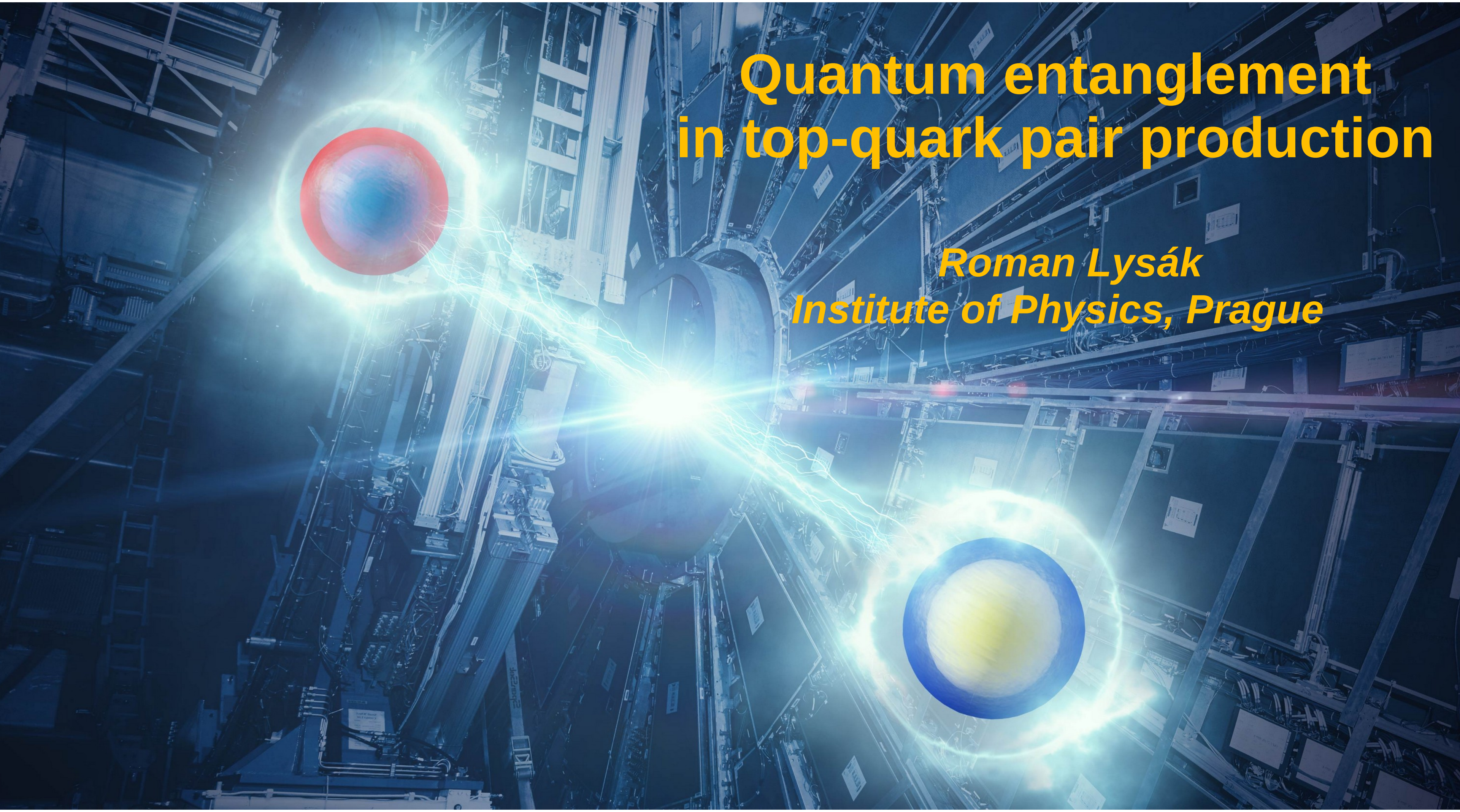


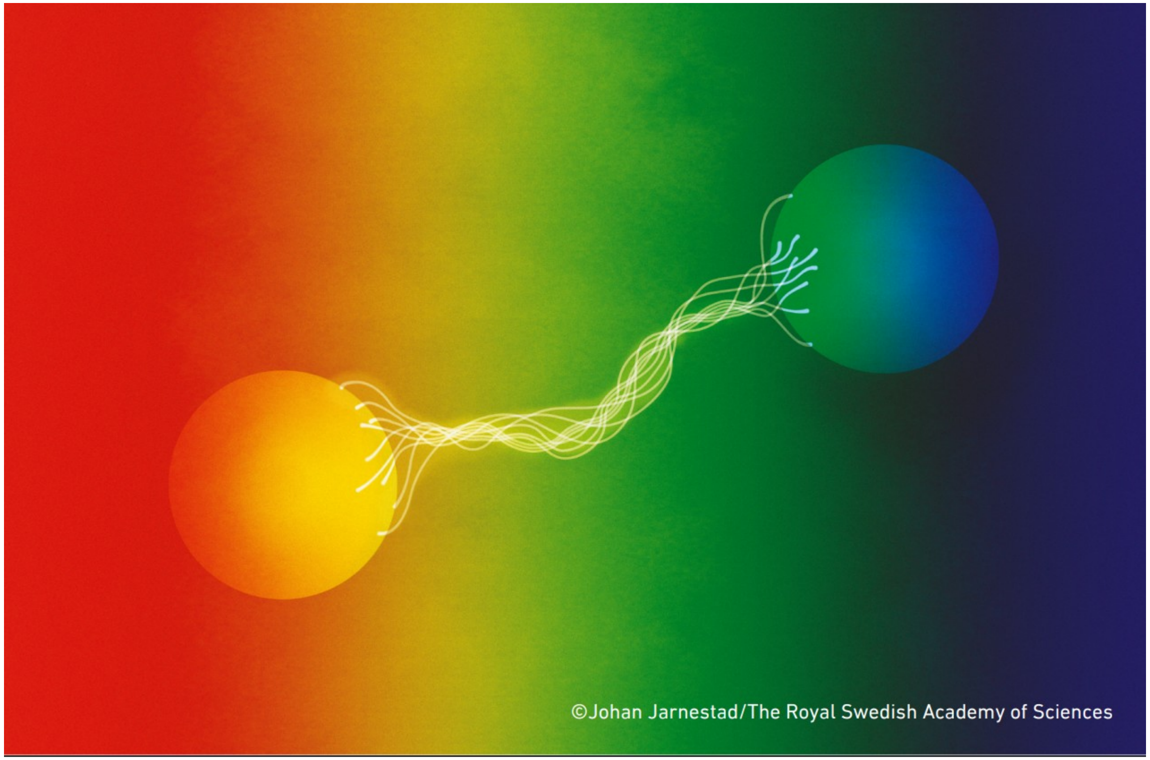
Quantum entanglement in top-quark pair production

Roman Lysák
Institute of Physics, Prague

Seminar of Division of elementary particle physics at FZU, Feb’24

Overview

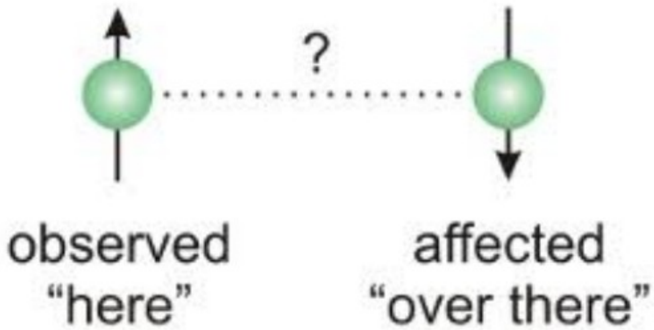
- Standard Model is a quantum field theory
  - based on quantum mechanics and special relativity
- Quantum entanglement is one of the most striking features of quantum mechanics
  - Not present in classical physics
  - Recently, proposals to measure it at particle colliders
- In this talk: the study of **spin** entanglement in **top quark pair production**



QUANTUM ENTANGLEMENT

What is quantum entanglement?

- Phenomenon when quantum state of one particle cannot be described independently from another particle
  - → there are **correlations** between observed physical properties of both particles
  - → **measurement** of one particle influence other particle entangled with it



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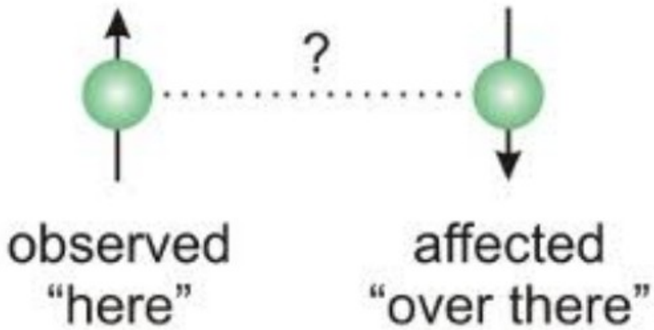


- For a pure quantum state, the general form of the wave function:
  - For the simplest case of two spinless particle moving along the line

$$\psi(x,y) = \sum_i c_i \psi_i(x)\psi_i(y)$$

- The quantum state is separable if wave function can be written as:

$$\psi(x,y) = \psi_1(x)\psi_2(y)$$

- **If the state is not separable → entangled state**

Quantum entanglement for two qubits

- Basic entanglement definition can be generalized also to what are called mixed states
  - Mixed states describe classical statistical mixtures of pure states
    - e.g. at the LHC we cannot control the initial state → mixed state
  - Mixed states described in general by ‘density matrix’ (ρ)

- A typical example of entanglement is provided by two qubits
  - qubit = a quantum system with two possible states
  - e.g. two particles with ½ spin
    - entanglement is characterized by their spin correlations

- The most general density matrix describing two qubits:

$$\rho = \frac{I_4 + \sum_i \left(B_i^+ \sigma^i \otimes I_2 + B_i^- I_2 \otimes \sigma^i\right) + \sum_{i,j} C_{ij} \sigma^i \otimes \sigma^j}{4}$$

I_4, I_2: unit (4x4), (2x2) matrices
$\sigma_{i,j}$: Pauli matrices

  - The state is described by 15 parameters $B_i^{\pm}$, C_{ij}
  - In case of two particles with ½ spin:
    - $B_i^{\pm}$: individual spin polarizations
    - C_{ij}: spin correlation matrix

A measure of quantum entanglement

- The Peres-Horodecki criterion is a necessary condition for entanglement in bipartite systems of dimension 2 × 2
- A quantitative measure of the degree of entanglement is obtained by 'concurrence' C[ρ]:

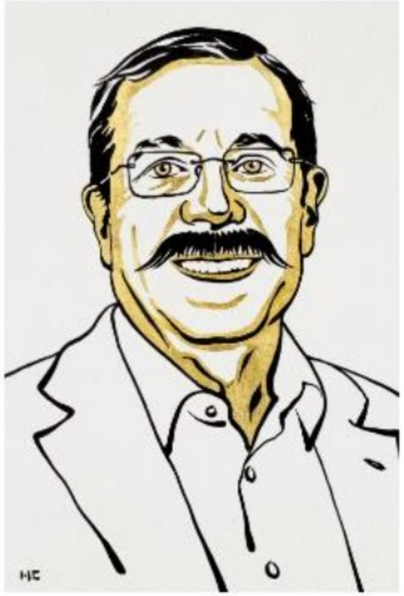
$$C[\rho] \equiv \max(0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4)$$

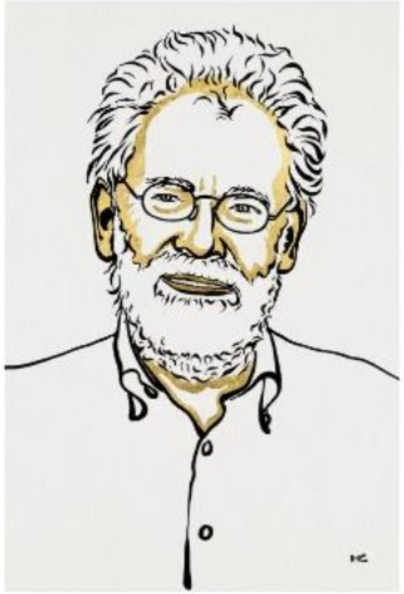
  - λ_i: eigenvalues of the matrix $\mathcal{C}(\rho) = \sqrt{\sqrt{\rho}\tilde{\rho}\sqrt{\rho}}$, with $\tilde{\rho} = (\sigma_2 \otimes \sigma_2)\ \rho^*\ (\sigma_2 \otimes \sigma_2)$
  - $0 \leq C[\rho] \leq 1$
- Quantum state is entangled if and only if **C[ρ] > 0**

The Nobel prize in physics in 2022

- "for experiments with **entangled** photons, establishing the violation of Bell inequalities and pioneering quantum information science"

Alain Aspect, John F. Clauser and Anton Zeilinger



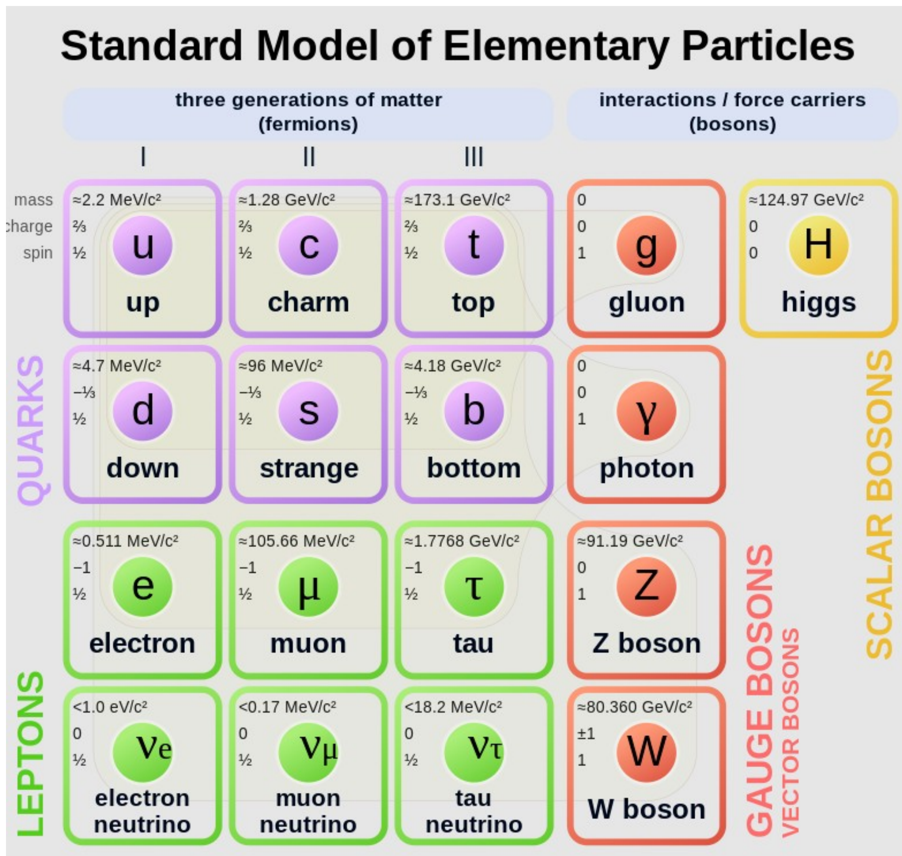


Quantum entanglement observed in many different systems such as photons, atoms, superconductors, neutrinos, macroscopic diamonds, etc.

