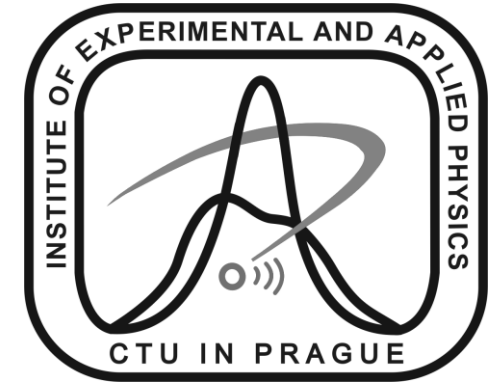


Regular seminar of
Division of Elementary Particle Physics
FZU - Institute of Physics of the Czech Academy of Sciences
Prague
June 6, 2024



Optical properties of cosmic void

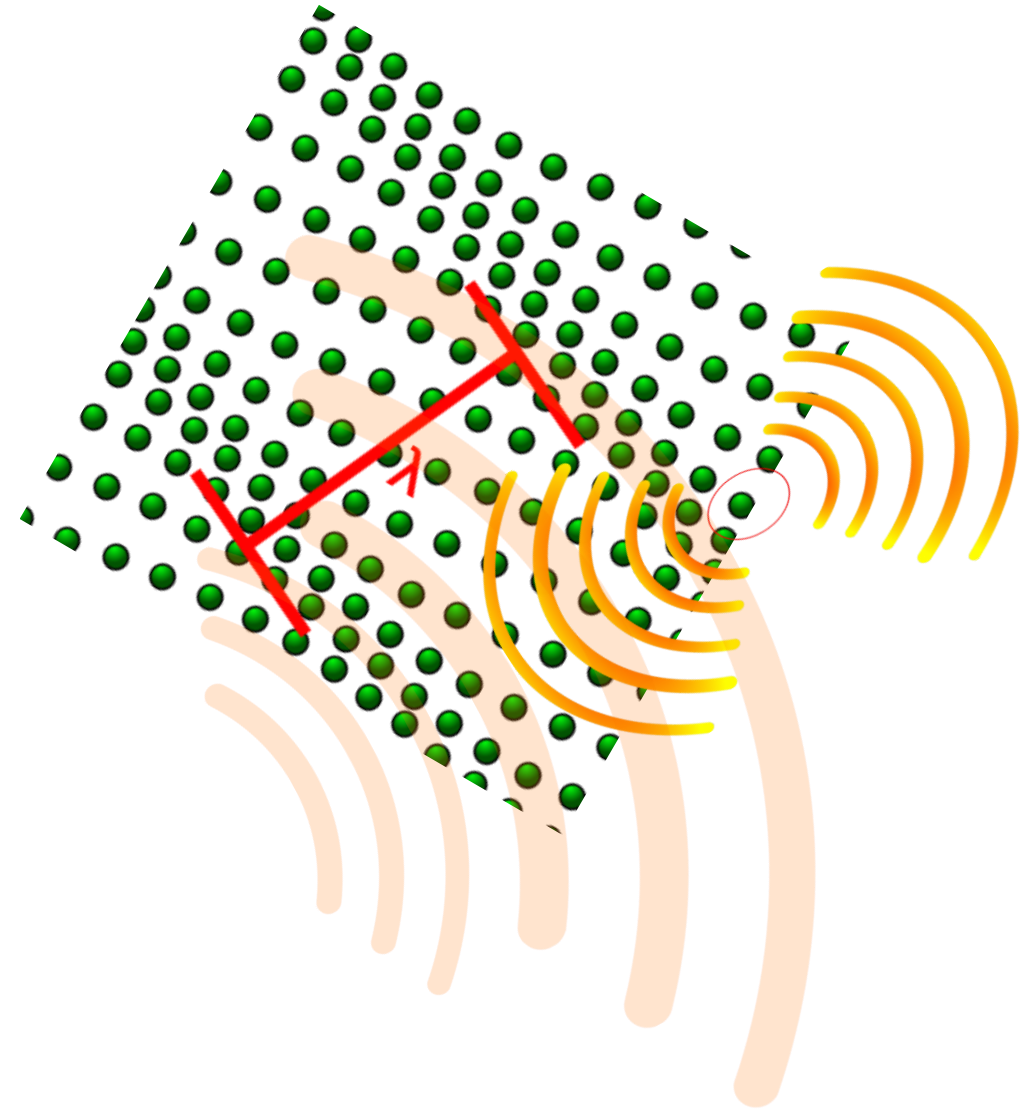
ADAM SMETANA

INSTITUTE OF EXPERIMENTAL AND APPLIED PHYSICS,
CZECH TECHNICAL UNIVERSITY IN PRAGUE, CZECH REPUBLIC

Optical properties of a medium

Optical properties are dominated by the most effective component of the medium in terms of

- the lightness of the particles
- their abundance
- their strength of electromagnetic interactions



