

Electroweak $SU(2)_L \times U(1)_Y$ gauge model
with ultimately calculable quark and lepton masses
and with theory-enforced astro-particle sector

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(P. Beneš, JH and A. Smetana, JHEP(2021) and to appear)

from CERN Courier 2017:

Asked what single mystery he would like to see solved in his lifetime, Weinberg doesn't have to think for long: *he wants to be able to explain the observed pattern of quark and lepton masses.*

Weinberg the Great



Sentimental overture:

I learned the Higgs mechanism in the SM with its then new concept of spontaneous symmetry breakdown here in Rez in late 60th and early 70th in numerous discussions with prominent low-T physicist Milan Odehnal. Since then for me **the Higgs mechanism is nothing but a clever phenomenology**: Lorentz-invariant and non-Abelian image of the phenomenological description of superconductivity (no electrons at all !!!) developed by Ginzburg and Landau. This raises the question: **What is the microscopic dynamics underlying the Higgs mechanism, known in superconductivity as BCS?**

Basic idea from BCS: fermion mass term is the R - L bridge:

$$\boxed{\bar{\Psi}_R m \Psi_L + \text{H.c.}} \quad \text{Strictly prohibited by } SU(2)_L \times U(1)_Y$$

Each fermion mass m in SM, if generated spontaneously by some **strong dynamics**, breaks spontaneously the electroweak $SU(2)_L \times U(1)_Y$ gauge symmetry down to $U(1)_{em}$.
1. By Goldstone theorem there must exist just three 'would-be' NG bosons giving rise to the masses of W and Z bosons proportional to the fermion mass m . 2. By Nambu there must be the composite Higgs boson as the symmetry partner of NG bosons. 3. No elementary, condensing Higgs fields.

Schematic (generate only one fermion mass), effective (non-renormalizable), suggestive (suggest some vector-boson exchanges) model which employs this idea is based on the renown model of Nambu and Jona Lasinio, clearly motivated by BCS:

$$L_{NJL} = G(\bar{t}_R q_L)(\bar{q}_L t_R) = -\frac{1}{2} G(\bar{q}_L \gamma^\mu q_L)(\bar{t}_R \gamma_\mu t_R)$$

JH, JINR preprint (1982); Delbourgo and Kenny (1982); JH, CERN preprint (1985); JH, PRD (1987); Bardeen, Hill and Lindner, PRD (1990).

Idea known to founders: S. Weinberg (2007): From BCS to the LHC

“There was (and still is) one outstanding issue: just how is the local electroweak symmetry broken? In the BCS theory, the spontaneous breakdown of electromagnetic gauge invariance arises because of attractive forces between the electrons near the Fermi surface. These forces don't have to be strong: the symmetry is broken however weak these forces may be. But this feature occurs only because of the existence of a Fermi surface, so in this respect the BCS theory is a misleading guide for particle physics. In the absence of a Fermi surface, dynamical spontaneous symmetry breaking requires the action of strong forces. **There are no forces acting on the known quarks and leptons that are anywhere strong enough to produce the observed breakdown of the local electroweak symmetry dynamically,** so Salam and I did not assume a dynamical symmetry breakdown; instead we introduced elementary scalar field into the theory, whose vacuum expectation value in the classical approximation would break the symmetry.”

Loophole: If force is strong at a huge scale $\Lambda \sim 10^{16}$ GeV and short-range.
Suggestion of Yanagida: His Lagrangian $SM + V_{FR}$ without Higgses. Name borrowed from Pagels: **QUANTUM FLAVOR DYNAMICS (QFD)**.