ENTANGLEMENT IN TOP-QUARK PAIR PRODUCTION

Top quark

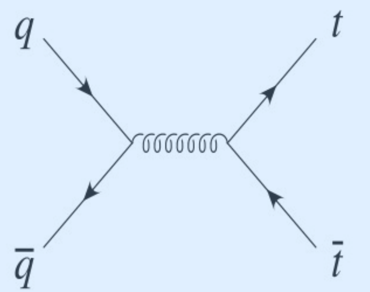
- The most massive fundamental particle, m(top) ~ 172.5 GeV
  - → Lifetime very short: ~10^{-25} s
    - – Hadronization time: ~10^{-23} s, spin-decorrelation time: ~10^{-21}s
      - → spin information transferred to its decay products



Top quark production

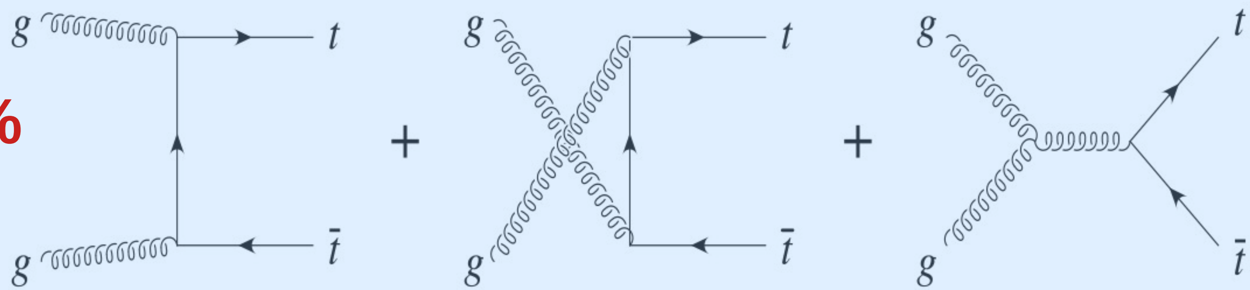
- Pair production via quantum chromodynamics:

~10%

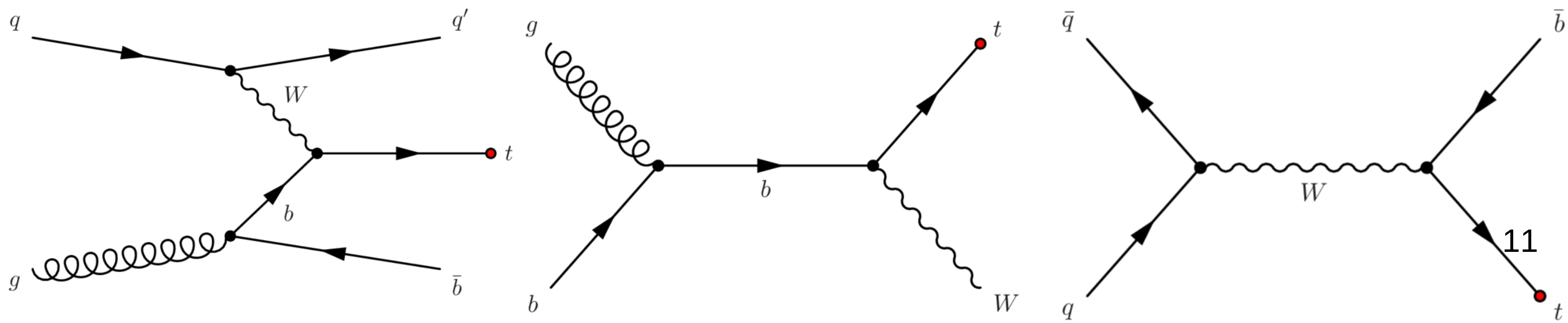


→ **this measurement**

~90%



- Single top quark production via electroweak interaction:

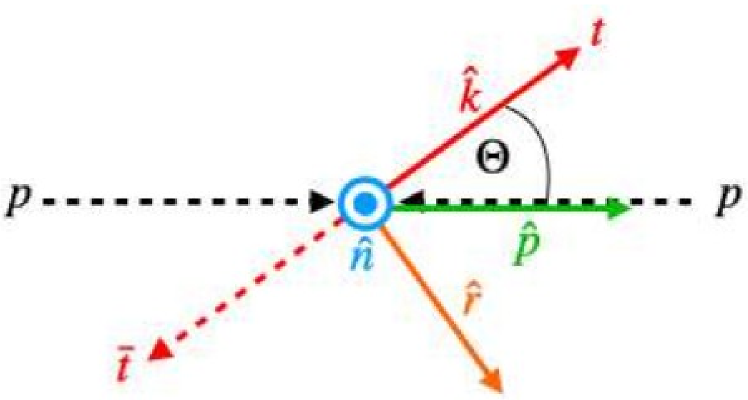


$t\bar{t}$ spin correlations

- Theoretically, $t\bar{t}$ spin correlations known quite well (@NLO in QCD)
- The angular differential cross-section (for dilepton decay):

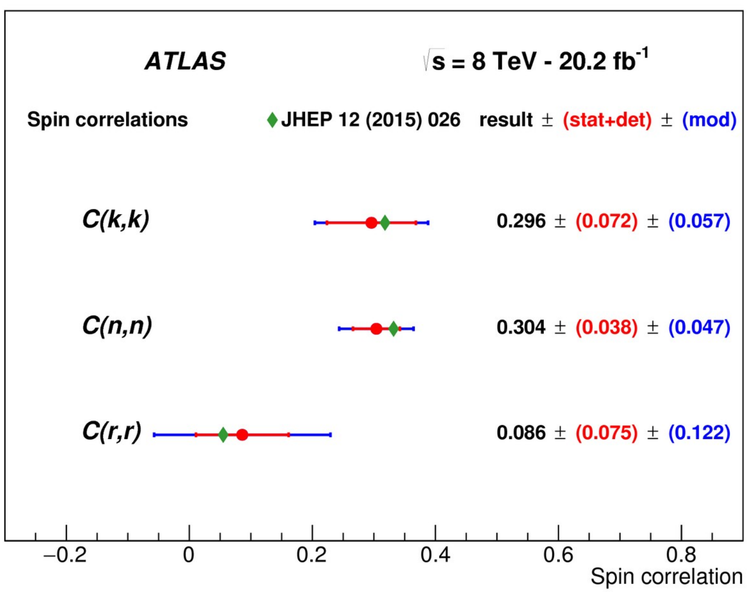
$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega_+ d\Omega_-} = \frac{1 + \mathbf{B}^+ \cdot \hat{\mathbf{q}}_+ - \mathbf{B}^- \cdot \hat{\mathbf{q}}_- - \hat{\mathbf{q}}_+ \cdot \mathbf{C} \cdot \hat{\mathbf{q}}_-}{(4\pi)^2}$$

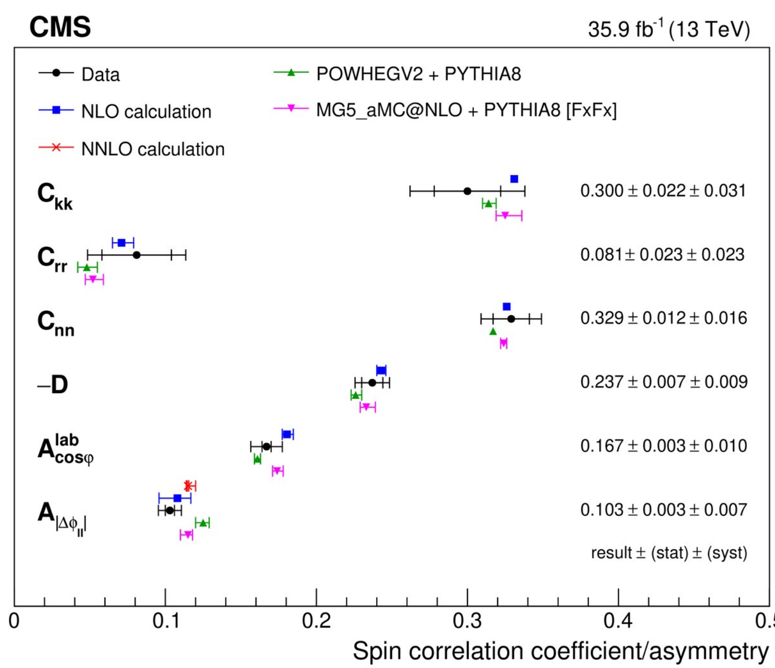
  - q+(q-) - direction of lepton (antilepton) in its parent top(antitop) quark rest frame
  - B^{+-} - top/antitop spin polarization
  - C – spin correlation matrix (3x3)
- Spin polarization and correlations are computed in orthonormal basis
  - Typically used 'helicity' basis: k,n,r
- Overall, **15 coefficients fully characterising the spin information in $t\bar{t}$ production**



Measurements of $t\bar{t}$ spin correlations and D

- studying $t\bar{t}$ spin correlations is an active area in both ATLAS and CMS
  - Measurements were performed already at 7, 8 and 13 TeV
- full spin density matrix (15 coefficients) measured by both
  - ATLAS at $\sqrt{s}$ = 8 TeV (arXiv:1612.07004)
  - CMS at $\sqrt{s}$ = 13 TeV (arXiv:1907.03729)
- CMS measured also entanglement observable 'D'
- **All measurements performed inclusively, i.e. in full $m(t\bar{t})$ range**

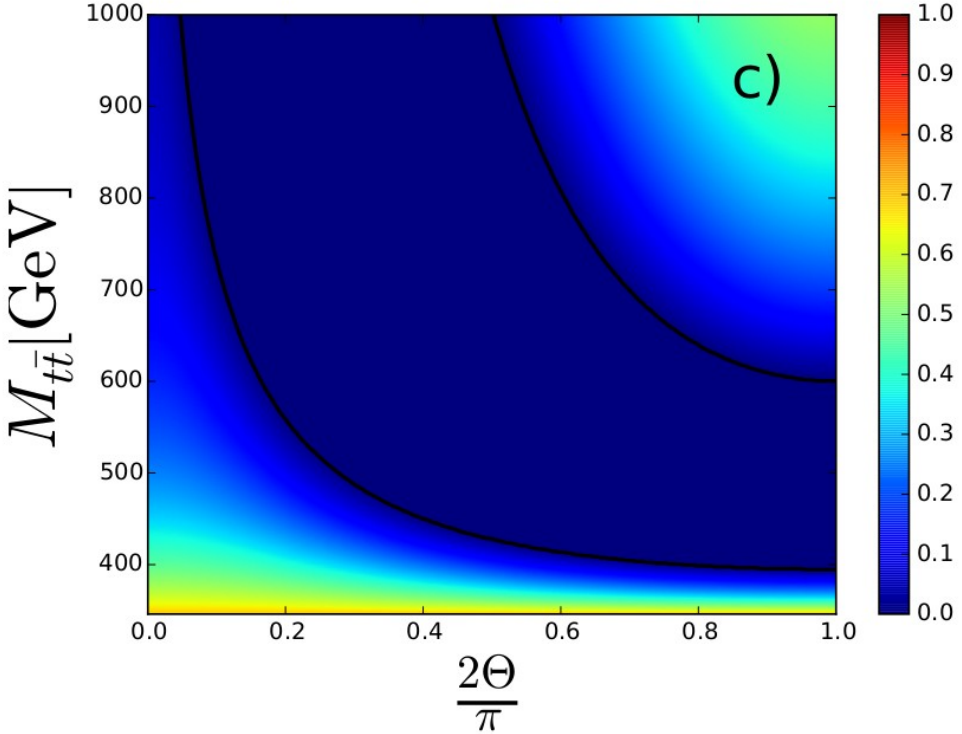




Entanglement condition

- Pair of top-quarks (spin ½ particles) is an example of two-qubit system
- Concurrence C[ρ] of the spin density matrix:

Eur. Phys. J. Plus (2021) 136:907



- Sufficient and necessary condition for entanglement: **C[ρ] > 0**
- Two regions with entanglement:
  - Low m($t\bar{t}$) → close to m($t\bar{t}$) threshold: gluon fusion produces a $t\bar{t}$ pair in a spin singlet
  - high m($t\bar{t}$) and theta ~pi/2 → high pT
- Equivalently, the sufficient condition obtained with spin correlation matrix: **-Tr[C] > +1**