Cosmic void

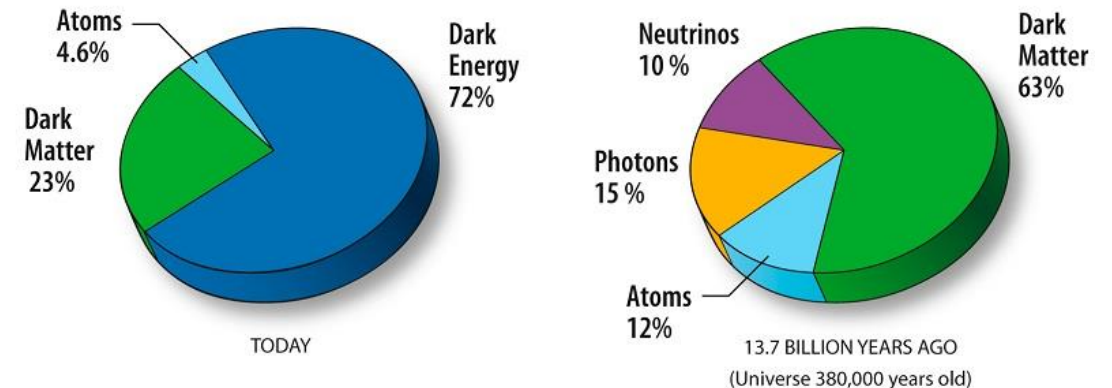
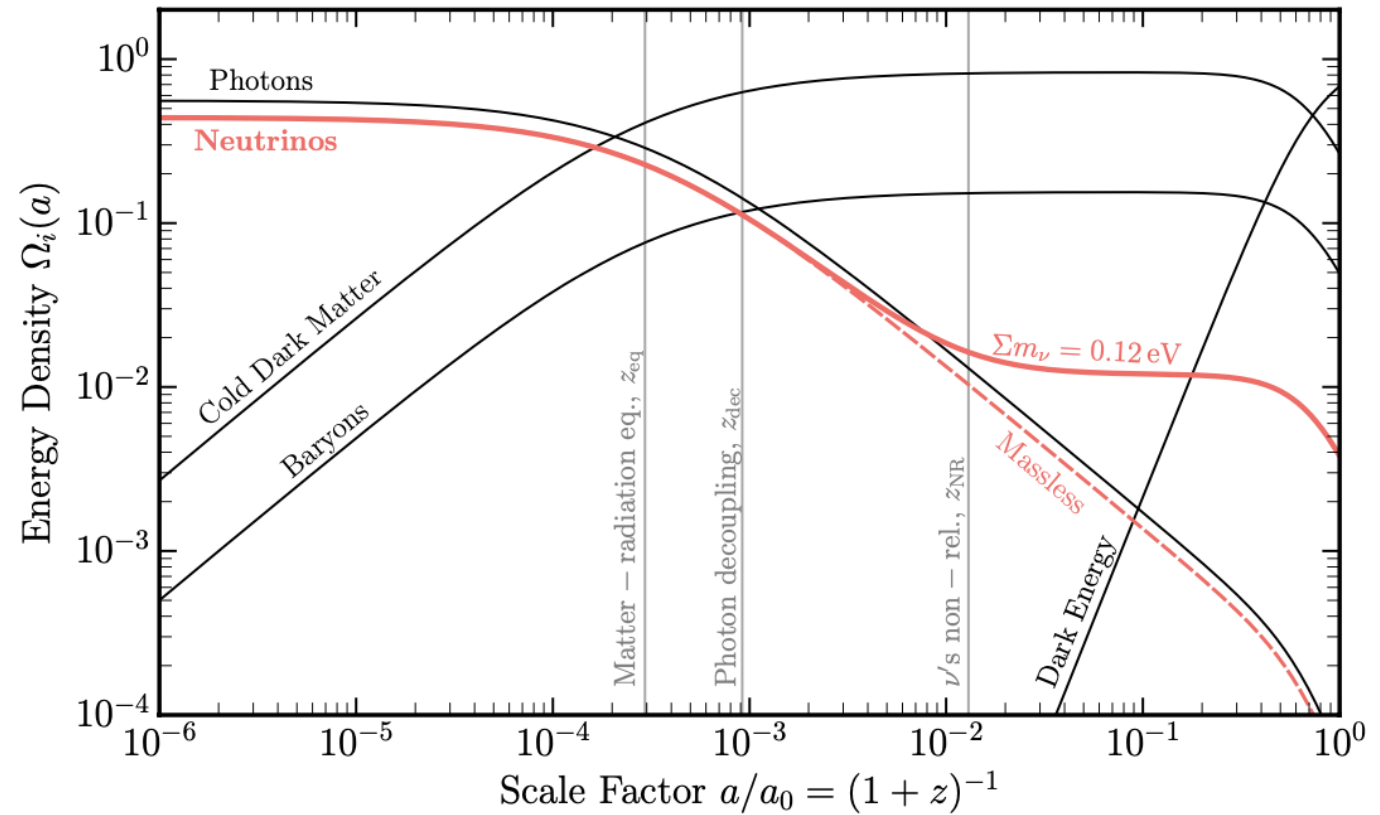
How **empty** is the „empty“ space in the Universe?

Energy density budget of the universe

- Neutrinos (contribute to DM) $\sim 0.5\%$
- Photons $\sim 0.01\%$
- Dark matter (DM) $\sim 23\%$
 $\sim 0.4 \text{ GeV/cc (local)}$

Number density budget of the universe

- Neutrinos $\sim 336 / \text{cc}$
- Photons $\sim 440 / \text{cc}$
- Protons/ electrons $\sim 3 \times 10^{-6} / \text{cc}$
- DM (WIMPs 100 GeV) $\sim 2 \times 10^{-7} / \text{cc}$
- DM (axion, 5 μeV) $\sim 10^{14} / \text{cc (local)}$

