen, Oldengott, Pierobon & Y[3]W 2022

Actually, T_{coverage} is only the first half of the story!

- **It is NOT the isotropisation time scale and here's the reason.**



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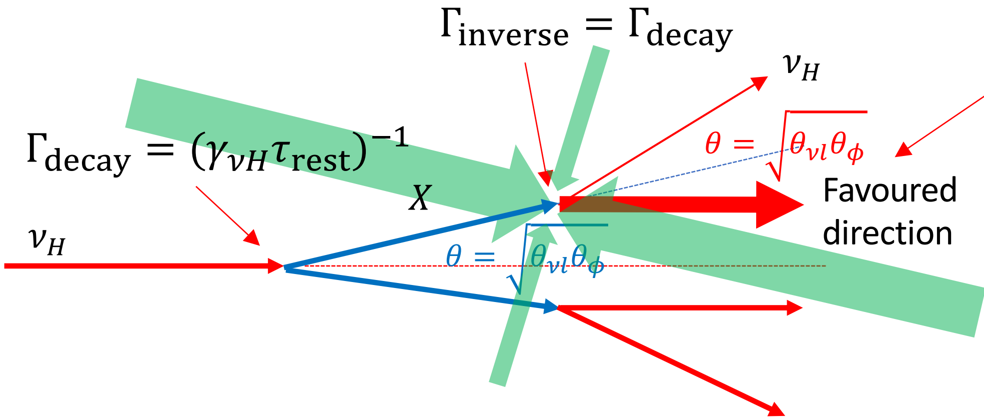

Emission direction of ν_H at inverse decay depends on the momentum anisotropy of the background X that recombines with the emitted X.
→ Random walk of ν_H in θ space is biased towards the anisotropy of X.

Is T_{coverage} the isotropisation time scale?

Barenboim, Chen, Hannestad, Oldengott, Tram & Y³W 2021
Chen, Oldengott, Pierobon & Y³W 2022