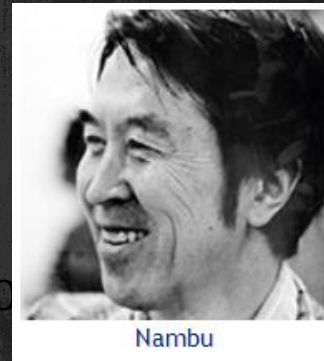
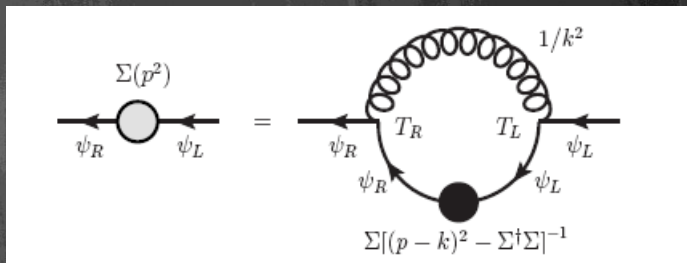
1. QUANTUM FLAVOR DYNAMICS instead of the Higgs sector

1. There are 3 SM fermion flavors (families). ASSUME: All chiral fermion species transform as triplets of gauge $SU(3)$. This implies that QFD does not distinguish between neutrinos, leptons, u - and d -quarks. **Fortunately, there are the EW interactions.**
2. Theoretical necessity: Anomaly freedom demands addition of one triplet of electroweakly sterile ν_{fR} .
3. The dynamics (like QCD) is asymptotically free, and strongly coupled in the infrared. **The only parameter is the (theoretically arbitrary) scale Λ .**
4. Unlike QCD it contains electrically neutral fermions (neutrinos) which can have (unlike quarks) the Majorana masses. This is why QFD is not vector-like, i.e. confining but it is effectively chiral.
5. Important hints from Yanagida's elementary Higgs sector (mainly symmetries) and the NJL models (naïve strong coupling).
6. QFD generates **spontaneously** soft fermion masses ($\Sigma(p^2)$).
7. Dynamical meaning of three fermion flavors (families).

II. Spontaneous generation of soft fermion masses by strong QFD strictly prohibited by QFD and EW interactions

$$S(p)^{-1} = (\gamma^\mu p_\mu - \Sigma(p^2)) \quad \text{the pole of } S(p) \text{ is the fermion mass: } \Sigma(p^2=m^2) \equiv m^2$$

Schwinger-Dyson (gap) equation (L-R bridge)



- Majorana masses of ν_{fR} : $\overline{\nu_{fR}} \Sigma_{fg}(p^2) (\nu_{gR})^c$ in QFD: $3^* \times 3^* = 3_a + 6^*_s$
- Self-consistent generation of all eight flavor gluon masses.

Dirac masses of SM fermions: $\overline{\Psi_{fR}} \Sigma_{fg}(p^2) \Psi_{gL}$ in QFD: $3^* \times 3 = 1 + 8$
in EW: $1 \times 2 = 2$

assume complete breakdown (hint from the Higgs version); different effective couplings g_i of the same order of magnitude !

Wick rotation, separable approximation, ignore flavor mixing

Ultimately: M_f and m_f are the calculable multiples of Λ (QCD)

$$\Sigma(p^2) = M_f^2/p$$

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functional form of Σ crucial for further computations

$$M_f \sim \Lambda$$

$$m_f \sim \Lambda \exp(-1/\alpha_f) \quad (\text{BCS-like formula})$$

where α_f are very small combinations of g_i in separable approximation.

I know no way of verification the reliability of these most important non-perturbative, strong-coupling results.

- L.Susskind argues that non-analytic dependences are natural (unlike using elementary Higgses which are perturbative)
- Using elementary Higgses (sextet Φ for Majorana, singlet ψ and octet ψ_a for Dirac) it is mathematically correct (trivial) but esthetically unnatural:

$$\langle \Phi \rangle \sim \Lambda, \quad \langle \psi \rangle \sim m_f, \quad \langle \psi_a \rangle \sim m_f \quad \text{by hands}$$

- it is interesting (supportive ?!) that there are three algebraically independent invariants which can be made of Φ ($\text{tr } \Phi^+\Phi$, $\text{tr } (\Phi^+\Phi)^2$, $\det \Phi^+\Phi$) and also of ψ and ψ_a (ψ^2 , $\psi_a \psi_a$, $d_{abc} \psi_a \psi_b \psi_c$).