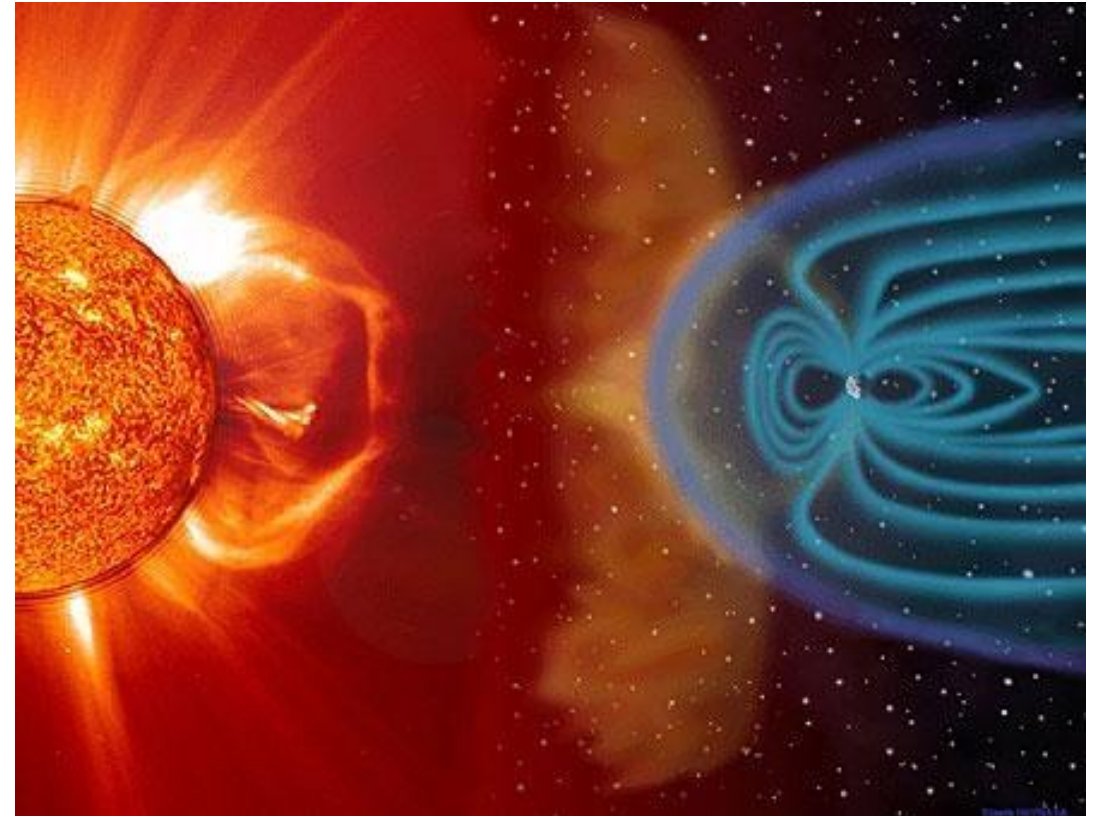


Solar wind

Locally, the Earth environment is given by the solar activity.

Solar wind (SW)

- Composition: electrons \sim protons $>$ alpha $>$ heavier ions
- at 1 AU from the Sun
 - average particle density $\sim 10 \text{ cm}^{-3}$ and quasi-neutrality
 $\rightarrow n_e \approx n_p \approx 5 \text{ cm}^{-3}$
 - at scales of optical wavelengths range $\sim 1 \mu\text{m}$, SW is practically a **collision-less ideal plasma**



Outline

- 1. Propagation of electromagnetic wave in medium**
- 2. Solar wind optical noise for LISA**
- 3. Optical properties of neutrino media**
- 4. Optical properties of axionic DM**

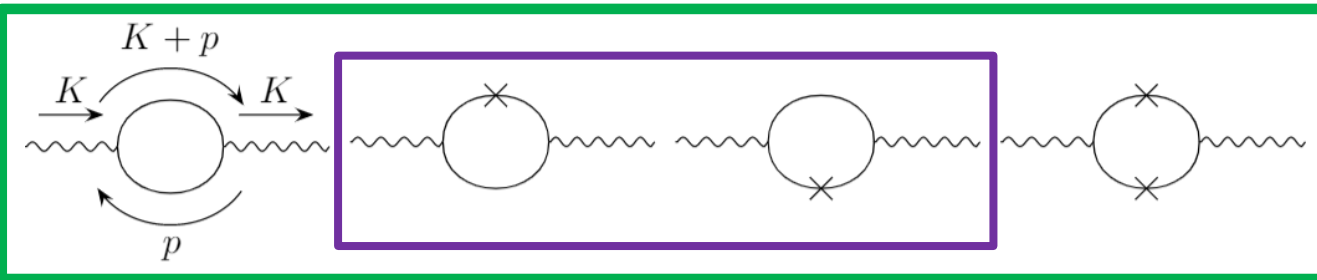
Propagation of electromagnetic wave in medium