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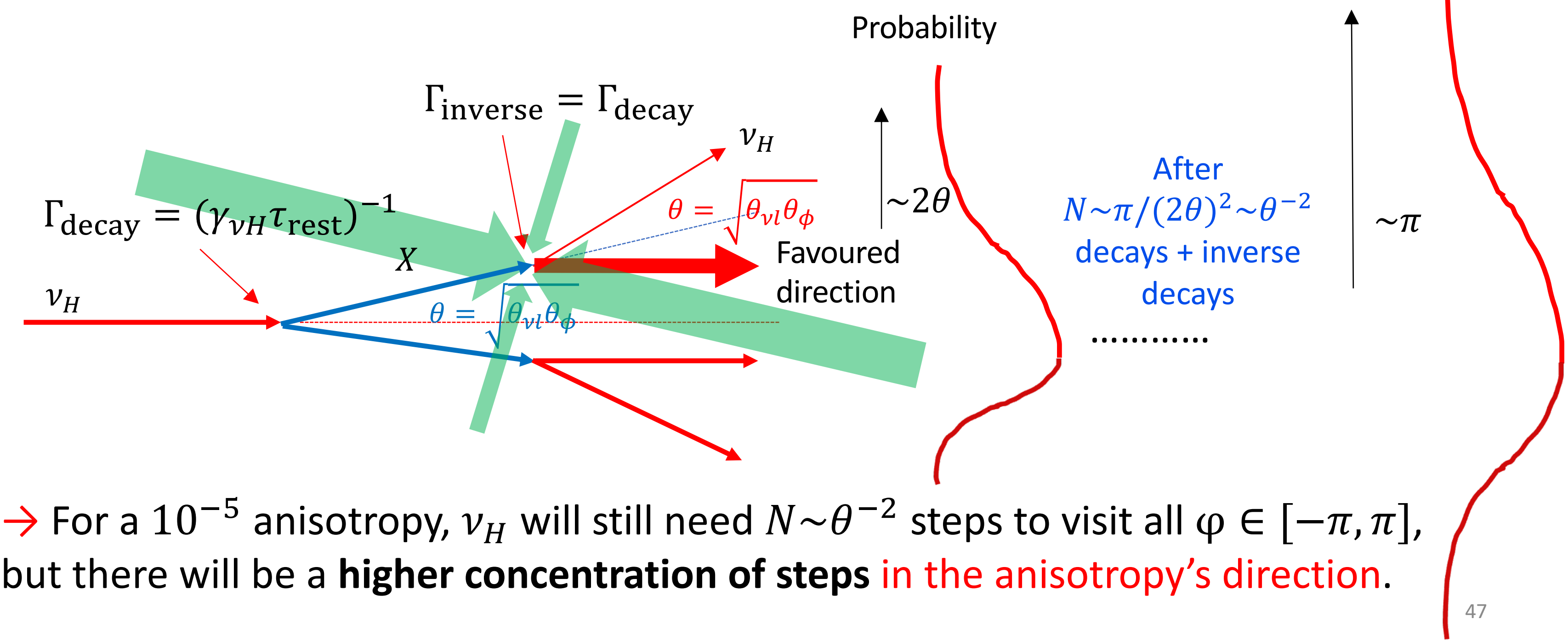
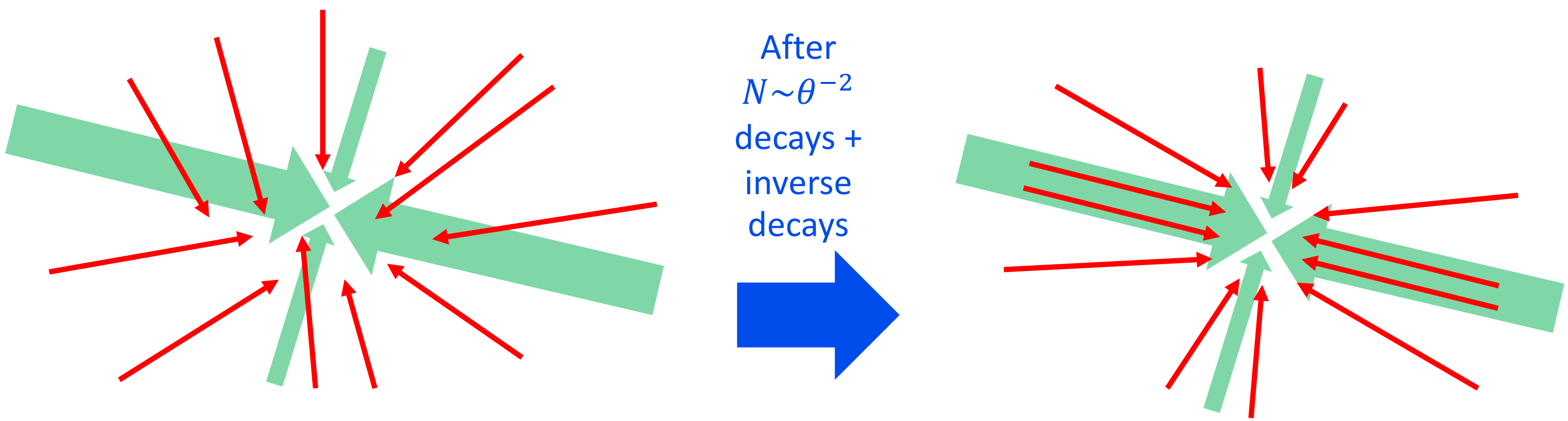


→ For a 10^{-5} anisotropy, ν_H will still need $N \sim \theta^{-2}$ steps to visit all $\varphi \in [-\pi, \pi]$, but there will be a **higher concentration of steps** in the anisotropy's direction.

Is T_{coverage} the isotropisation time scale?

That was for just one particle ν_H.