Entanglement observable (‘D’)

- ‘D’ can be obtained from the differential distribution:

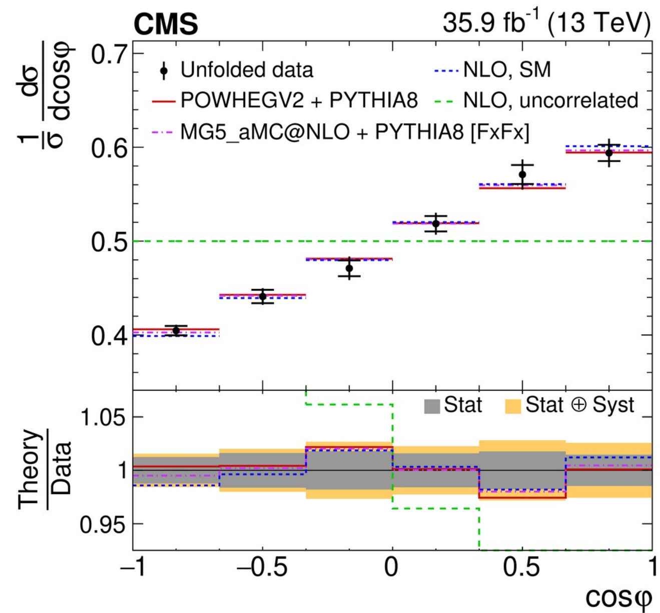
$$\frac{1}{\sigma_{\ell\bar{\ell}}}\frac{\mathrm{d}\sigma_{\ell\bar{\ell}}}{\mathrm{d}\cos\varphi} = \frac{1}{2}(1 - D\cos\varphi), \;\; D = \frac{\mathrm{tr}[\mathbf{C}]}{3}$$

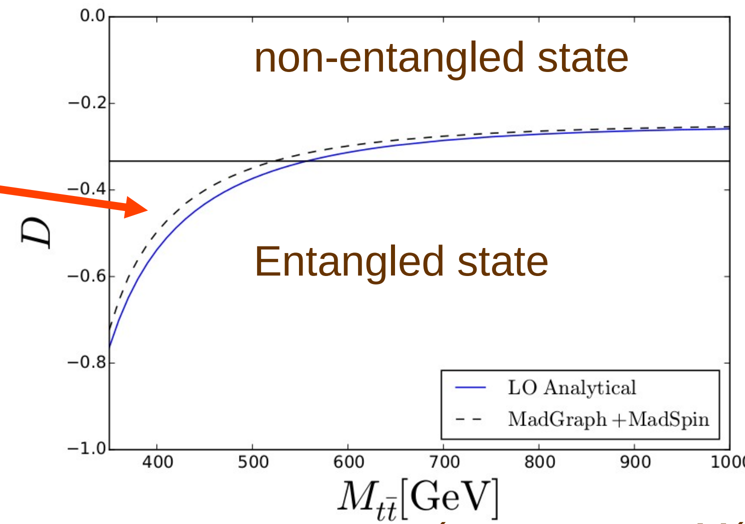
ϕ: angle between the two lepton directions measured in their parent top quark and antiquark rest frames

- Entanglement condition: -Tr[C] > +1

  → translates to **D < -1/3**

- It’s crucial to measure ‘D’ at **low invariant mass m($t\bar{t}$)**

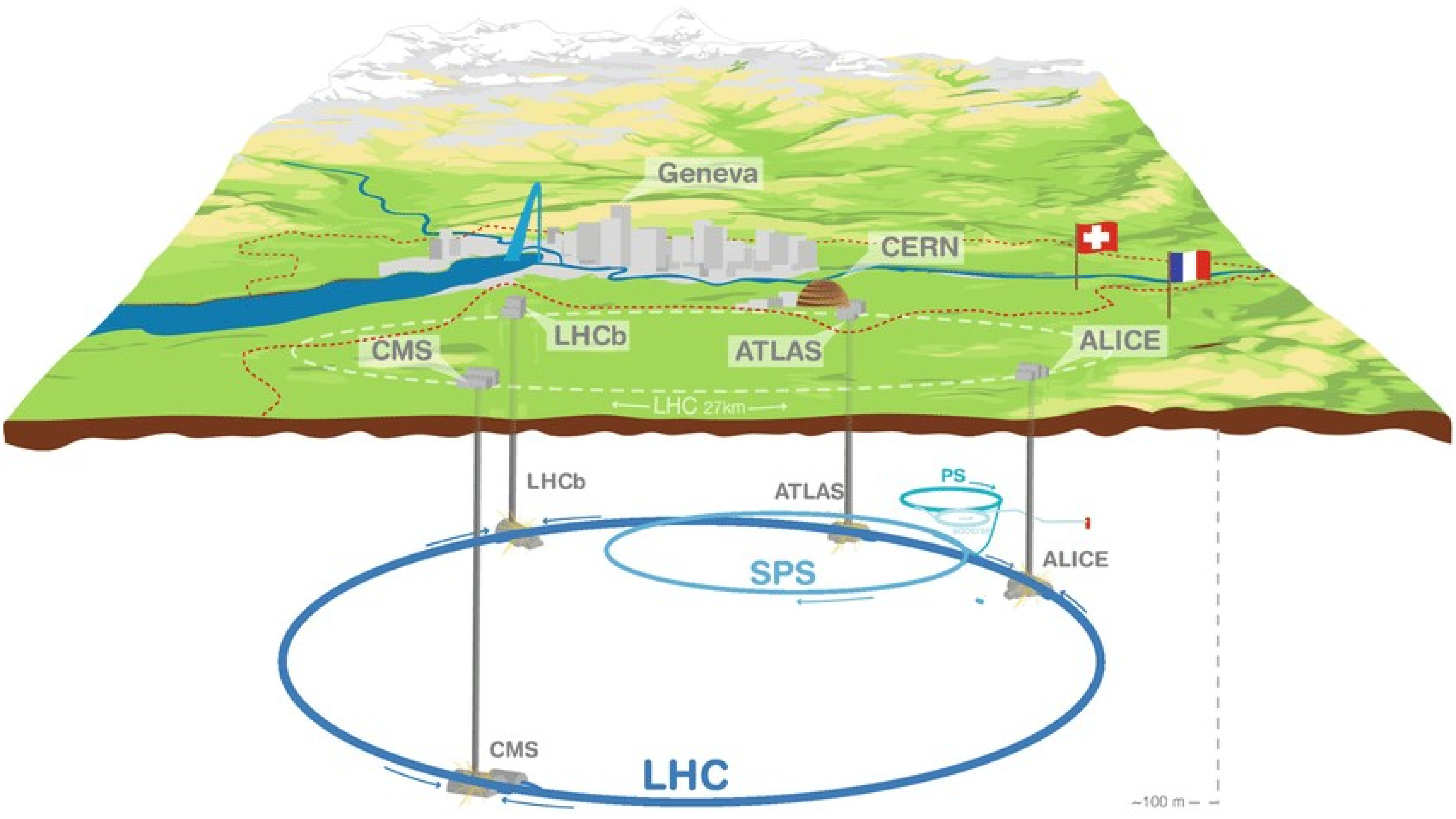




(upper cut on M($t\bar{t}$))

THE EXPERIMENT

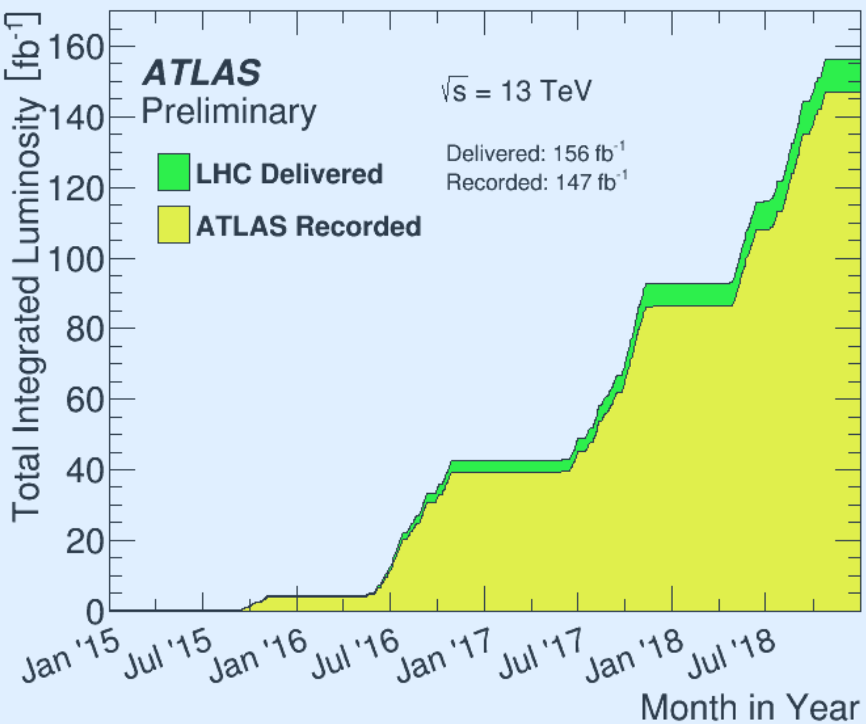
Large Hadron Collider (LHC)



- LHC: proton-proton collisions at sqrt(s) ≤ 14 TeV
- Eventual luminosity by experiments: ~3000/fb (2010-2041?)

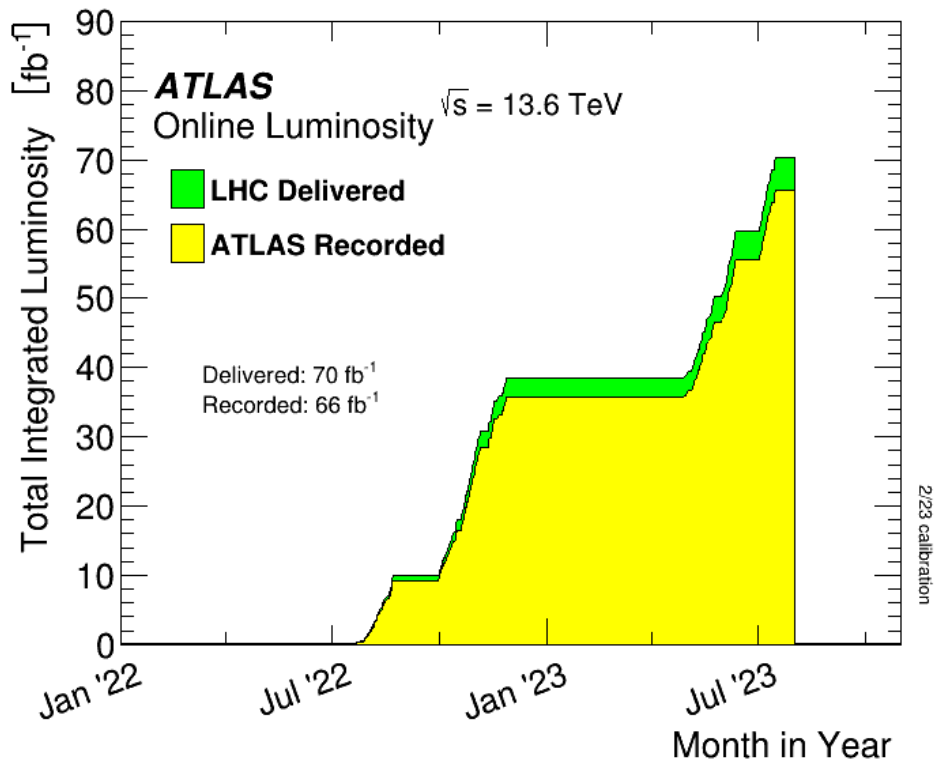
LHC data

Run 2 (2015-2018)
√s = 13 TeV



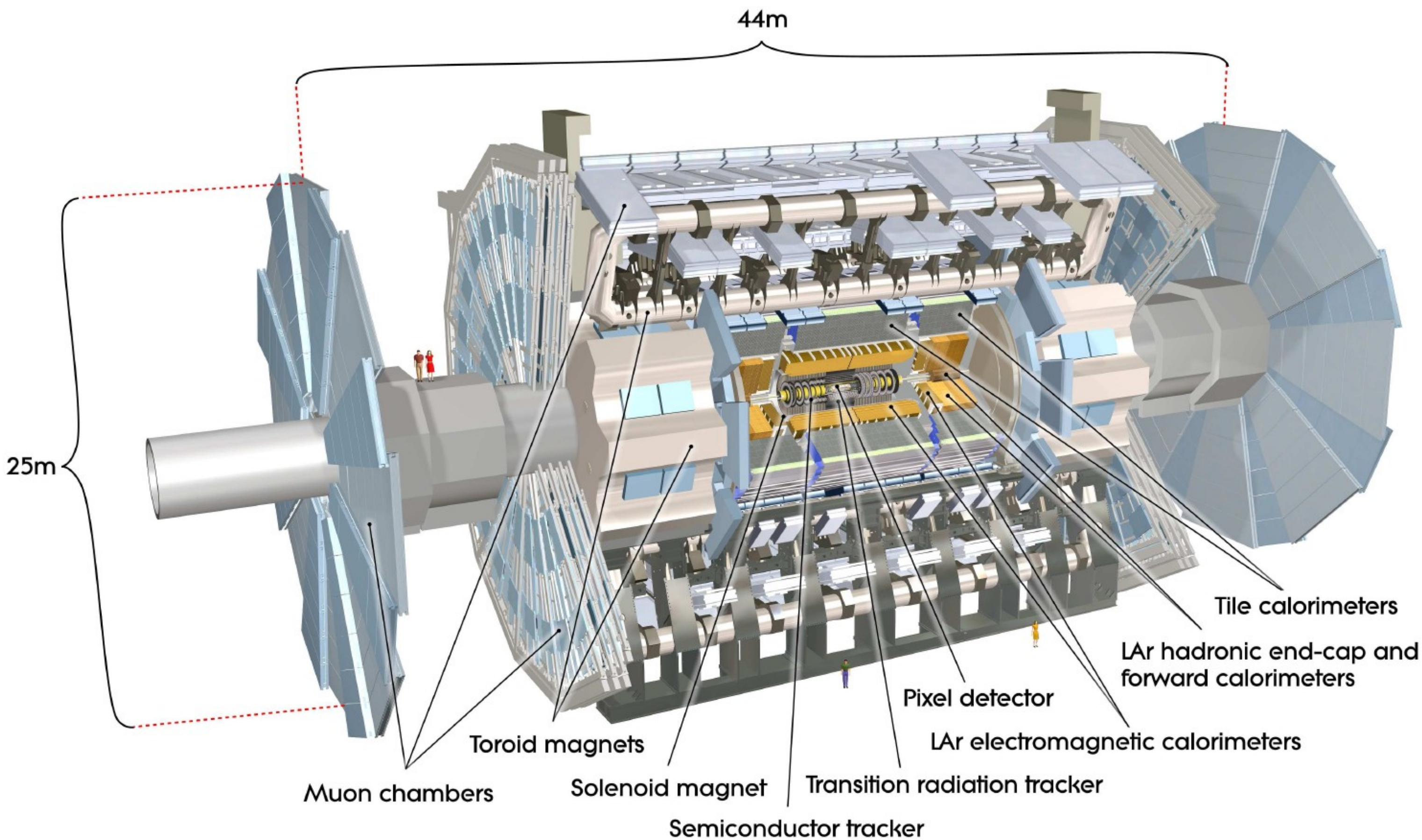
→ **this measurement (140/fb of 'good' data)**