Self-consistent flavor gluon mass generation due to spontaneous generation of Majorana masses M_f

Sterile neutrino current $j_a^\mu = \frac{1}{2} \bar{n} \gamma^\mu \frac{1}{2} \Lambda_a n$

Where $n = \nu_R + (\nu_R)^c$ $\frac{1}{2} \Lambda_a = \frac{1}{2} \lambda_a P_R + \frac{1}{2} (-\lambda_a^T) P_L$

Use Dirac equation with $\Sigma(p^2)$:

Pole part of the WT identity is $\Gamma_{a,pole}^\mu(p,p) = \frac{q^\mu}{q^2} P_a(p,p)$

Here $P_a(p,p)$ are effective vertices between eight ν_R -composite 'would-be NG bosons and the neutrino pair:

$$P_a(p,p) \sim \left\{ \Sigma(p), \frac{1}{2} \lambda_a \right\} \gamma_5 \quad a=1,3,4,6,8$$

$$P_a(p,p) \sim \left[\Sigma(p), \frac{1}{2} \lambda_a \right] \quad a=2,5,7$$

All eight flavor gluons acquire masses $\sim M_f$. They are mixed-parity.

Four symmetry partners of eight 'would-be' NG bosons in the complex symmetric sextet Φ remain in the spectrum

Under $U(3)$ $\Phi \rightarrow U\Phi U^T$: Hence it can be parameterized as

$$\Phi = \exp(i\lambda_a \theta_a) \frac{1}{\sqrt{2}} \exp(i\theta) \text{diag}[v + \chi] \exp(i\lambda_a^T \theta_a) \approx [\text{diag}(v + \chi) + i\theta + i\theta_a \{\text{diag}(v + \chi), \frac{1}{2} \lambda_a\}_{\pm}]$$

θ_a are 8 'would-be' NG bosons giving masses to C_a

θ is the neutrino-composite Majoron

χ_i are three neutrino composite Higgses with masses $\sim \Lambda$.

No idea how to do reliable computations.

PS dynamical perturbation theory using vertices of WT identities

$$\Gamma_A^\mu(p', p) = eQ_i[\gamma^\mu - (p' + p)^\mu \Sigma'(p', p)]$$

$$\Gamma_W^\mu(p', p) = \frac{e}{2\sqrt{2}\sin\theta_W} \{[\gamma^\mu - (p' + p)^\mu \Sigma'(p', p)]T^+ - [\gamma^\mu \gamma_5 T^+ - \frac{(p'-p)^\mu}{(p'-p)^2}(\Sigma(p') + \Sigma(p))\gamma_5 T^+]\}$$

$$\Gamma_Z^\mu(p', p) = \frac{e}{\sin 2\theta_W} \{[\gamma^\mu - (p' + p)^\mu \Sigma'(p', p)](T_{3L}^i - 2Q_i \sin^2\theta_W) - [\gamma^\mu \gamma_5 - \frac{(p'-p)^\mu}{(p'-p)^2}(\Sigma(p') + \Sigma(p))\gamma_5]T_{3L}^i\}$$

I. Pseudo-vector vertices imply W,Z masses (Pagels-Stokar formula):

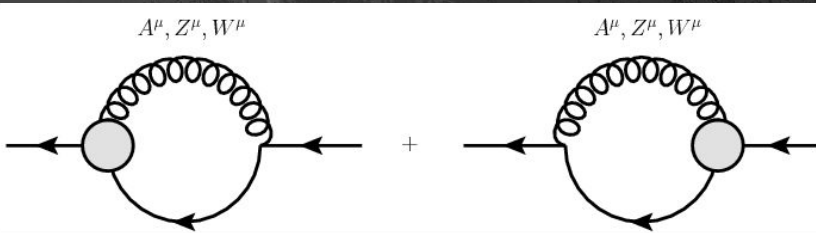
$$m_W^2 = \frac{1}{4}g^2 \frac{5}{4\pi} \sum m_f^2 \quad m_Z^2 = \frac{1}{4}(g^2 + g'^2) \frac{5}{4\pi} \sum m_f^2$$

Saturation by one mass: $m = 390 \text{ GeV}$.