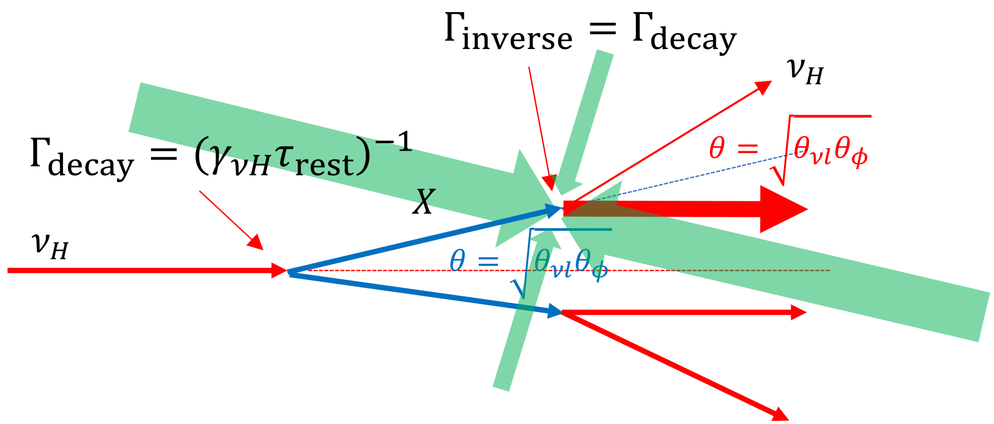
- Suppose now we have a whole ensemble of ν_H's random-walking in the same anisotropic background.



- After T_{coverage}, the ν_H ensemble **will not become isotropic**, but will end up **almost** as anisotropic as the background…

Almost as anisotropic (or how long part 2)…

After one coverage time, the anisotropy of ν_H will be smeared over $\sim\theta = \sqrt{\theta_{\nu l}\theta_\phi}$ relative to the anisotropy of X, because ν_H is **always emitted at an angle $\pm\boldsymbol{\theta}$** relative to X in an inverse decay.

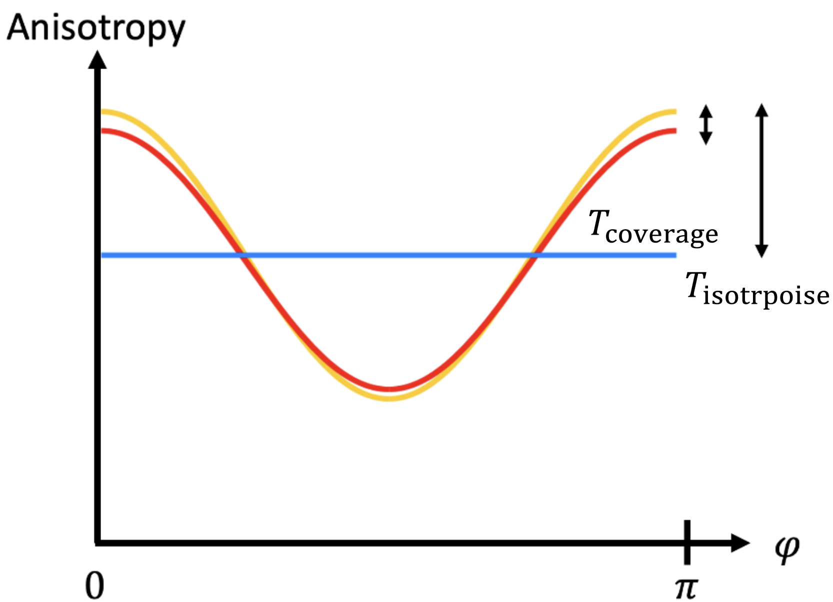


→ Even though total isotropisation of ν_H is not possible after one coverage time, a small amount of anisotropy is inevitably lost as a result.

Almost as anisotropic (or how long part 2)...

Smearing over $\sim\theta$ **reduces the peak anisotropy** after one coverage time by an amount:

$$\text{Peak}_{\text{new}} - \text{Peak}_{\text{old}} \sim O(\theta^2)$$



→ Need to **repeat** coverage $M \sim \theta^{-2} = (\theta_{\nu l}\theta_{\phi})^{-1}$ times to completely rid the (ν_H, ν_l, ϕ) ensemble of anisotropy.

→ **True isotropisation time scale**:

$$T_{\text{isotropise}} \sim (\theta_{\phi}\theta_{\nu l})^{-1} T_{\text{coverage}} \sim (\theta_{\phi}\theta_{\nu l})^{-2} \gamma_{\nu H}\, \tau_{\text{rest}}$$