Run 3 (2022-2025)
√s = 13.6 TeV



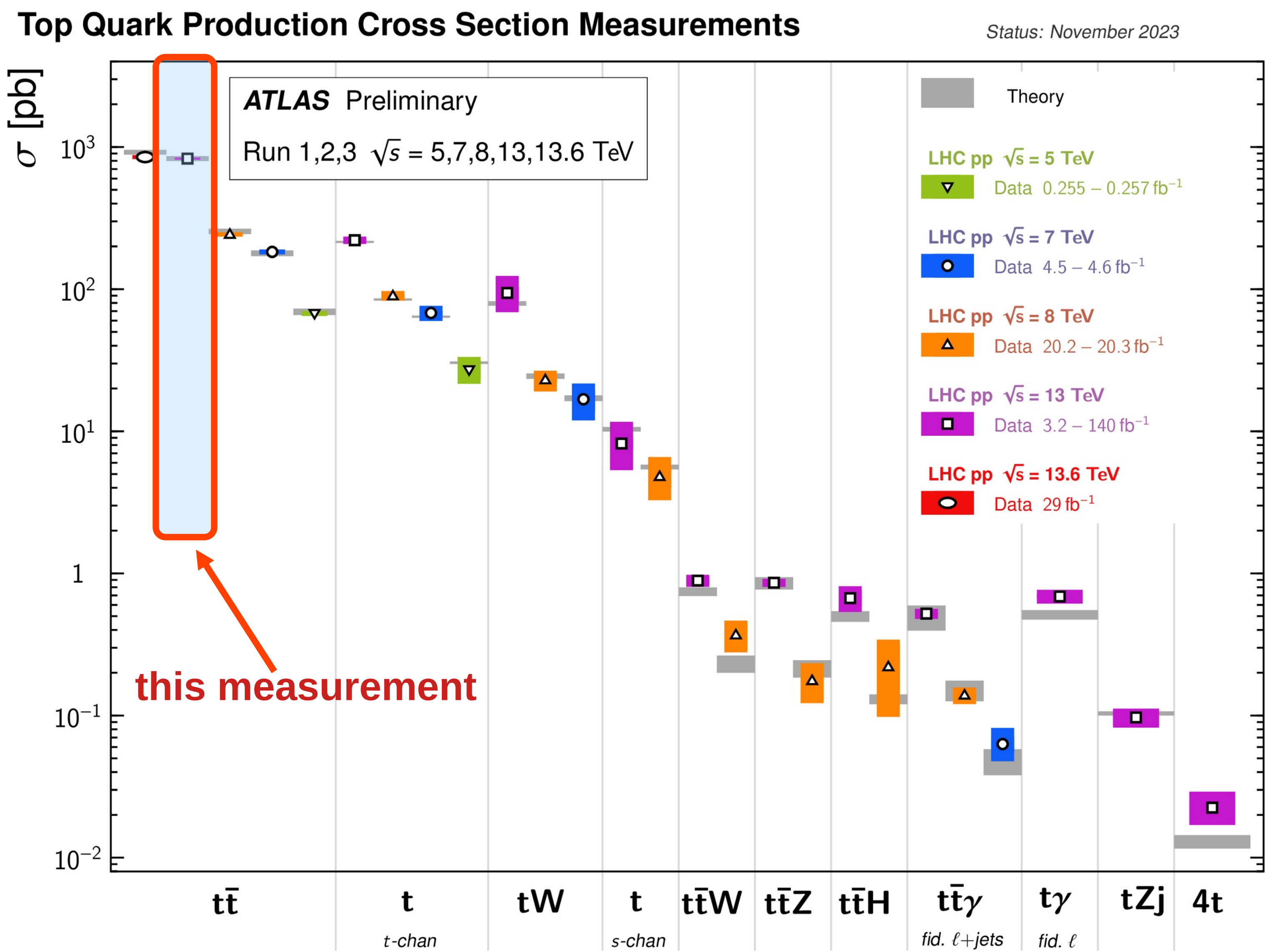
- Run 3 goal: collect ~250-300/fb

ATLAS detector



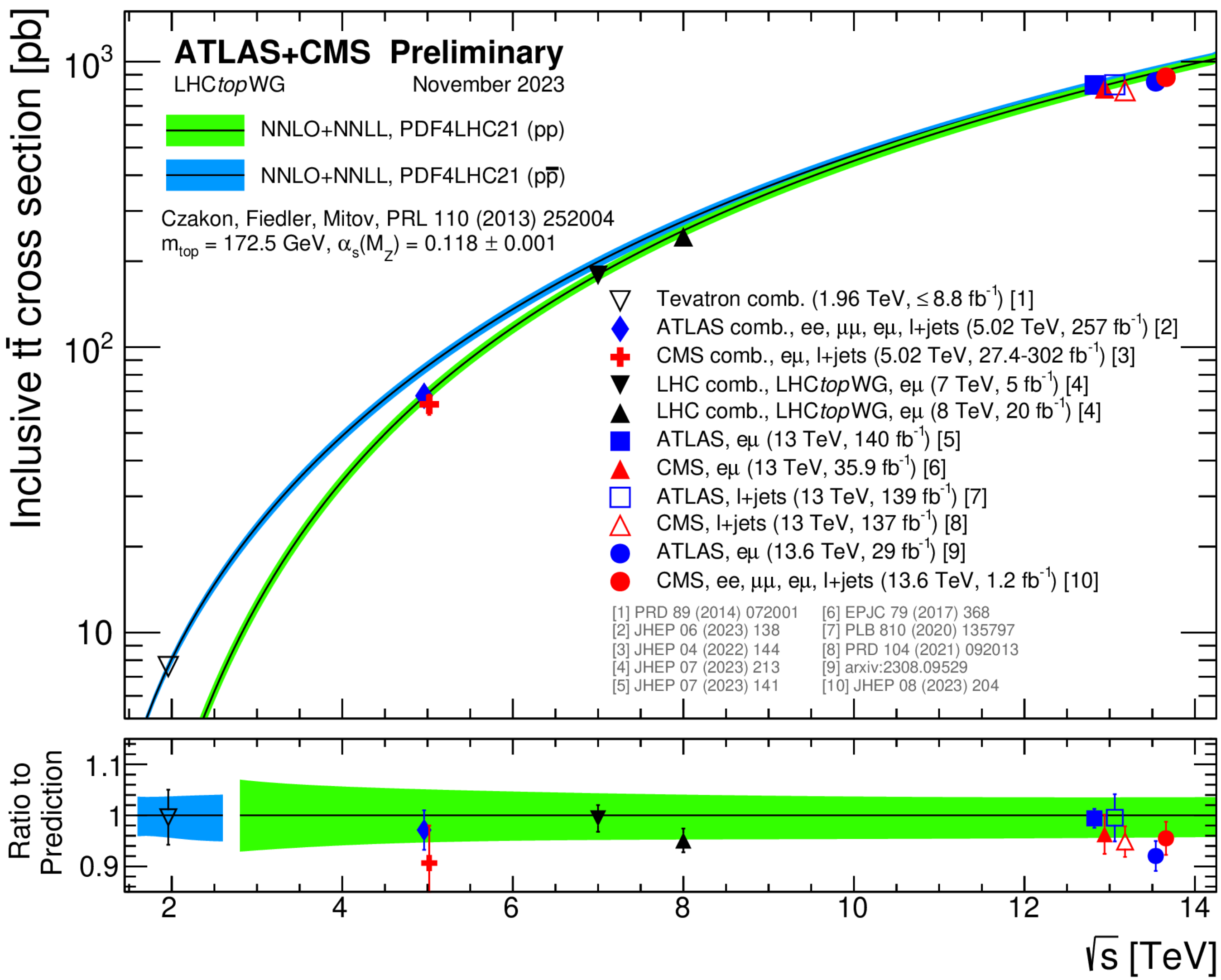
- Top-quark pair events: complex topologies
  → all major detector sub-systems important for top-quark physics

Top quark production at LHC



- spread of the cross-sections: 5 orders of magnitude

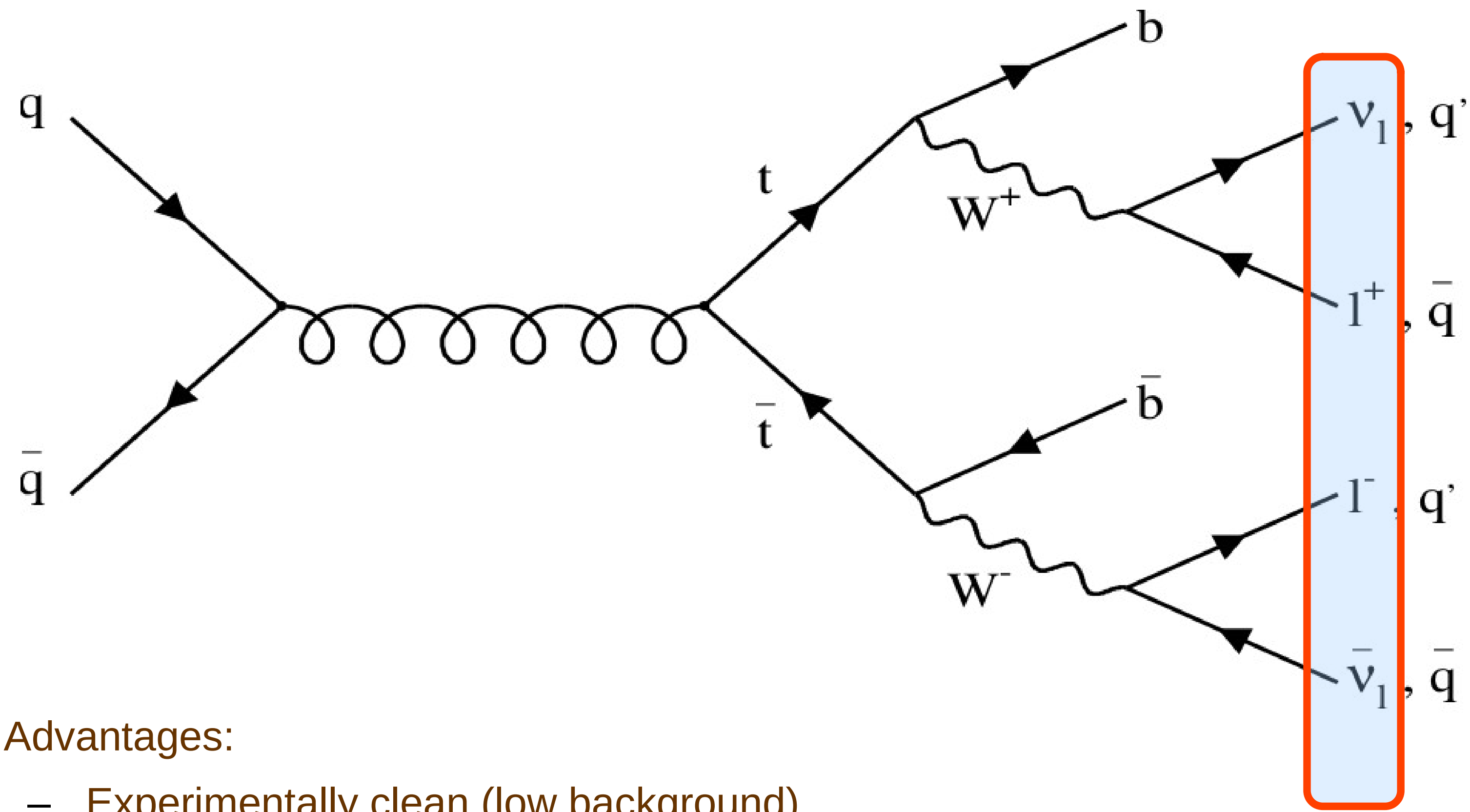
Top quark pair production



- ~120M of top-quark pairs produced in ATLAS experiment in Run 2
- Inclusive cross-sections known with the relative precision ~2–4%