Forward photon scattering

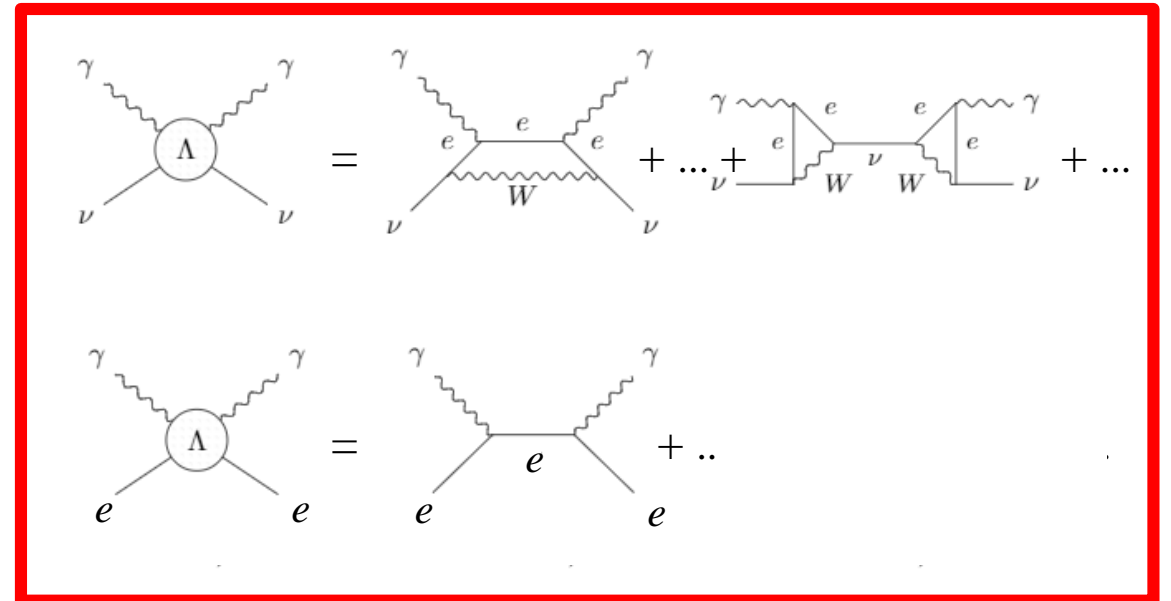
The essence of the medium effect on the propagation is that photon is experiencing the **forward scattering** on the particles of the medium.

The quantity of interest is the photon polarization tensor:

$$i\Pi_{\mu\nu} = (-1) \int \frac{d^4p}{(2\pi)^4} S(p+K) \Lambda_\mu(K) S(p) \Lambda_\nu(K)$$



Example of scattering on neutral particles (neutrinos) vs charged particles (electrons)



Real time formalism

$$S(p) = S^{vac}(p) + (\hat{p} + m)\Gamma_F(p, \mu)$$

$$S^{vac}(p) = \frac{1}{\hat{p} - m + i\epsilon}$$

$$\Gamma(p) = 2\pi i \delta(p^2 - m^2) [\theta(p_0) n_F(p, \mu) + \theta(-p_0) n_F(p, -\bar{\mu})]$$

Refractive index of a medium

Refractive index of a medium $n = \frac{k}{\omega}$. where $K^2 = \omega^2 - k^2$ is given by

$$n = \sqrt{\varepsilon(K)\mu(K)}$$

where permittivity is $\varepsilon(K) = 1 - \frac{\Pi_{\parallel}}{K^2}$,

and permeability is $\frac{1}{\mu(K)} = 1 + \frac{K^2\Pi_{\perp} - \omega^2\Pi_{\parallel}}{k^2K^2}$.

The in-medium photon polarization tensor is

$$\Pi_{\mu\nu} = \Pi_{\perp}\mathcal{P}_{\mu\nu} + \Pi_{\parallel}\mathcal{Q}_{\mu\nu}$$

It results in the **EQUATION** for K^2 : $\Pi_{\perp}(K^2) = K^2$

If $\Pi_{\perp}(K^2 \rightarrow 0) \rightarrow \text{const.}$ (the case of charged particles) the solution is straightforward and leads to

$$n \simeq 1 - \frac{\Pi_{\perp}}{2\omega^2}$$

which is in accord with Sellmeier equation for refractive index n for elmag wave of frequency ω

$$n = \sqrt{1 - \frac{\omega_p^2}{\omega^2}} \doteq 1 - \frac{1}{2} \frac{\omega_p^2}{\omega^2} + \dots$$