OK, that was hand-waving. But…

The isotropisation rate is calculable…

Given an interaction Lagrangian, the collision integral for $f_i(x^\mu, P^i)$ is

$$m_i \left(\frac{\mathrm{d}f_i}{\mathrm{d}\sigma}\right)_C = \frac{1}{2}\left(\prod_j^N \int g_j \frac{\mathrm{d}^3\mathbf{n}_j}{(2\pi)^3 2E_j(\mathbf{n}_j)}\right)\left(\prod_k^M \int g_k \frac{\mathrm{d}^3\mathbf{n}_k}{(2\pi)^3 2E_k(\mathbf{n}_k)}\right)$$
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$$\times [f_{k_1}\cdots f_{k_N}(1\pm f_i)(1\pm f_{j_1})\cdots(1\pm f_{j_N}) - f_i f_{j_1}\cdots f_{j_N}(1\pm f_{k_1})\cdots(1\pm f_{k_M})]$$

- **To compute the isotropisation rate**, follow the previous procedure of linearisation and decomposition into a Legendre series.

→ The damping rate of the quadrupole ($\ell = 2$) moment represents the lowest-order isotropisation rate of the neutrino ensemble.

Tedious stuff, but this is really the only correct way to calculate these things, else you can get it very wrong…

The isotropisation rate is calculable…

With some reasonable approximations (e.g., separation of scales), we have calculated the damping rate of the ℓth neutrino kinetic moment from relativistic $\nu_H \to \nu_l + \phi$ and its inverse process:

$\sim(\theta_\phi \theta_{\nu l})^2$

T_0 = comoving neutrino temperature

$$\frac{d\mathcal{F}_{\ell\geq 2}}{dt} = -\alpha_\ell \tilde{\Gamma}_{\rm dec} \left(\frac{a m_{\nu H}}{T_0}\right)^4 \Phi\left(\frac{m_{\nu l}}{m_{\nu H}}\right) \mathfrak{F}\left(\frac{a m_{\nu H}}{T_0}\right) \mathcal{F}_{\ell \geq 2}$$

O(1) prefactor

Boosted decay rate, $\sim(\gamma_{\nu H} \tau_{\rm rest})^{-1}$

Phase space factor $\sim \frac{1}{3}\left(\frac{\Delta m_\nu^2}{m_{\nu H}^2}\right)^2$

Bonus: Relativistic to non-relativistic transition:
~ 1-10 when relativistic;
drops to 0 when non-relativistic

Barenboim, Chen, Hannestad, Oldengott, Tram & Y[3]W 2021
Chen, Oldengott, Pierobon & Y[3]W 2022

The isotropisation rate is calculable…

With some reasonable approximations (e.g., separation of scales), we have calculated the damping rate of the ℓth neutrino kinetic moment from relativistic $\nu_H \to \nu_l + \phi$ and its inverse process:

It's model-independent; any dependence on the interaction structure is contained in $\Gamma_{\rm dec}$.

$\sim(\theta_\phi \theta_{\nu l})^2$

T_0 = comoving neutrino temperature

$$\frac{d\mathcal{F}_{\ell\geq 2}}{dt} = -\alpha_\ell \tilde{\Gamma}_{\rm dec} \left(\frac{a m_{\nu H}}{T_0}\right)^4 \Phi\left(\frac{m_{\nu l}}{m_{\nu H}}\right) \mathfrak{F}\left(\frac{a m_{\nu H}}{T_0}\right) \mathcal{F}_{\ell\geq 2}$$

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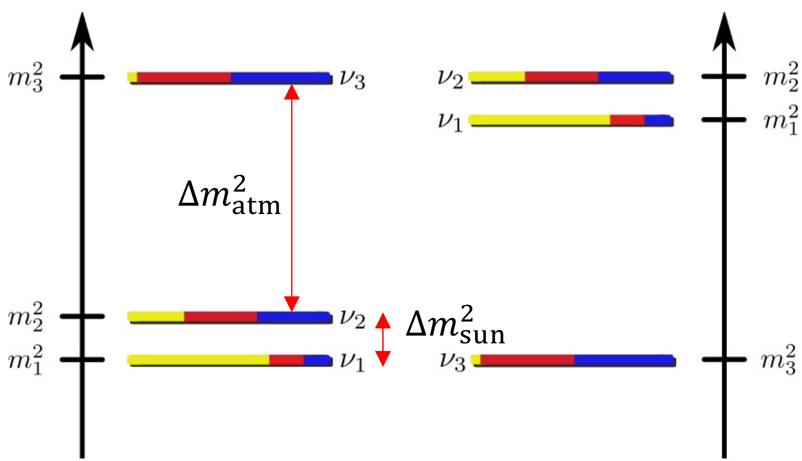
Barenboim, Chen, Hannestad, Oldengott, Tram & Y³W 2021
Chen, Oldengott, Pierobon & Y³W 2022

Revised constraints on the neutrino lifetime…

Decay scenarios…

Global neutrino oscillation data currently point to two possible orderings of neutrino masses → **several possible decay/free-streaming patterns**.