THE ANALYSIS

ttbar dilepton channel



- Advantages:
  - Experimentally clean (low background)
  - For spin-correlation/QE measurement: precise reconstruction of leptons
- Disadvantages:
  - Relatively low stats (~5% of the whole production)
  - Difficult kinematic reconstruction of top quark pair (2 undetected neutrinos)

The event selection

2 leptons, 2 neutrinos, and 2 b-quarks in the final state → apply cuts:

- Require 1 electron and 1 muon: $p_T > 25 - 28$ GeV (|eta| < ~2.5)
- Opposite-sign electric charge of leptons
- No cut on missing transverse momentum from neutrinos
- ≥ 2 jets with $p_T > 25$ GeV
  - ≥ 1 jet coming from b-quark ('b-tagged jet', 85% efficiency)

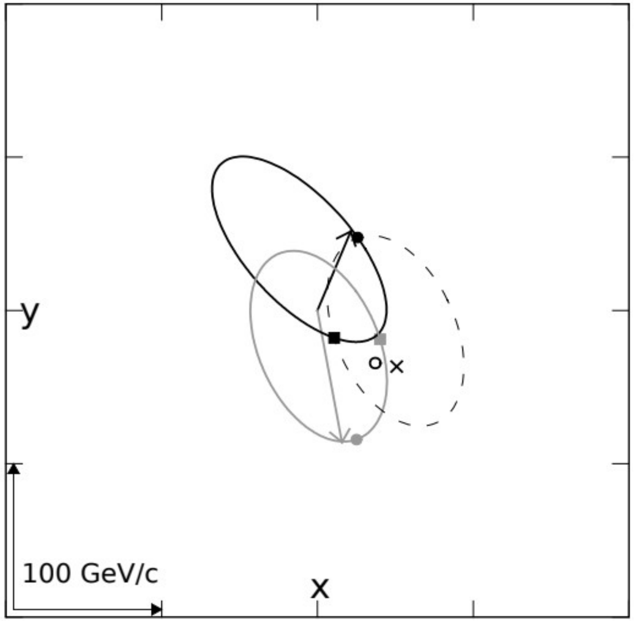
→ **About 1.1M events after full event selection**

- About 90% of these are expected to be top-quark pair signal events
- the backgrounds: **single top**, **Z/W+jets**, $t\bar{t}$+X (X=W,Z,H), VV (V=W,Z), fakes
  - All but 'fakes' (data-driven) estimated by simulation

Top quark pair reconstruction

- Reconstruction of top quarks momenta complicated due to 2 neutrinos
  - Numerous methods developed and used in the past ~30 years
    - all(?) use m(top) and m(W) as inputs

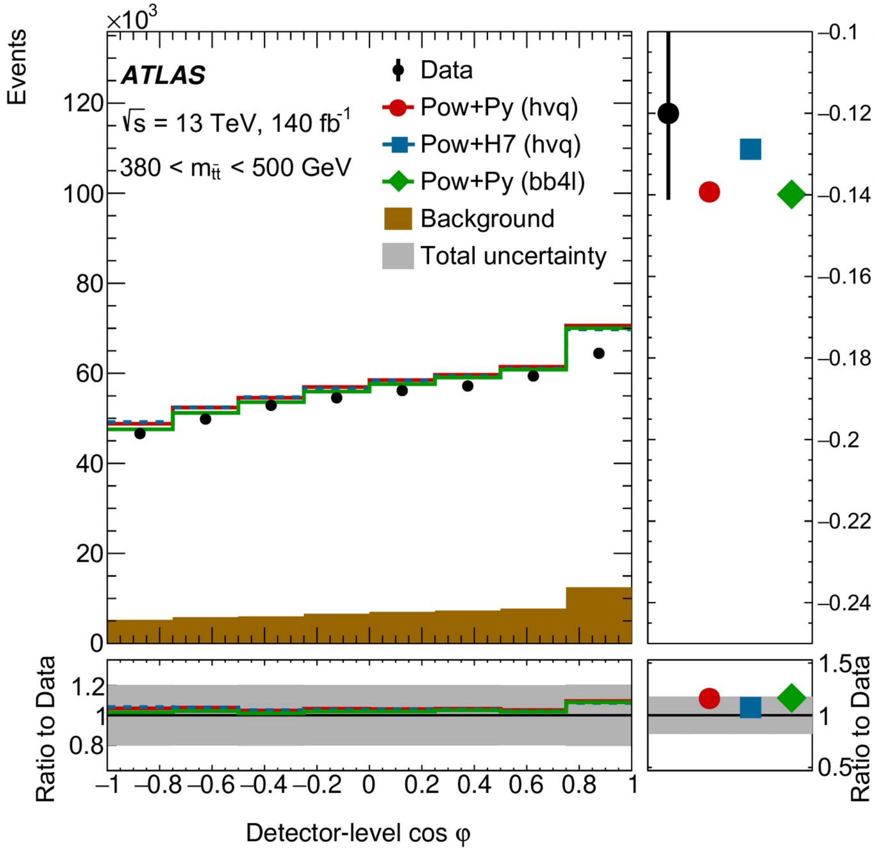
- The main method used: ‘Ellipse’ method (85% efficiency)
  - Analytically calculate two ellipses for $p_T(\nu)$ and find intersections (solutions)



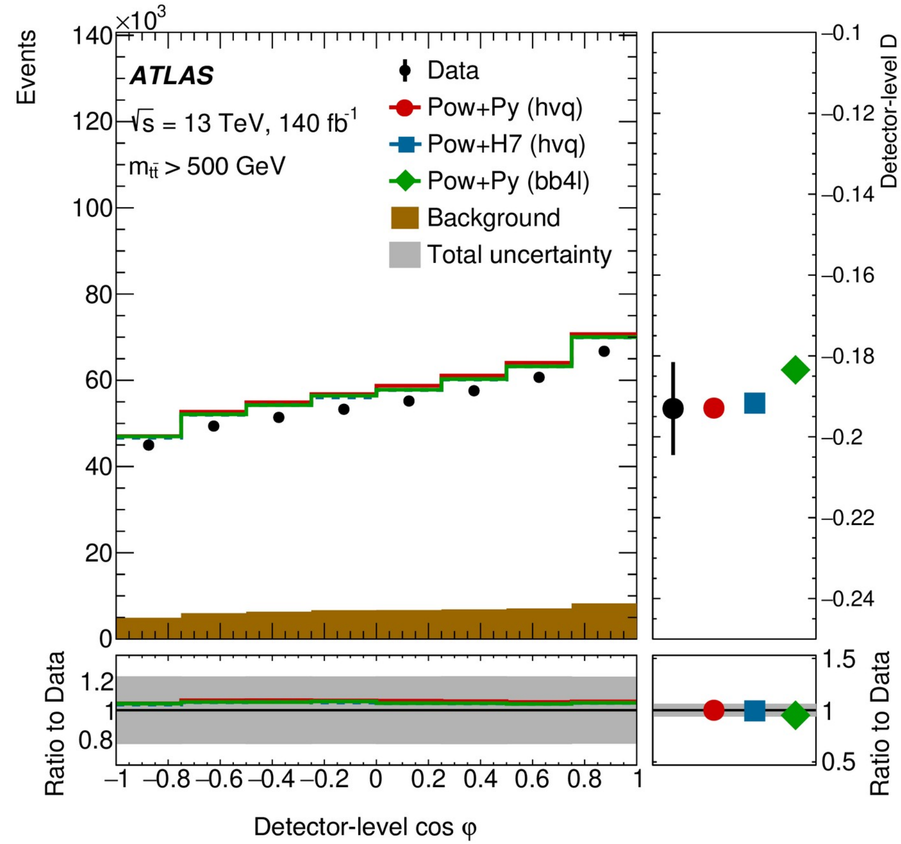
- Sample divided into based on $m(t\bar{t})$:
  - **Signal region: 340 GeV < $m(t\bar{t})$ < 380 GeV**
  - 2 validation regions: 380 GeV < $m(t\bar{t})$ < 500 GeV, $m(t\bar{t})$ > 500 GeV

Reconstructed cos φ: validation regions

380 < m($t\bar{t}$) < 500 GeV:



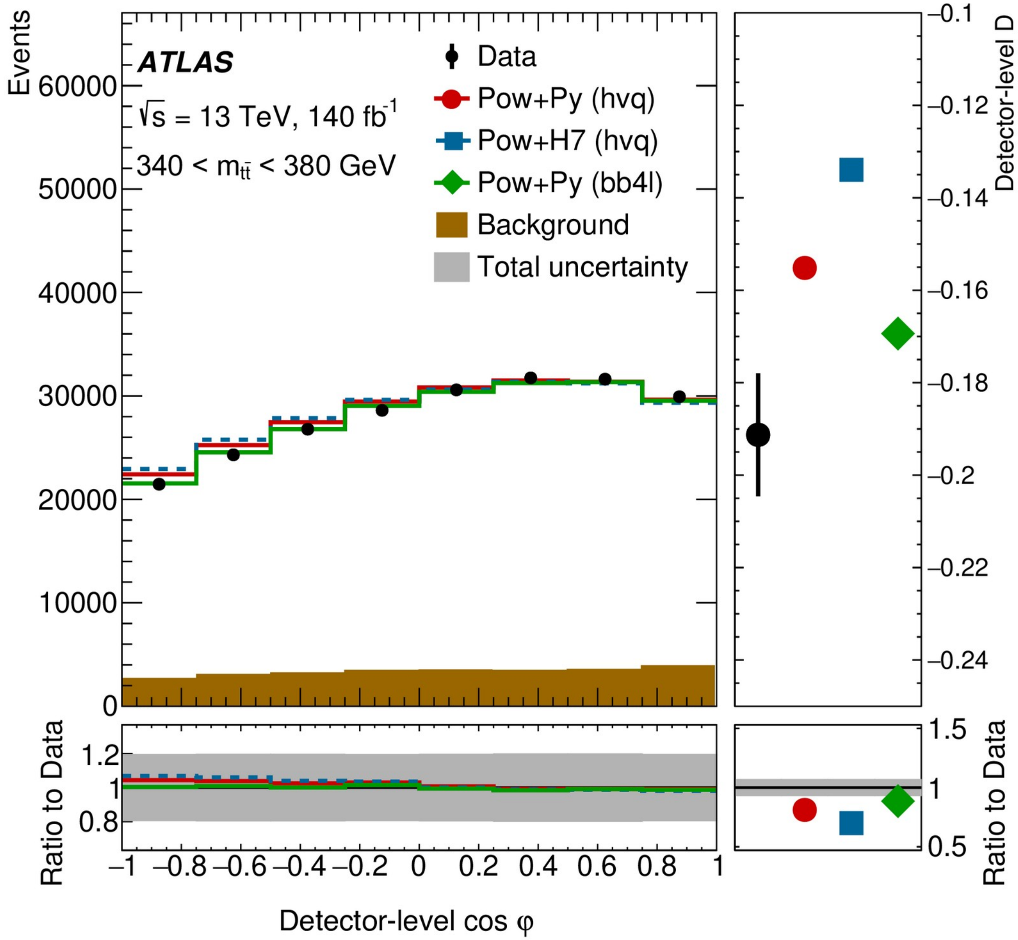
m($t\bar{t}$) > 500 GeV:



- Data agree with predictions

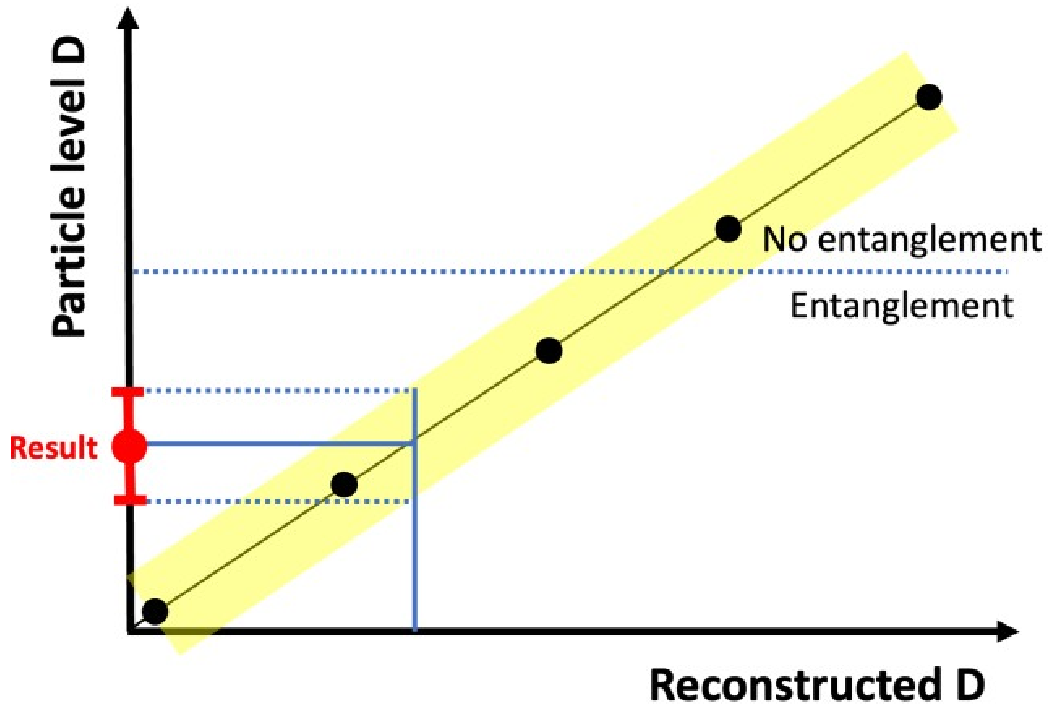
Reconstructed cos φ: signal region

340 < m($t\bar{t}$) < 380 GeV:



- The reconstructed value of 'D' is below predictions

Calibrating the observable



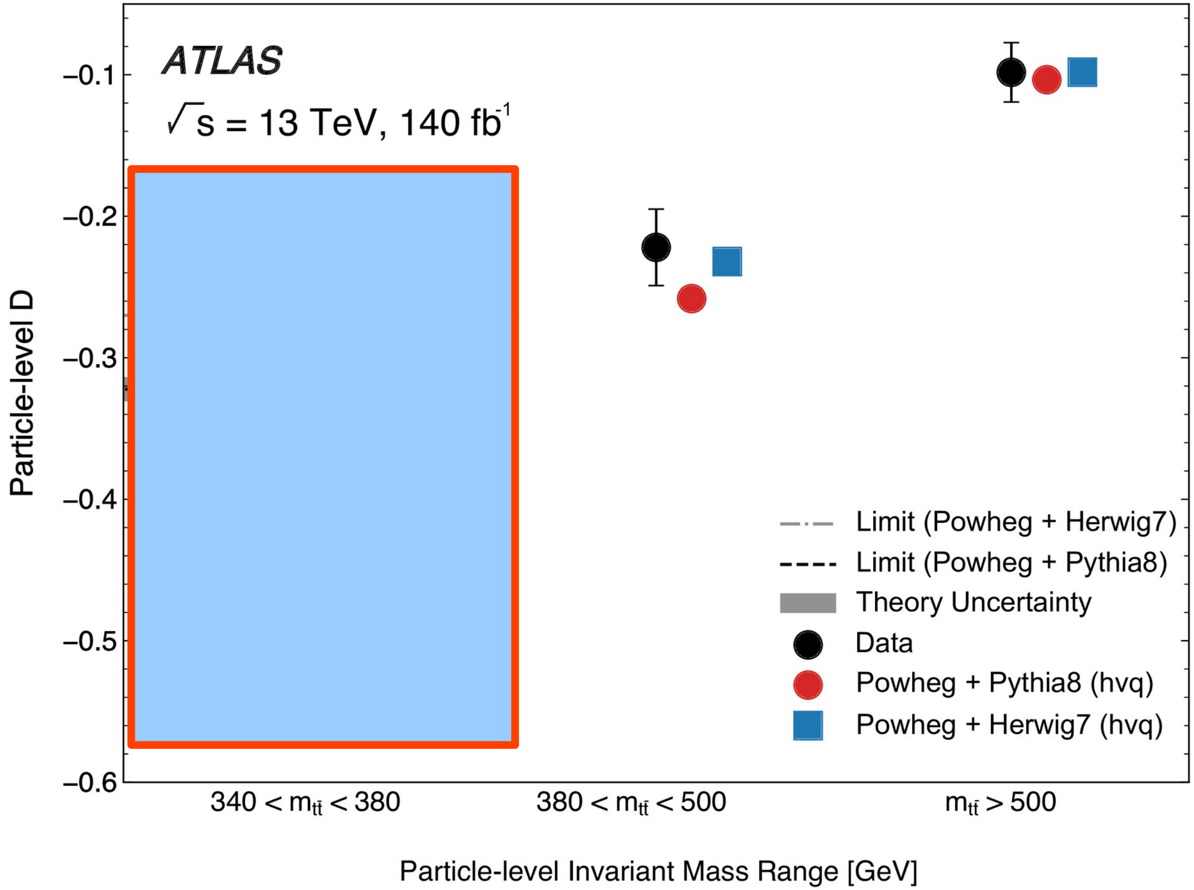
- The measured data (reconstructed level) are corrected to the truth (particle) level
  - The particle level cuts are similar to reconstructed level
- Calibration curve created by reweighting simulation based on 'truth' 'D' value

Systematic uncertainties

- 3 main categories:
  - Signal modelling → dominant
    - main component: top quark decay
  - Background modelling
  - Object reconstruction
- For each systematic uncertainty: create a new calibration curve

Systematic source	$\Delta D_{\text{expected}} (D = -0.470)$	ΔD (%)
Signal Modelling	0.015	3.2
Electrons	0.002	0.4
Muons	0.001	0.1
Jets	0.004	0.8
b-tagging	0.002	0.4
Pile-up	< 0.001	< 0.1
$E_{\text{T}}^{\text{miss}}$	0.002	0.4
Backgrounds	0.009	1.8
Total Statistical Uncertainty	0.002	0.4
Total Systematic Uncertainty	0.018	3.9
Total Uncertainty	0.018	3.9

Particle-level ‘D’ in validation regions



- measured ‘D’ in validation regions:

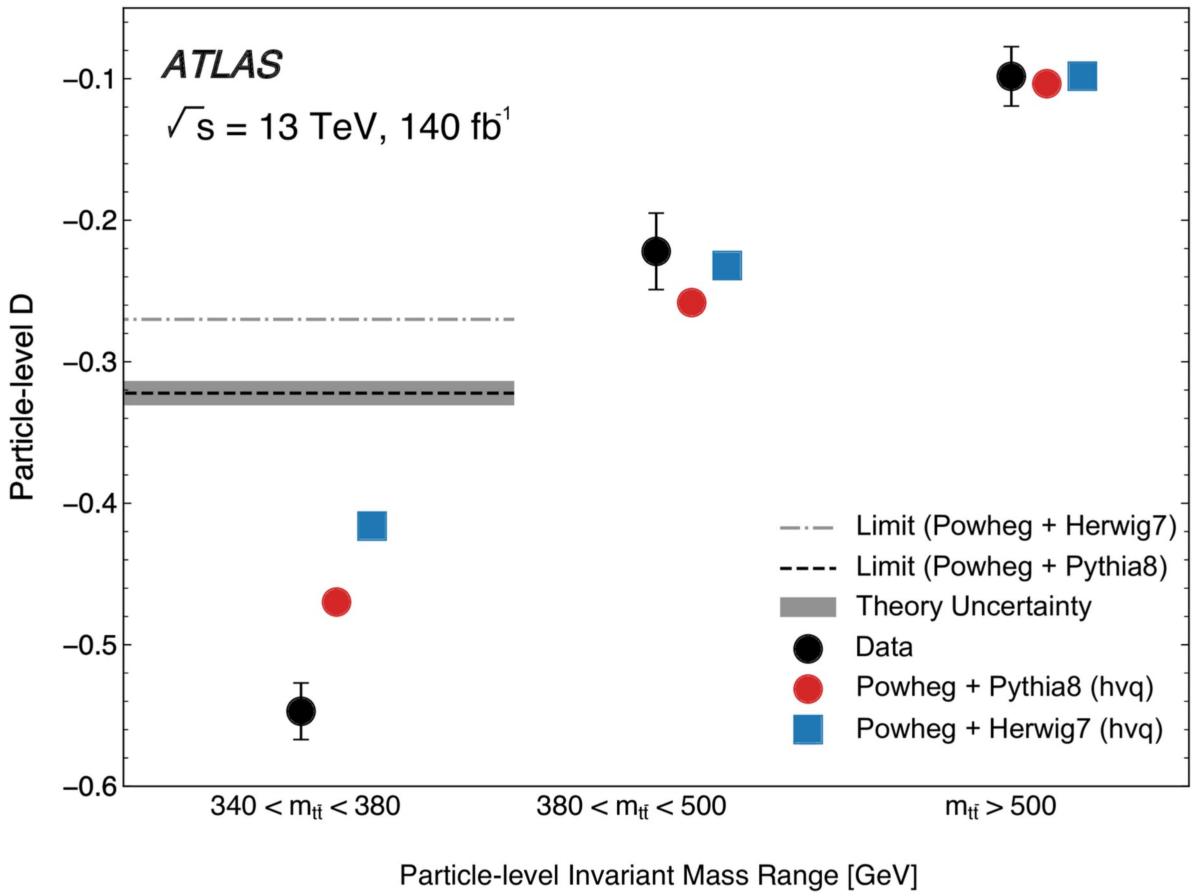
380 < m($t\bar{t}$) < 500 GeV:

$$D = -0.222 \pm 0.001\ [\text{stat.}] \pm 0.027\ [\text{syst.}]$$

m($t\bar{t}$) > 500 GeV:

$$D = -0.098 \pm 0.001\ [\text{stat.}] \pm 0.021\ [\text{syst.}]$$

‘D’ in signal region

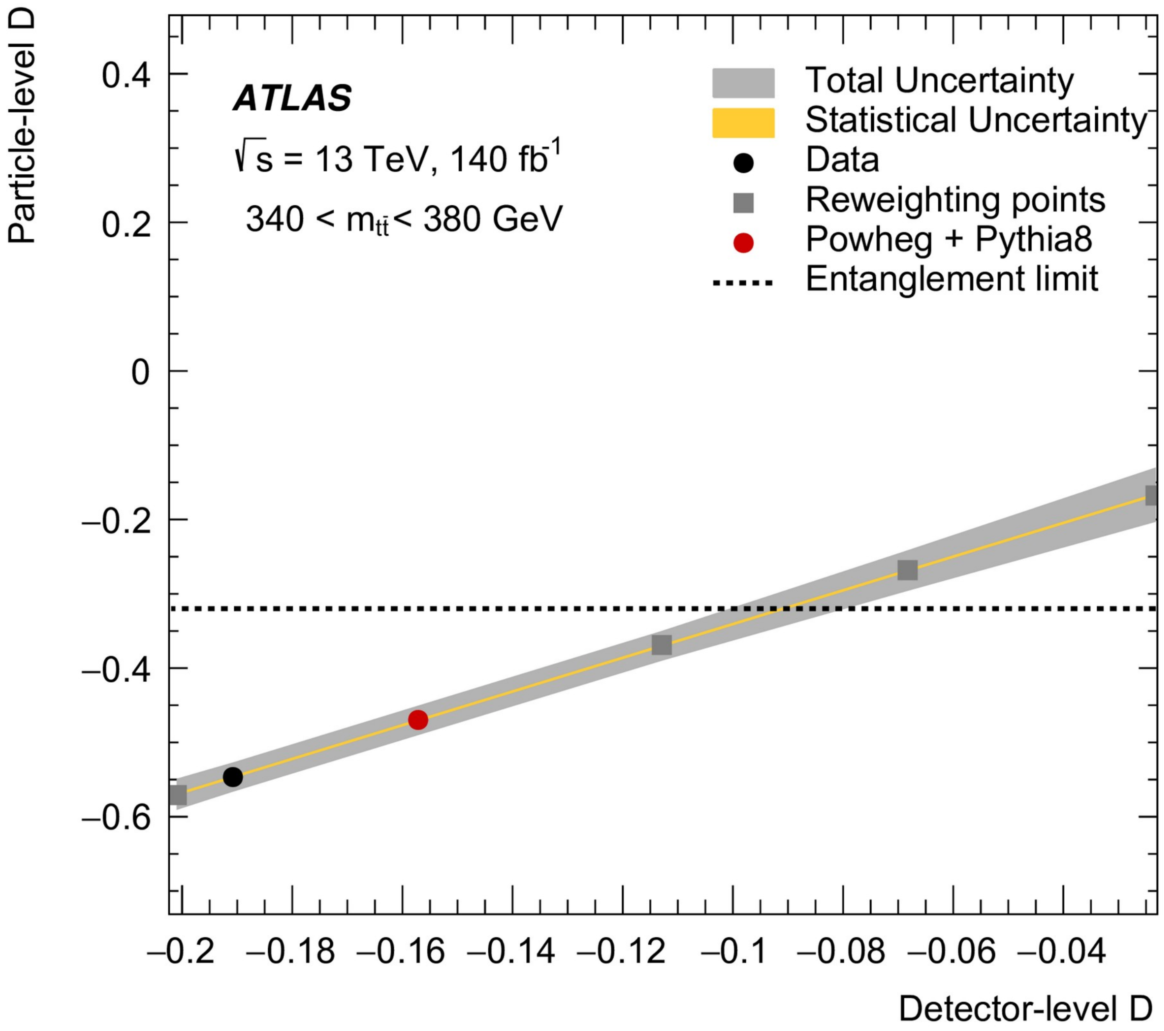


- Measured ‘D’ in signal region:

$$D = -0.547 \pm 0.002\ [\text{stat.}] \pm 0.021\ [\text{syst.}]$$

- Measured value significantly (>> 5 standard deviations) below entanglement limit (−0.322 ± 0.009 for Powheg+Pythia 8) → **observation of entanglement**