Case of neutral particles

Due to momentum dependence of the effective electromagnetic vertices

magnetic dipole moment

$$-f_M(q^2)i\sigma_{\mu\nu}q^\nu$$

electric dipole moment

$$f_E(q^2)\sigma_{\mu\nu}q^\nu\gamma_5$$

charge radius

$$f_Q(q^2)\gamma_\mu$$

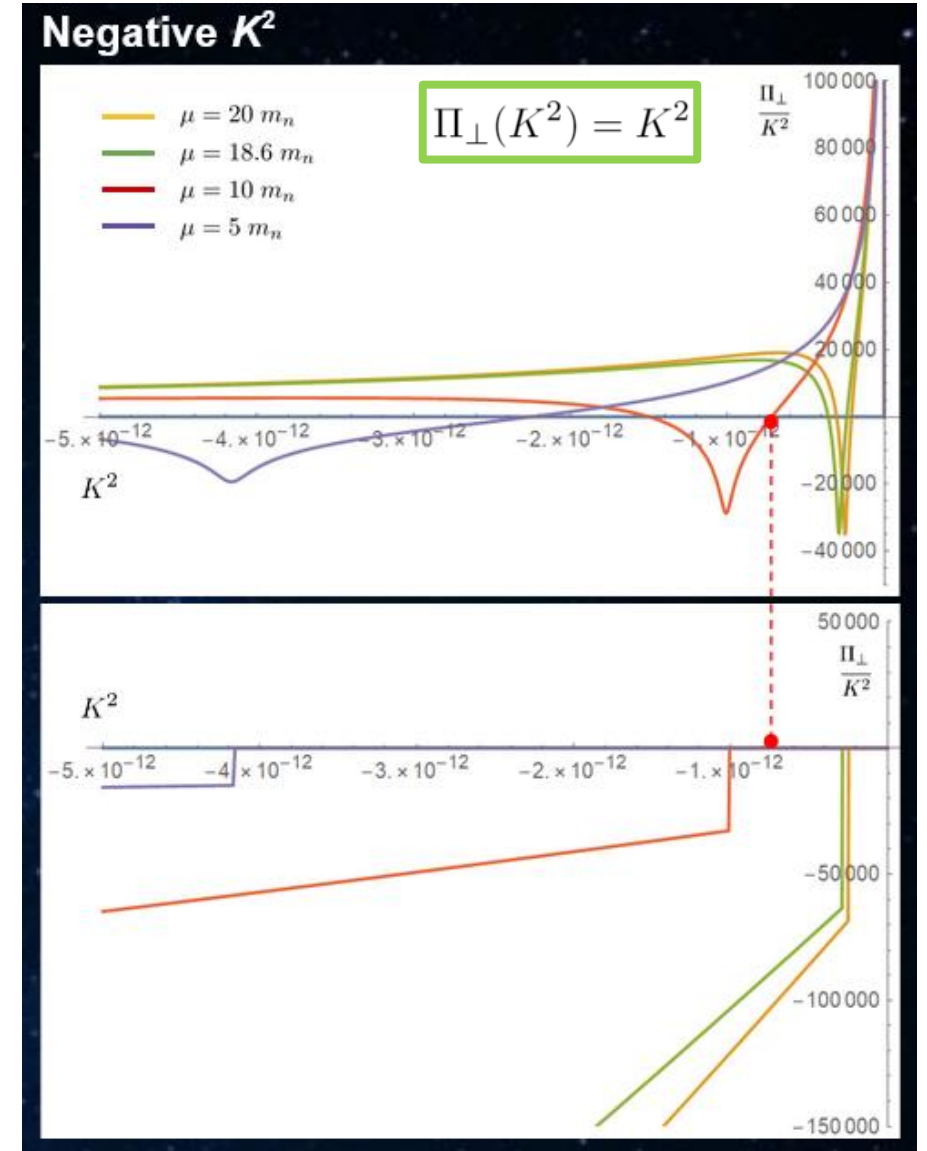
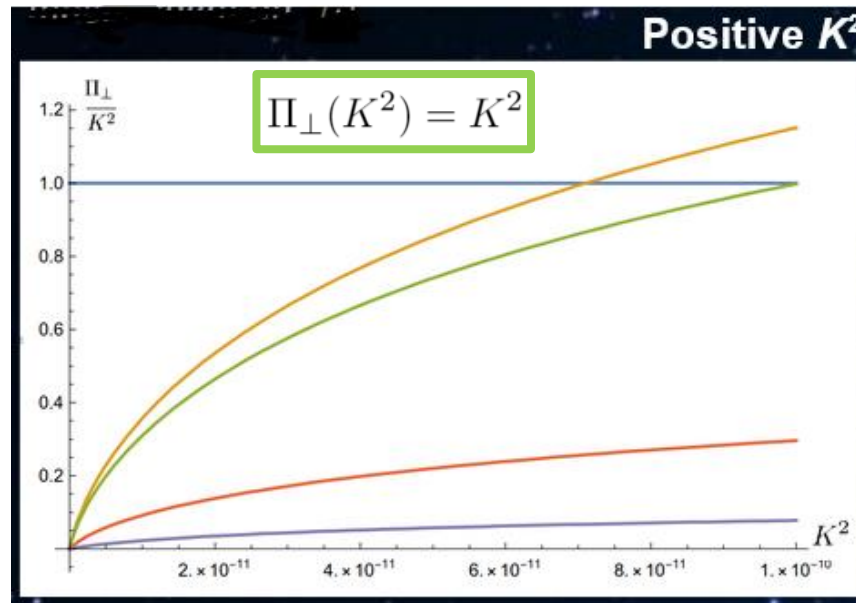
anapole moment

$$f_A(q^2)(q^2\gamma_\mu - q_\mu\not{q})\gamma_5$$

the resulting polarization function behaves as

$$\Pi_\perp(K^2 \rightarrow 0) \rightarrow \text{const.}$$

and solving for n is **not trivial**, as shown on graphs for the case of neutrons and their magnetic moment.



General formalism (covariant) & Parity violation

In medium the longitudinal photon polarization is developed:

$$e_3^\mu = \frac{\tilde{u}^\mu}{\sqrt{-\tilde{u}^2}}$$

where:
$$\tilde{u}^\mu = u^\mu - \frac{(k \cdot u)}{k^2} k^\mu - \frac{(e_1 \cdot u)}{e_1^2} e_1^\mu - \frac{(e_2 \cdot u)}{e_2^2} e_2^\mu$$

is the fully transverse projection of the medium 4-velocity.

The Lorentz invariant „momentum“ quantity is

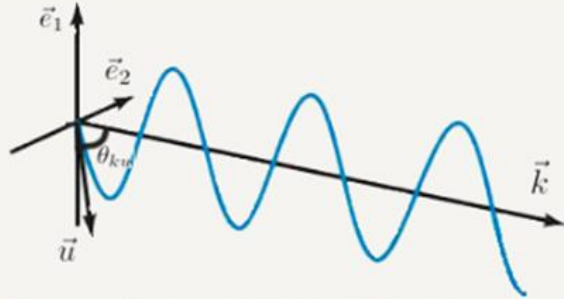
$$K = \sqrt{(k \cdot u)^2 - k^2(1 + (e_1 \cdot u)^2 + (e_2 \cdot u)^2)} = \omega \gamma |n - \beta \cos \theta_{ku}|$$

This is in contradiction with the expression used in literature that leads to **incorrect** angular dependence.

$$K' = \sqrt{(k \cdot u)^2 - k^2} = \sqrt{K^2 - (1 - n^2)\omega^2}$$

Arguments in favor of our result:

- $K \in \mathcal{R}, \geq 0$.
- $K = 0$ if $k^\mu \parallel u^\mu$, $n < 1$ and $\beta = n$