Normal mass ordering | Inverted mass ordering

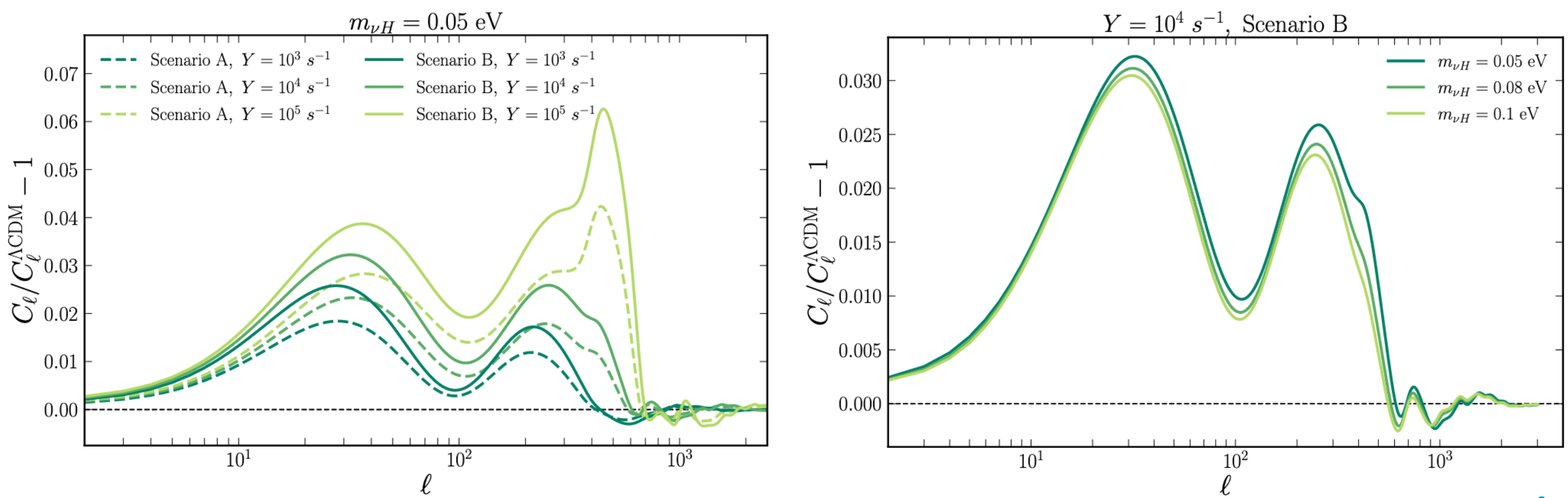
		FS	Decay	Gap	Min $m^2_{\nu H}$
Scenario A: one decay channel					
A1	NO	ν_1	$\nu_3 \to \nu_2$	$\Delta m^2_{32}\|_{\rm N}$	$\|\Delta m^2_{31}\|_{\rm N}$
		ν_2	$\nu_3 \to \nu_1$	$\|\Delta m^2_{31}\|_{\rm N}$	
	IO	ν_2	$\nu_1 \to \nu_3$	$\|\Delta m^2_{31}\|_{\rm I}$	$\|\Delta m^2_{31}\|_{\rm I}$
		ν_1	$\nu_2 \to \nu_3$	$\Delta m^2_{23}\|_{\rm I}$	$\Delta m^2_{23}\|_{\rm I}$
A2	NO	ν_3	$\nu_2 \to \nu_1$	Δm^2_{21}	Δm^2_{21}
A3	IO	ν_3	$\nu_2 \to \nu_1$		$\Delta m^2_{23}\|_{\rm I}$
Scenario B: two decay channels					
B1	NO	–	$\nu_3 \to \nu_2, \nu_1$	$\|\Delta m^2_{31}\|_{\rm N}$	$\|\Delta m^2_{31}\|_{\rm N}$
B2	IO	–	$\nu_1, \nu_2 \to \nu_3$	$\|\Delta m^2_{31}\|_{\rm I}$	$\|\Delta m^2_{31}\|_{\rm I}$

Free-streaming

Decay pairs

Signatures in the CMB TT power spectrum…

Fractional deviations in the CMB TT power spectrum from ΛCDM for various the effective isotropisation rate Y and ν_H masses.