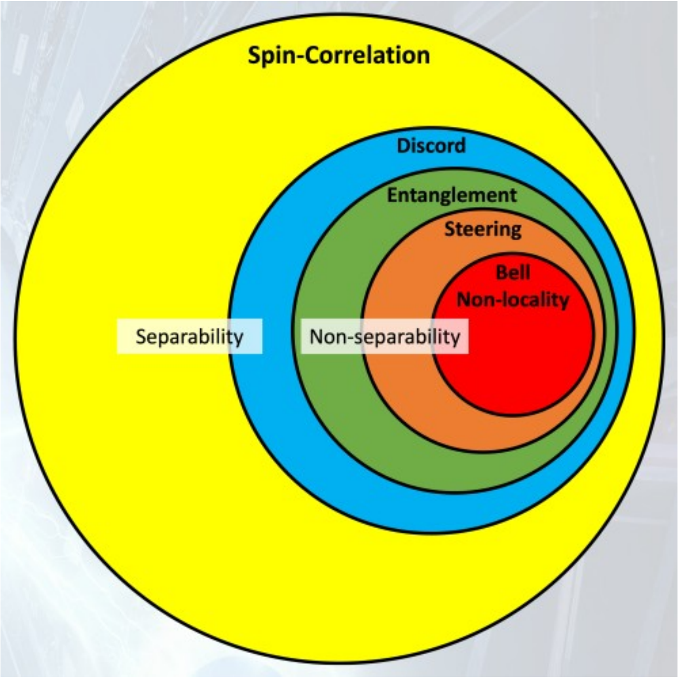
Calibration curve in signal region



Outlook

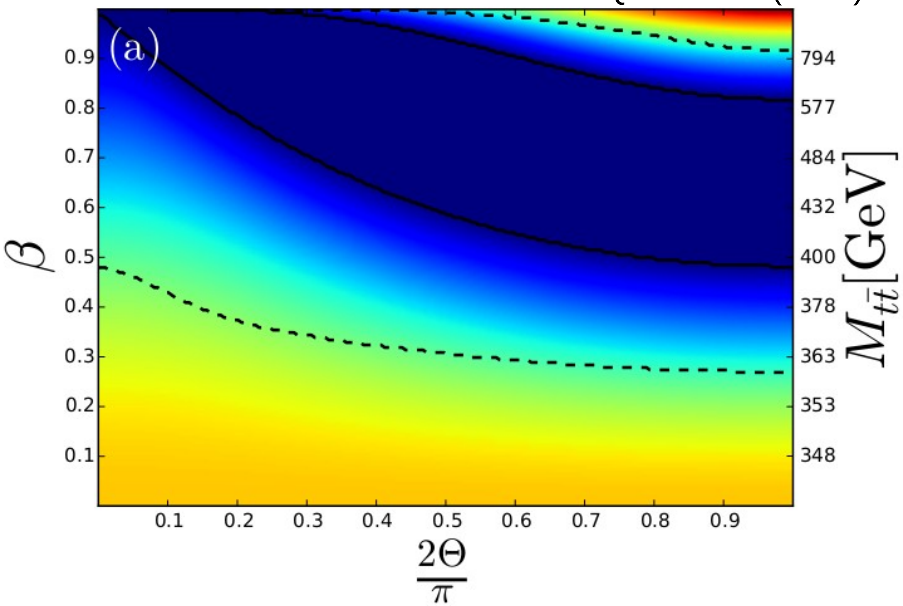
- This measurement can pave the way for measurements of various quantum information concepts at colliders in the future
  - e.g. test Bell inequalities in $t\bar{t}$ production (Quantum 6 (2022) 820)
  - perform tests in other system of particles
    - e.g. test Bell inequalities in Higgs boson decays H → WW (arXiv:2106.01377)

Y.Avik, TOP2023:



tt production, concurrence:

Quantum 6 (2022) 820



Conclusion

- ATLAS experiment observed quantum entanglement using top-quark pairs
  - experimental test in relativistic environment
  - far from the typical length/time/energy-scales of existing measurements
  - first time in quark-pair system
- Up to 20x more data expected with full LHC program
- Potentially, many quantum information measurements at the colliders in the future

Result is described in:
- *ATLAS physics briefing*
- *ATLAS public web page*
- *arxiv:2311.07288*
- Paper was submitted to 'Nature' journal

BACKUP

Monte-Carlo simulated samples

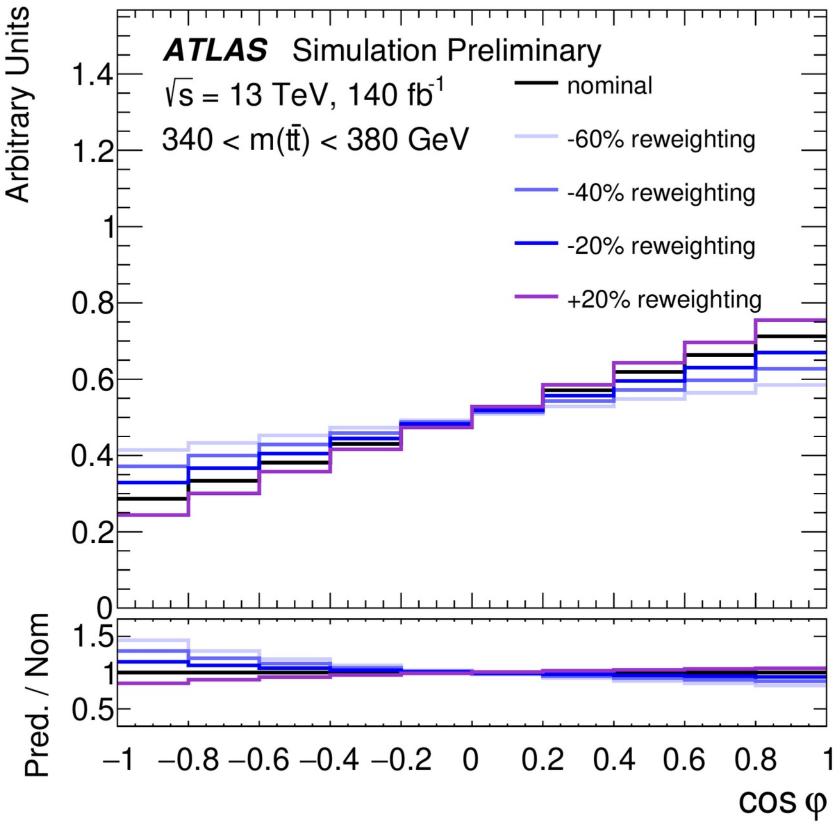
- ttbar signal modelling:
  - Nominal sample: PowhegBox (v2,hvq), hdamp=1.5*m(top)
  - Alternative: PowhegBox-RES (bb4l)
    - Includes off-shell and non-resonant effects
  - Alternative parton-shower sample: Powheg(v2) + Herwig 7.21 (default tune)

- Background processes modelling:
  - Single-top quark, tW-channel: PowhegBox(v2); 5flavor,DR schemes
  - ttbar+X(X=W,Z): MadGraph5_aMC@NLO 2.3.3
  - ttbar+H: PowhegBox(v2), 5 flavor scheme,
  - W/Z + jets: Sherpa 2.2.11, including off-shell effects, NNPDF3.0NNLO
    - @NLO in QCD for ≤ 2 additional partons and @LO for ≤ 5 partons
  - VV(V=W,Z): Sherpa 2.2.2, NNPDF3.0NNLO
    - @NLO in QCD for ≤ 1 parton and @LO for ≤ 3 partons
  - Fakes: data driven

- Typically:
  - precision of the modelling: NLO in QCD
  - for ME generations: NNPDF3.0NLO PDF
  - for parton shower: Pythia 8.230, A14 tune, NNPDF2.3LO

Reweighting of cos(phi) distribution

- The effects of quantum entanglement are fundamental to the calculations in the MC generators and cannot be easily changed
- However, the effects of entanglement can be directly accessed via the observable D in the event
- each event is reweighted according to its parton level values of m(ttbar) and cos φ in order to change $D = -3 \cdot \langle \cos\varphi \rangle$



Signal modelling systematics uncertainties

- Top quark decay: nominal vs. Madspin decay
- Recoil to top: different schemes where partons recoil against b-quark vs. top-quark
- ISR: using the Var3c up/down variants of the A14 tune
- FSR: change by x2 or ½ mu(R) for emissions from the parton shower
- Scales: change mu(R) and mu(F) by factor 2 or ½
- pThard1: Powheg parameter which regulates the definition of the region of phase-space that is vetoed in the showering when matched to a parton shower (nominal vs. pThard=1)
- hdamp: *h*damp is a resummation damping factor that controls the matching of ME to PS and thus effectively regulates the high-pT radiation against which the ttbar system recoils (1.5*m(top) vs. 3*m(top))

Systematic	Relative Size [D = SM (−0.47)]
Top-quark Decay	1.6 %
Parton Distribution Function	1.2 %
Recoil To Top	1.1 %
Final State Radiation	1.1 %
Scale Uncertainties	1.1 %
NNLO Reweighting	1.1 %
pThard1 Setting	0.8 %
Top-quark Mass	0.7 %
Initial State Radiation	0.2 %
Parton Shower	0.2 %
$h_{\rm damp}$ Setting	0.1 %

Most significant systematic uncertainties

- Z → tautau: 20% cross-section uncertainty to account for the uncertainty in the cross-section prediction as well as to account for possible mismodelling of the rate of associated heavy flavour production
- Single-top Wt: cross-section uncertainty (5.3%) and Wt-ttbar diagram overlap subtraction scheme (diagram removal vs. diagram subtraction)

Leading Systematics	Relatvie Size [D = SM (−0.47)]
Top-quark decay	1.6 %
$Z \to \tau\tau$ Cross-section	1.5 %
Recoil To Top	1.1 %
Final State Radiation	1.1 %
Scale Uncertainties	1.1 %
NNLO Reweighting	1.1 %
Parton Distribution Function (5)	0.8 %
pThard1 Setting	0.8 %
Top-quark Mass	0.7 %
Single Top Quark Wt Cross-section	0.4 %

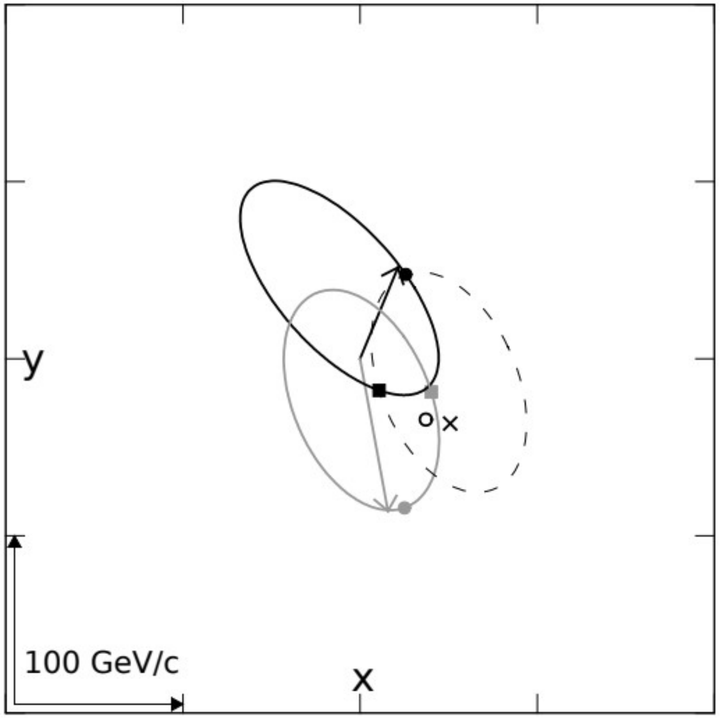
Yields

Process	Inclusive	340 – 380 GeV	380 – 500 GeV	> 500 GeV
$t\bar{t}$	1030000 ± 40000	202000 ± 8000	408000 ± 16000	417000 ± 17000
tW	59800 ± 1100	10330 ± 200	23800 ± 500	25700 ± 500
Z+jets	38000 ± 4000	9300 ± 400	19000 ± 4000	9730 ± 270
$WW/WZ/ZZ$	9140 ± 340	1320 ± 50	3280 ± 120	4540 ± 170
$t\bar{t}X$	2959 ± 6	437.7 ± 2.1	1080.1 ± 3.4	1441 ± 4
fakes	17700 ± 8900	3600 ± 1900	7100 ± 3800	7000 ± 3700
Expectation	1150000 ± 40000	227000 ± 8000	462000 ± 17000	466000 ± 17000
Data	1105403	225056	441196	439151
data/MC	0.96 ± 0.03	0.99 ± 0.04	0.95 ± 0.04	0.94 ± 0.04

Top quark pair reconstruction

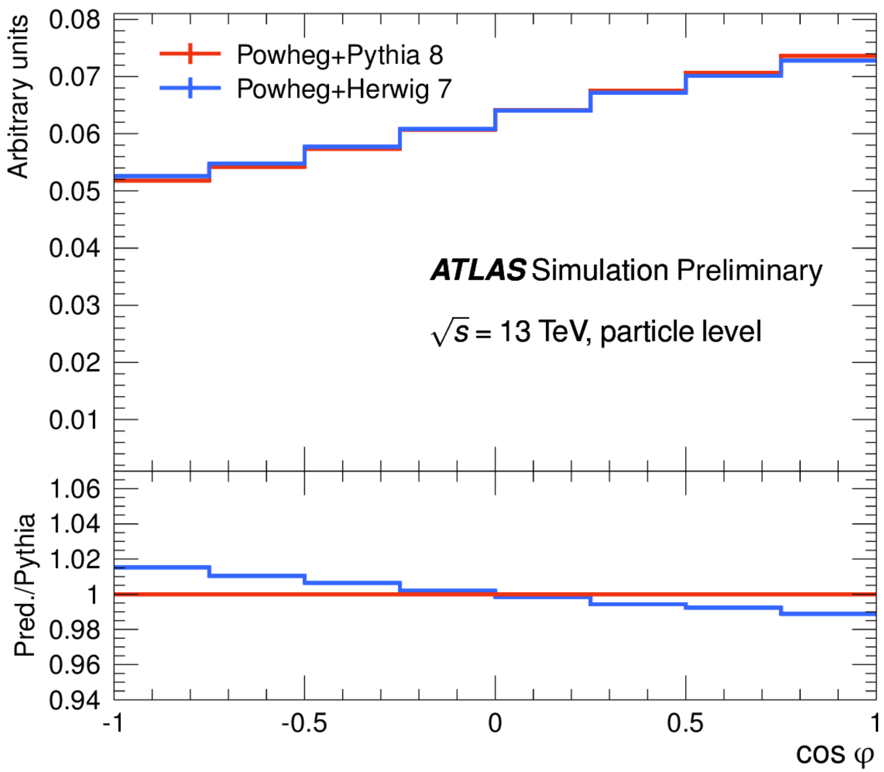
- Reconstruction of top quarks momenta complicated due to 2 neutrinos
  - Several methods developed and used before
    - all(?) use m(top) and m(W) as inputs