No fundamental EW scale !!!

$$F_f^2 = 8N \int \frac{d^4p}{(2\pi)^4} \frac{\Sigma_f^2(p^2) - \frac{1}{4}p^2(\Sigma_f^2(p^2))'}{(p^2 + \Sigma_f^2(p^2))^2}$$

II. Vectorial vertices are used for calculating the fermion mass splitting in terms of $e, Q_i, \sin\theta_W, m_f/m_{W,Z}$. Not surprisingly the result is not good.

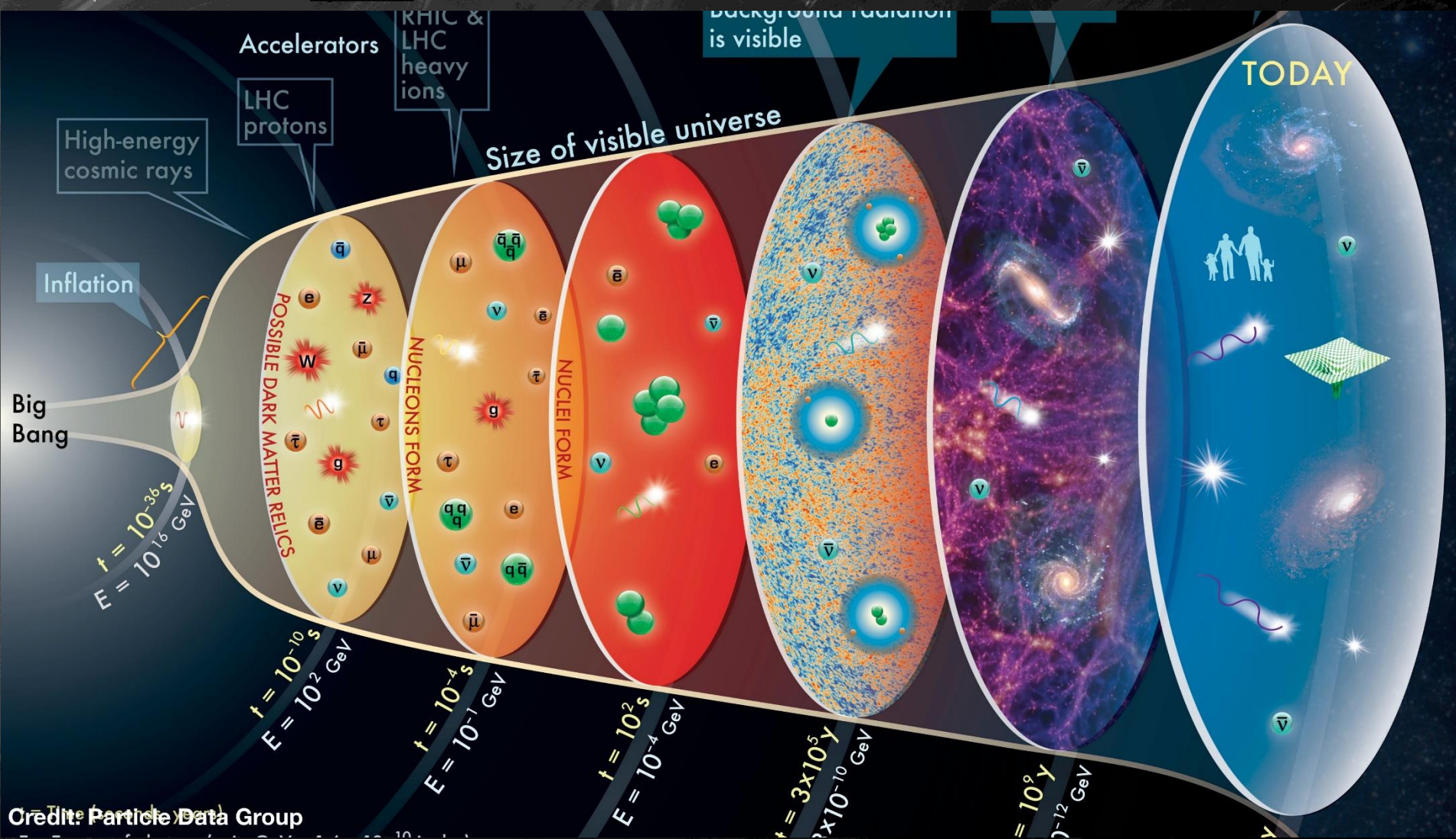


$$\Sigma'(p', p) \equiv \frac{\Sigma(p') - \Sigma(p)}{p'^2 - p^2}$$

III. The composite Higgs boson h (symmetry partner of 'would-be' NG bosons) is visualized within NJL. Couplings of h with A, W, Z are f.-loop-generated.

Value of the rigid (reductionist's) "scenario" ?

1. Why "ultimately": The dynamics (QFD) we look for has to be strong i.e., *difficult to handle (no reliable computations)*. Fortunately, we are accustomed in SM to the strong dynamics: *IN QCD WE TRUST*: all hadron masses of first generation in the chiral limit are, ultimately, the calculable multiples of $\Lambda_{\text{QCD}} \sim 200 \text{ MeV}$. In QFD we generate three masses m_f which, also ultimately, must be the calculable multiples of $\Lambda_{\text{QFD}} \sim 10^{16} \text{ GeV}$.
2. We use the idea of Pagels and Stokar, and compute (ultimately) the fermion mass splitting within flavors in terms of Σ in WT identities in terms of known weak hypercharges and $m_f/m_{W,Z}$.