Chen, Oldengott, Pierobon & Y[3]W 2022

Effective isotropisation rate: $Y = 6.55C \times 10^{10} \Phi(m_{\nu l}/m_{\nu H}) \left(\frac{m_{\nu H}}{0.05eV}\right)^5 \tau_{\mathrm{rest}}^{-1}$

Scenario A = 2 neutrinos participate in decay/inverse decay; Scenario B = all 3 participate

CMB lower bounds on the neutrino lifetime…

Implementing our new isotropisation rate in CLASS and using the Planck 2018 CMB TTTEEE+low+lensing data, **our revised lifetime constraint** is:

Rel to non-rel factor

$$\tau_{\text{rest}} \gtrsim 1.2\times10^{6}\, \mathfrak{F}\left[0.12\left(\frac{m_{\nu H}}{0.05\text{ eV}}\right)\right] \Phi\left(\frac{m_{\nu l}}{m_{\nu H}}\right)\left(\frac{m_{\nu H}}{0.05\text{ eV}}\right)^{5}\text{ s}$$

Phase space factor $\sim \frac{1}{3}\left(\frac{\Delta m_{\nu}^{2}}{m_{\nu H}^{2}}\right)^{2}$

Chen, Oldengott, Pierobon & Y[3]W 2022

- Or equivalently:

$\nu_3 \to \nu_{1,2} + \phi$ (NO)
$\nu_{1,2} \to \nu_3 + \phi$ (IO)

$$\tau_{\text{rest}} \gtrsim (6 - 10)\times10^{5}\text{s}$$

$\nu_2 \to \nu_1 + \phi$

$$\tau_{\text{rest}} \gtrsim (400 - 500)\text{s}$$

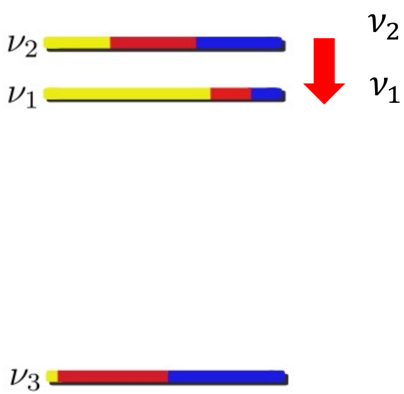
Cf old constraints (which misidentified T_{coverage} with $T_{\text{isotropise}}$):

$$\tau_{\text{rest}} \gtrsim 10^{9}\left(\frac{m_{\nu H}}{0.05\text{ eV}}\right)^{3}\text{ s}$$

Hannestad & Raffelt 2005

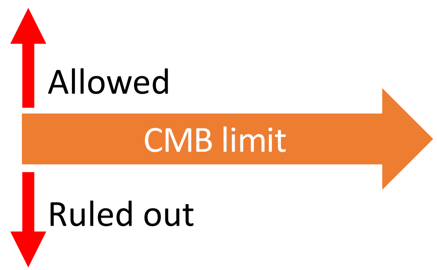
CMB lower bounds on the neutrino lifetime…

… currently the best limits on invisible neutrino decay $\nu_H \to \nu_l + \phi$.