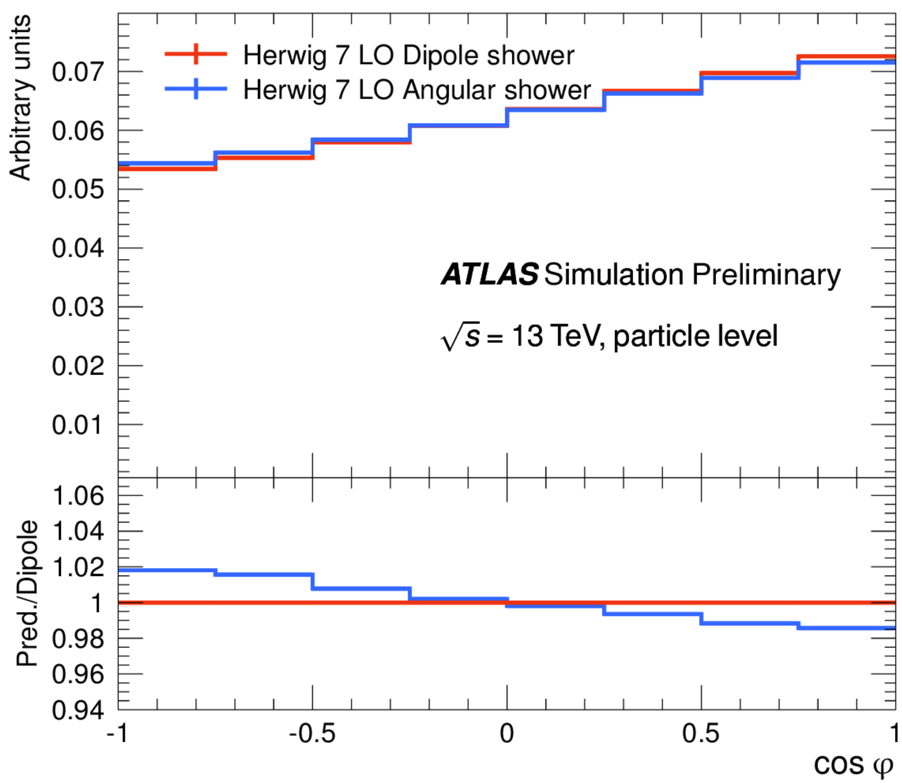
- A combination of various methods used here:
  - The main method: ‘Elipse’ method
    - Calculate two elipses for $p_T(\nu)$ and find intersections
    - ~85% efficiency
  - If ‘Ellipse’ fails → ‘Neutrino Weighting’ method
    - Scans eta(nu), eta(nubar) phase-space
    - Assigns weight to possible solutions by calculating compatibility between p_T of neutrinos and missing transverse momentum
    - Additonal 5 %
  - If both methods fail: simple pairing of leptons with the closest b-jets
    - 10% of events
    - Use highest-p_T jet if only 1 b-tagged jet



Parton shower and hadronization effects

- Large difference between Powheg+Pythia 8 and Powheg+Herwig 7
  - difference at particle and reco. level, while similar at parton level
  - Two main differences:
    - hadronisation model (Lund-string vs. cluster model)
    - shower ordering (pT-ordered vs. angular-ordered shower)
- the majority of the differences seem to originate from the different ordering in the parton shower
- The treatment of spin effects in MC Carlo generators combining the ME with PS requires special attention for future higher-precision quantum information studies

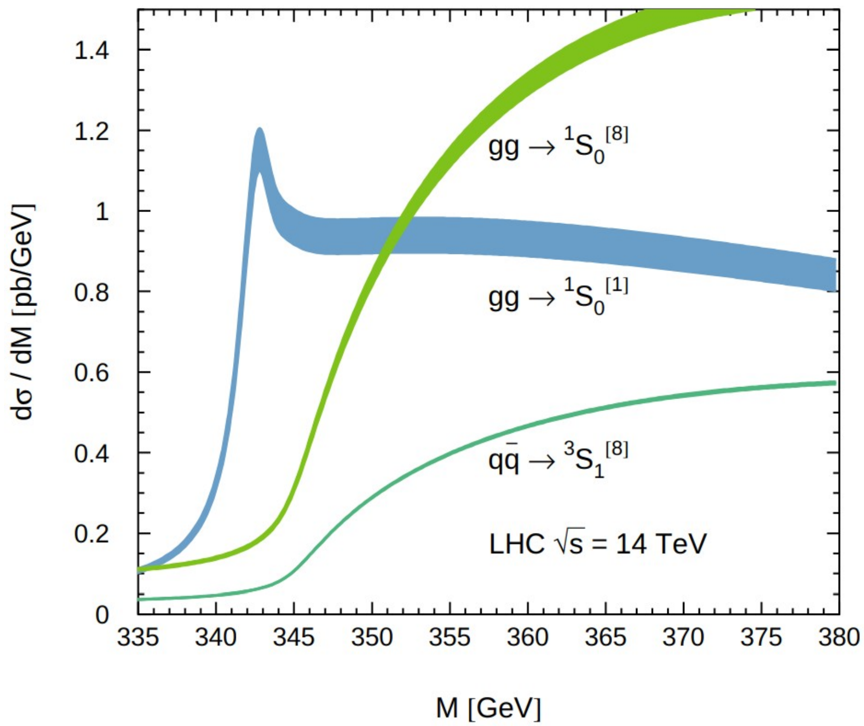




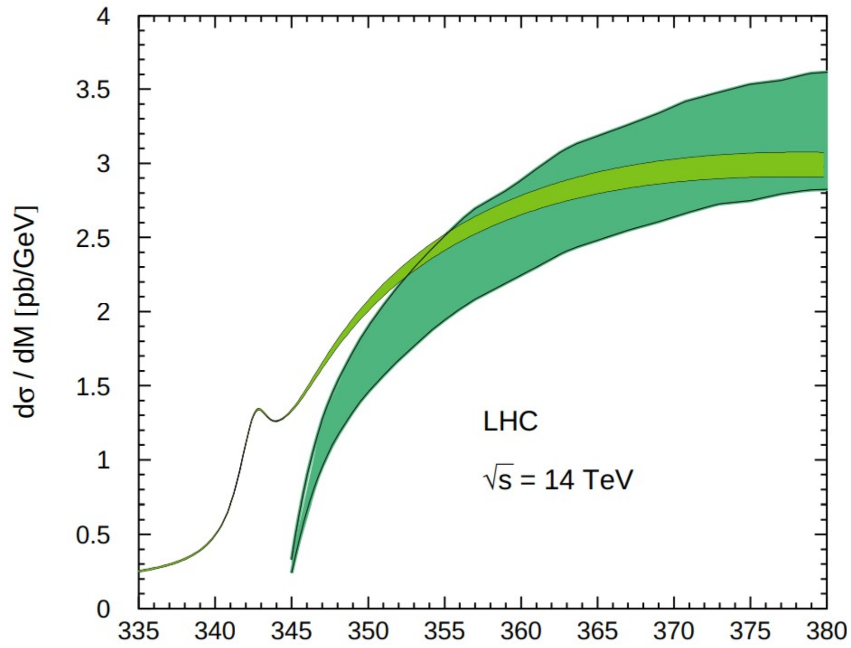
Threshold effects

- Non-relativistic threshold (bound state) effects change the m($t\bar{t}$) dependence of the $t\bar{t}$ cross section
  - Also EWK corrections can have an effect (e.g virtual Higgs correction)
  - These are not included in MC generators

non-relativ. QCD



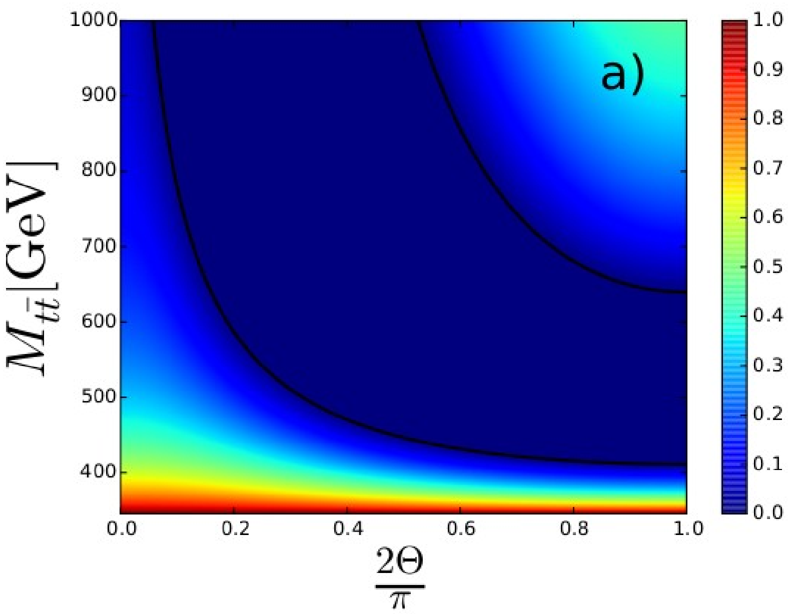
non-relativ. QCD + NLO QCD



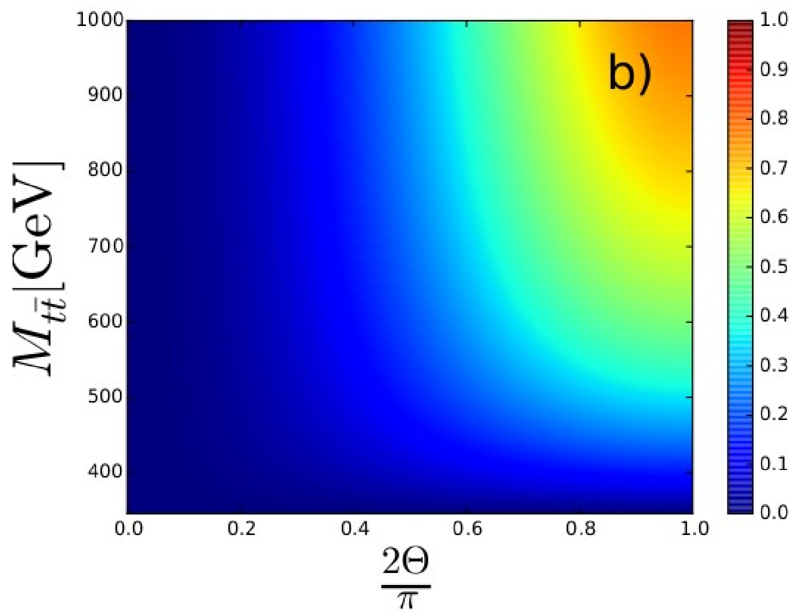
Checks performed that such contributions can be important but should not change the conclusion (observation of entanglement)

Differences for parton level processes

gg → tt



qq → tt



Quantum discord, steering

- Quantum discord:
  - the most basic form of quantum correlations
  - asymmetric between different subsystems
- Quantum Steering:
  - measurements on one subsystem can be used to "steer" the other one
  - a non-local feature that lies between entanglement and Bell non-locality

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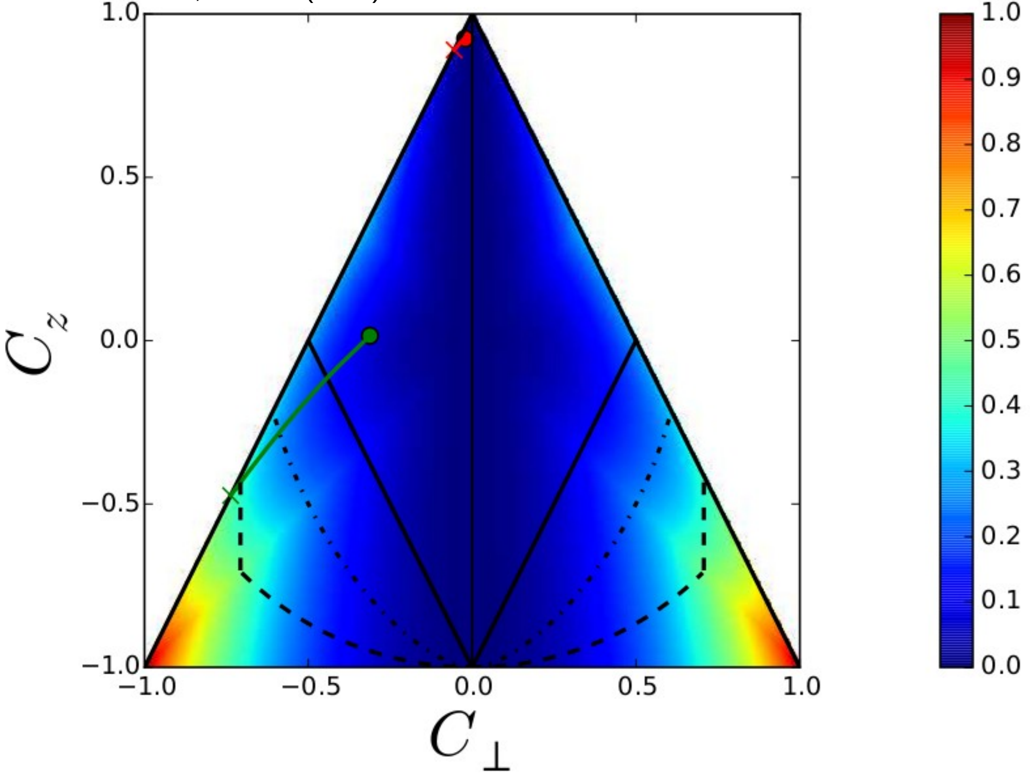

Correlation matrix in the beam basis:

$$C_1 = C_2 = C_\perp \text{ and } C_3 = C_z$$