3. REMARKABLY, there is *theory-enforced sector*, which contains degrees of freedom needed *for the description of the astro-particle physics phenomena*, otherwise postulated phenomenologically: sterile neutrinos (for seesaw, for baryo-genesis via leptogenesis), candidate for dark matter, inflaton, Starobinsky inflation (quadratic gravity).



Astro-particle (BSM) phenomena correlated by QFD

1. Explain tiny neutrino masses: ν_R necessary. (M. Peskin(2022): an oversight of the founders ?!)
 - (i) Introduce tiny Higgs Yukawa couplings with ν_{FR}
 - (ii) Postulate huge Majorana masses (seesaw) of ν_{FR} .
 - (iii) In our model ν_R enforced by anomaly freedom. Seesaw follows.
2. Explain baryogenesis in the Universe (A. Sakharov).
 - (i) Postulate GUT.
 - (ii) Baryogenesis without GUT (Fukugita, Yanagida): Postulate massive decaying Majorana neutrinos ν_{FR} .
 - (iii) Our model apparently has all properties required by the F-Y baryogenesis.

Astro-particle (BSM) phenomena correlated by QFD

3. Explain cosmic inflation.

- (i) Postulate one (or several) scalar inflaton field(s) Φ (Guth, Linde,...)
- (ii) Postulate a NJL-like strong interaction of v_{FR} forming the composite inflaton(s) (Barenboim).
- (iii) Quadratic gravity (Starobinsky) gives rise to inflation.
- (iv) Emergent gravity (Sakharov(1968)): QFT (in our case QFD) at a huge scale induces quadratic gravity, popular candidate for quantum gravity (Donoghue).