Inverted mass ordering

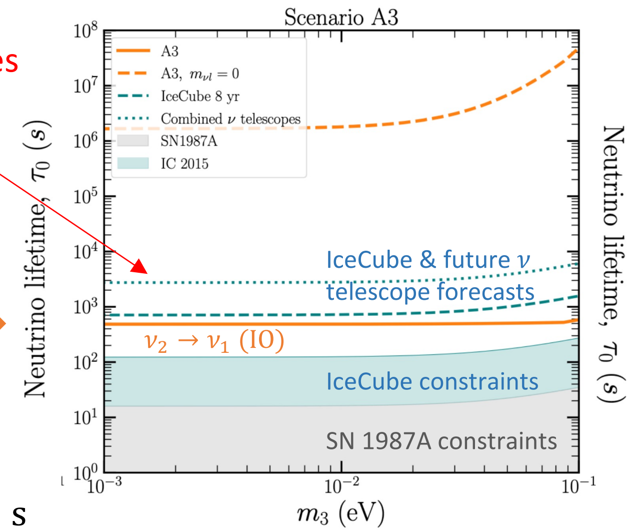
However, depending on the exact decay scenario, neutrino telescopes may become competitive in the future! Watch this space!



BBN: $\tau_0 \gtrsim 10^{-2} \to 10^{-1}$ s

Solar ν: $\tau_0 \gtrsim 10^{-5} \to 10^{-4}$ s

Lab ν: $\tau_0 \gtrsim 10^{-13} \to 10^{-11}$ s



Chen, Oldengott, Pierobon & Y[3]W 2022

* IceCube constraints & forecasts from Song et al. 2021

CMB lower bounds on the neutrino lifetime…

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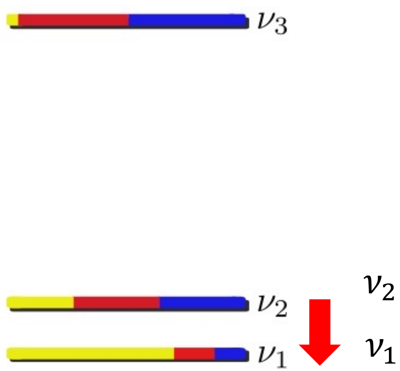
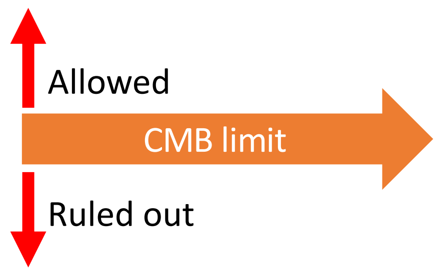
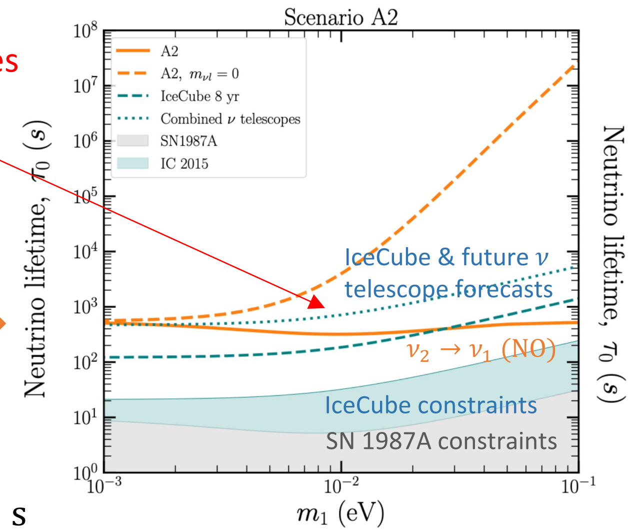


Normal mass ordering

However, depending on the exact decay scenario, neutrino telescopes may become competitive in the future! Watch this space!





BBN: $\tau_0 \gtrsim 10^{-2} \to 10^{-1}$ s
Solar ν: $\tau_0 \gtrsim 10^{-5} \to 10^{-4}$ s
Lab ν: $\tau_0 \gtrsim 10^{-13} \to 10^{-11}$ s

Chen, Oldengott, Pierobon & Y[3]W 2022

* IceCube constraints & forecasts from Song et al. 2021

Summary…

- The **cosmic neutrino background** is a fundamental prediction of standard hot big bang cosmology.
  - We have indirect evidence from precision cosmological observations that it exists and has properties consistent with standard expectations.

- Given this, we can contemplate using **precision cosmological observables** to constrain non-standard neutrino properties like invisible neutrino decay.

- But **mapping the decay rate** to the isotropisation rate that ultimately changes the CMB observable can be a tricky task.

- We have calculated the isotropisation rate from first-principles and revised the CMB constraint on the neutrino lifetime by many orders of magnitude.