- Electromagnetic wave 4-momentum:
 $k = (\omega, \mathbf{k}) = \omega(1, n\hat{\mathbf{k}})$, $k^2 = \omega^2(1 - n^2)$,
- medium 4-velocity:
 $u = (u_0, \mathbf{u}) = \gamma(1, \beta\hat{\mathbf{u}})$, $u^2 = 1$,
- Transverse polarization:
 $e_{1,2} = (e_{1,2}^0, \mathbf{e}_{1,2}) = (0, \hat{\mathbf{e}}_{1,2})$, $e_{1,2}^2 = -1$,
- Longitudinal polarization: e_3
 is obtained from transversality $k_\mu e_i^\mu = 0$. $e_{\mu i} e_j^\mu = -\delta_{ij}$,

General formalism & Parity violation

In parity-violating media, the polarization tensor develops a parity-violating part:

$$\Pi^{\mu\nu} = \Pi_T P_T^{\mu\nu} + \Pi_L P_L^{\mu\nu} + \Pi_P P_P^{\mu\nu}$$

which is spanned the parity-violating projector, so the full set is:

$$P_{P\nu}^\mu e_{1,2}^\nu = \pm i e_{2,1}^\mu$$

$$P_{T\nu}^\mu e_{1,2}^\nu = e_{1,2}^\mu, \quad P_{T\nu}^\mu e_3^\nu = 0,$$

$$P_{L\nu}^\mu e_3^\nu = e_3^\mu, \quad P_{L\nu}^\mu e_{1,2}^\nu = 0,$$

$$P_L^{\mu\nu} = \frac{\tilde{u}^\mu \tilde{u}^\nu}{\tilde{u}^2},$$

$$P_T^{\mu\nu} = g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2} - \frac{\tilde{u}^\mu \tilde{u}^\nu}{\tilde{u}^2},$$

$$P_P^{\mu\nu} = \frac{i}{K} \varepsilon^{\mu\nu\alpha\beta} k_\alpha \tilde{u}_\beta$$

Birefringence

Birefringence arises in gyroelectromagnetic media such as cosmic plasma exposed to intergalactic magnetic fields (IGMF), or in chiral media such as asymmetric neutrino gases, etc.

Royer, Phys. Rev. 174, 1719 (1968)

It arises from parity-violating part of the photon polarization tensor

$$\Pi_P^{\mu\nu} = \frac{i}{K} \varepsilon^{\mu\nu\alpha\beta} k_\alpha u_\beta \Pi_P.$$

Nieves & Pal, Phys. Rev. D 39, 652 (1989)

Consequently, R- and L-polarizations get different dispersion relations

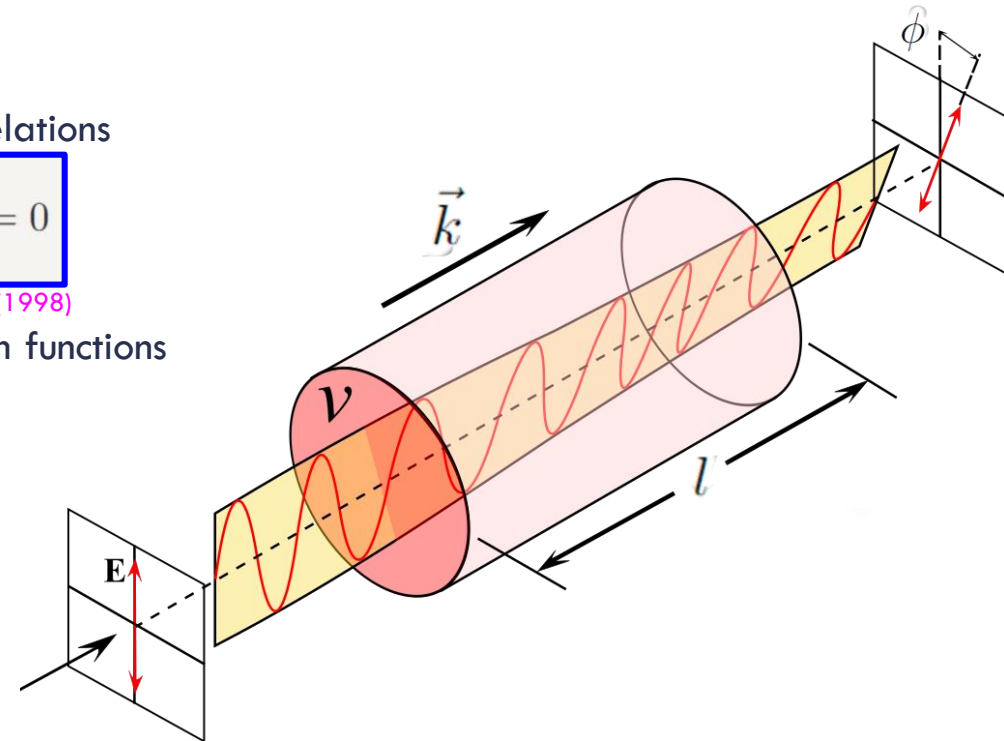
$$k^2 = \Pi_T \pm \Pi_P \Leftrightarrow \begin{pmatrix} -k^2 + \Pi_T & -i\Pi_P \\ i\Pi_P & -k^2 + \Pi_T \end{pmatrix} \begin{pmatrix} A_1 \\ A_2 \end{pmatrix} = 0$$

Mohanty et al, Phys. Rev. D 58, 093007 (1998)

where Π_T, Π_P are transverse and parity-violating polarization functions

A_1, A_2 are transverse components of photon field.