4. Explain the dark matter. Many candidates, e.g.:

- (i) Postulate WIMPs.
- (ii) Suggest as a candidate the primordial black holes.
- (iii) Suggest as a candidate the pseudo NG axion (Wilczek, SW, Sikivie)
- (iv) Suggest as a candidate the pseudo NG Majoron (Berezinsky, Valle)
- (v) In our model Majoron is the necessary consequence of the Goldstone theorem.

Why I am here: (Berezhiani and Khoury):

1. Universe without dark energy: Cosmic acceleration from dark matter-baryon interactions (PRD95(2017)).
2. Theory of dark matter superfluidity (PRD92(2015)).

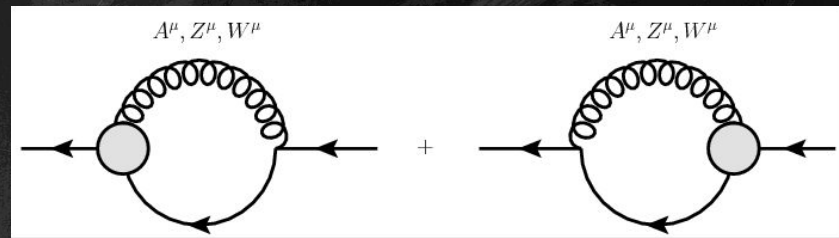
Thanks for your attention



"Before I came here I was confused about this subject. Having listened to your lecture I am still confused. But on a higher level."

Enrico Fermi

(2) Use the new polar-vector vertices for computing the fermion mass splitting in f (Pagels)



$$\Sigma_f^i(p^2) = -i \frac{m_f^2}{\sqrt{p^2}} A_f^i(p^2) + i \sqrt{p^2} B_f^i(p^2)$$

$$\Sigma_f^i \equiv \Sigma_f + \delta_{A,Z,W}^i \Sigma_f$$

where $A_f^i(p^2)$ and $B_f^i(p^2)$ are the explicit well-defined functions.

?1 How can the weakly coupled EW interactions give rise to the observed huge mass $m_t - m_b$ splitting? (hope: nonlinear pole equation★)

?2 How can the EW interactions, having the IDENTICAL couplings for all three families produce mass splitting not identical for all three families? (hope: dependence upon $m_f/m_{W,Z}$).

Unfortunately, the numerical solution of the pole equation

$$m_f^{i2} = \Sigma_f^i + \Sigma_f^i(p^2 = m_f^{i2}) \star$$

yields only the small unrealistic fermion mass splitting

(Bona fide) conclusions (more of a framework or scenario)
(We know no way of knowing whether $M_{fR} \gg m_f$
is the inherent property of QFD at strong coupling or not)

1. There is no generic electroweak (Fermi) scale. Only huge Λ .
2. Three active neutrinos are the light Majorana fermions.
3. Three superheavy Majorana neutrinos (seesaw, leptogenesis) are on the same footing with other SM fermions.
Ordinary matter: QCD nucleons qqq ;
dark matter: QFD composites $\nu_R \nu_R \nu_R$.
4. Calculation (post-dictions) of fermion masses hampered by theoretical uncertainties. Neutrino masses predictable in terms of m_f^ν and M_{fR} by the seesaw formula.
5. We post-dict the composite Higgs h and predict h_3 and h_8 at Fermi scale: Not all three families are alike.
6. We don't ask forgiveness yet.

II. SPONTANEOUS GENERATION OF m_f and M_f by QFD

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1. Gauge symmetry of the model is $SU(3) \times SU(2)_L \times U(1)_Y$.
2. Majorana masses of ν_{fR}



STRICTLY PROHIBITED by QFD: $3^* \times 3^* = 3_a + 6^*_s$ does not contain unity

3. Dirac masses of SM fermions

$$\Psi_{fR} M_f \Psi_{fL}$$

$$(3^* \times 3 = 1 + 8)$$

STRICTLY PROHIBITED by EW symmetry.

4. If dynamically generated by QFD M_f and m_f must be the calculable multiples of Λ .