It expresses as a rotation of the linear polarization of the transverse electromagnetic wave:



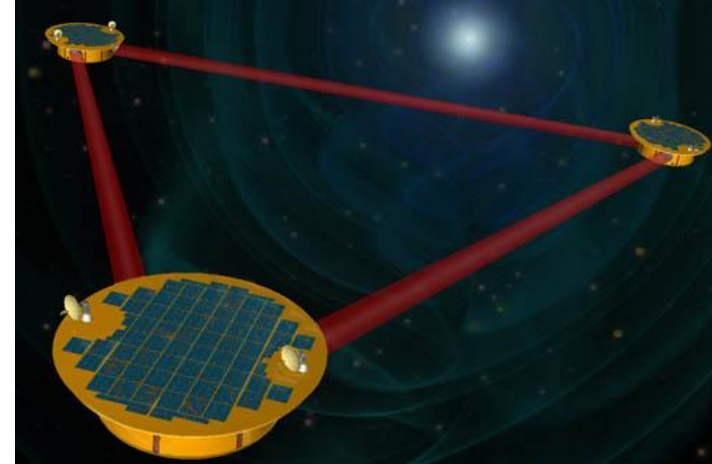
Solar wind optical noise for LISA

Gravitational wave antennas



Ground-based GW antennas (LIGO, VIRGO)

- Size limitation
- Seismic noise
- Vacuum technology
- Ultra-high vacuum ($\sim 10^7$ molecules/cm³)
- Vacuum (im)purity under control

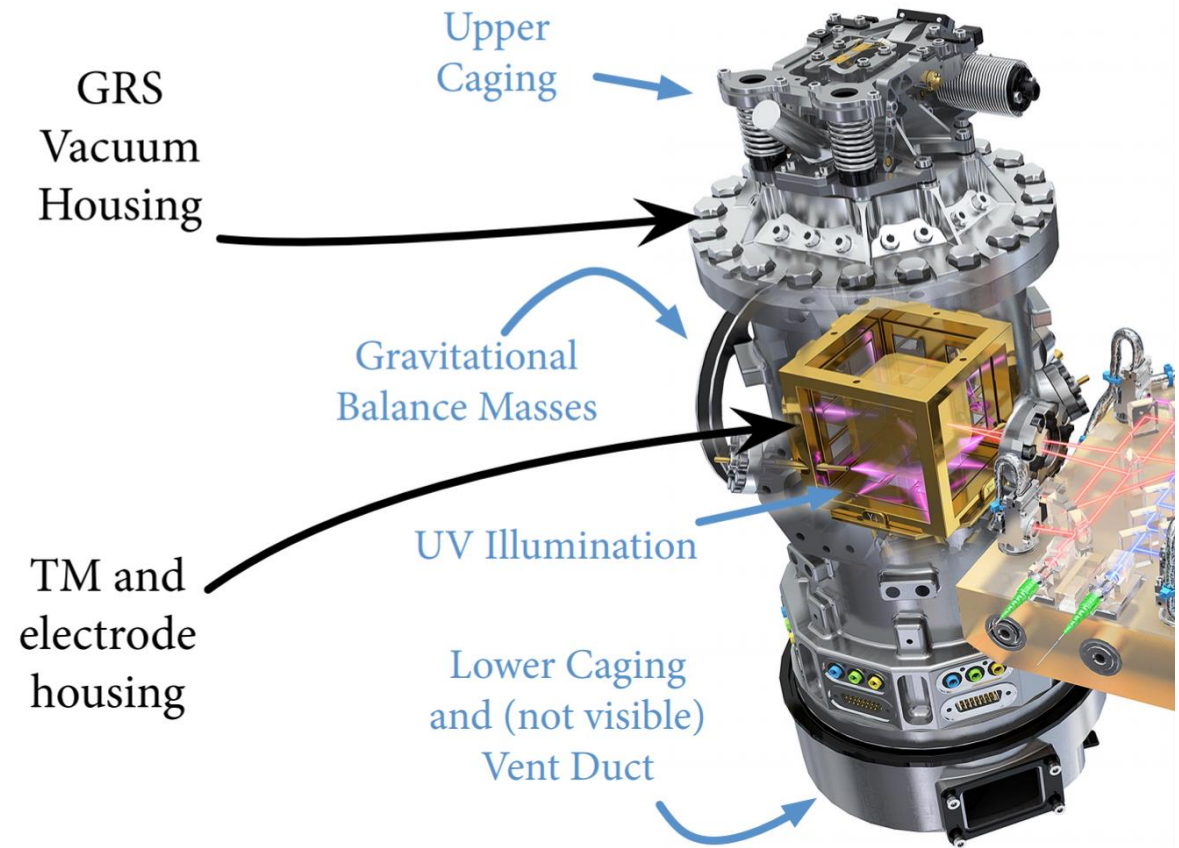
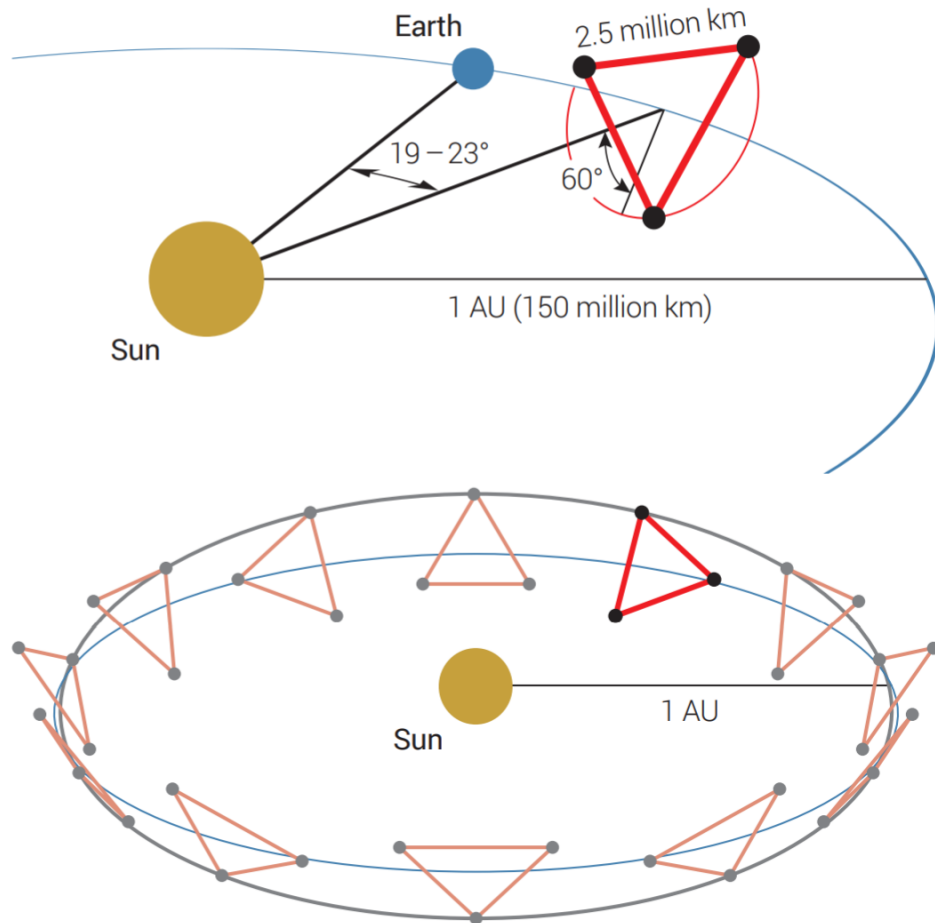


Space-based GW antennas (LISA)

- Free of size limitation
- No seismic noise
- Vacuum for free
- Better vacuum (~ 10 particles/cm³)
- Vacuum (im)purity **NOT** under control

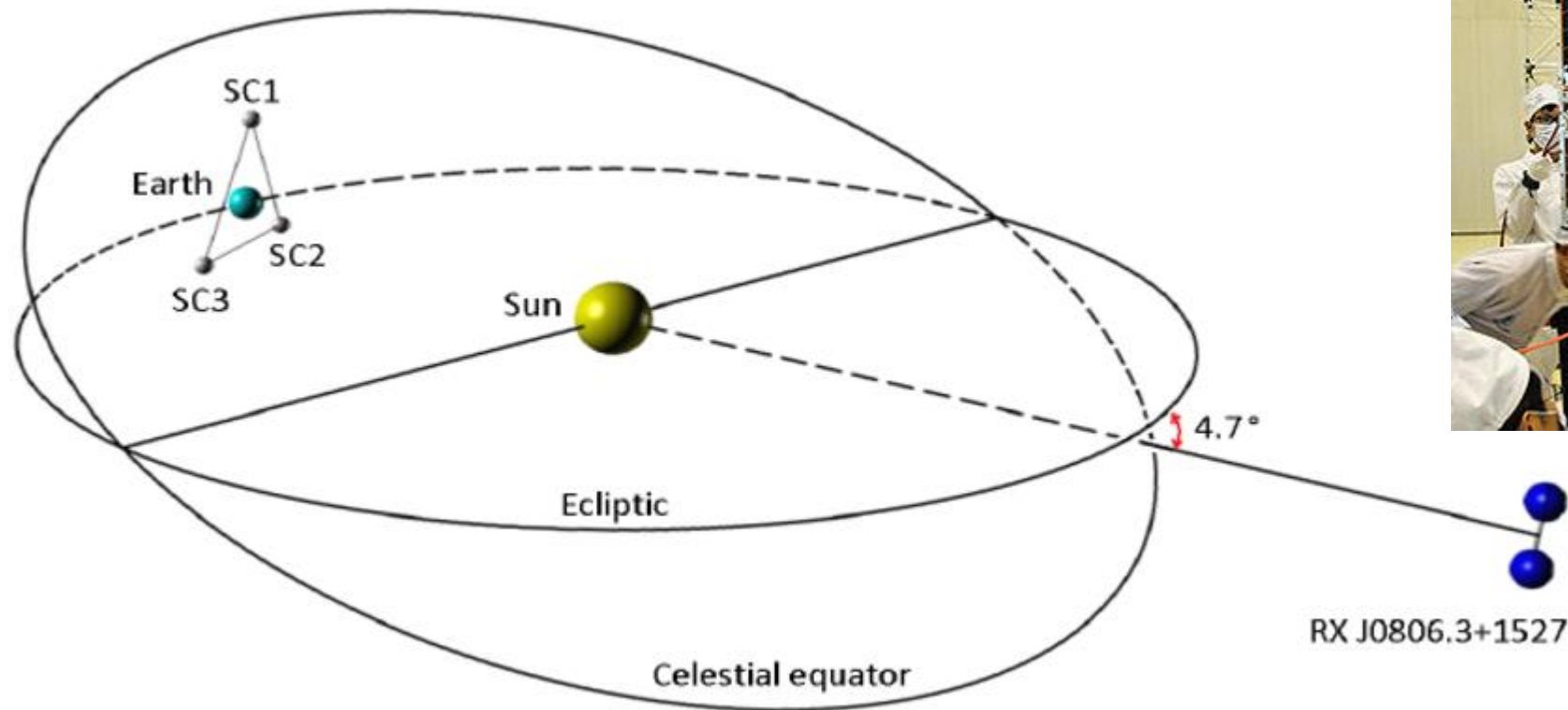
LISA – Laser Interferometer Space Antenna

Amaro-Seoane P., et al., 2017, Laser Interferometer Space Antenna (arXiv:1702.00786)



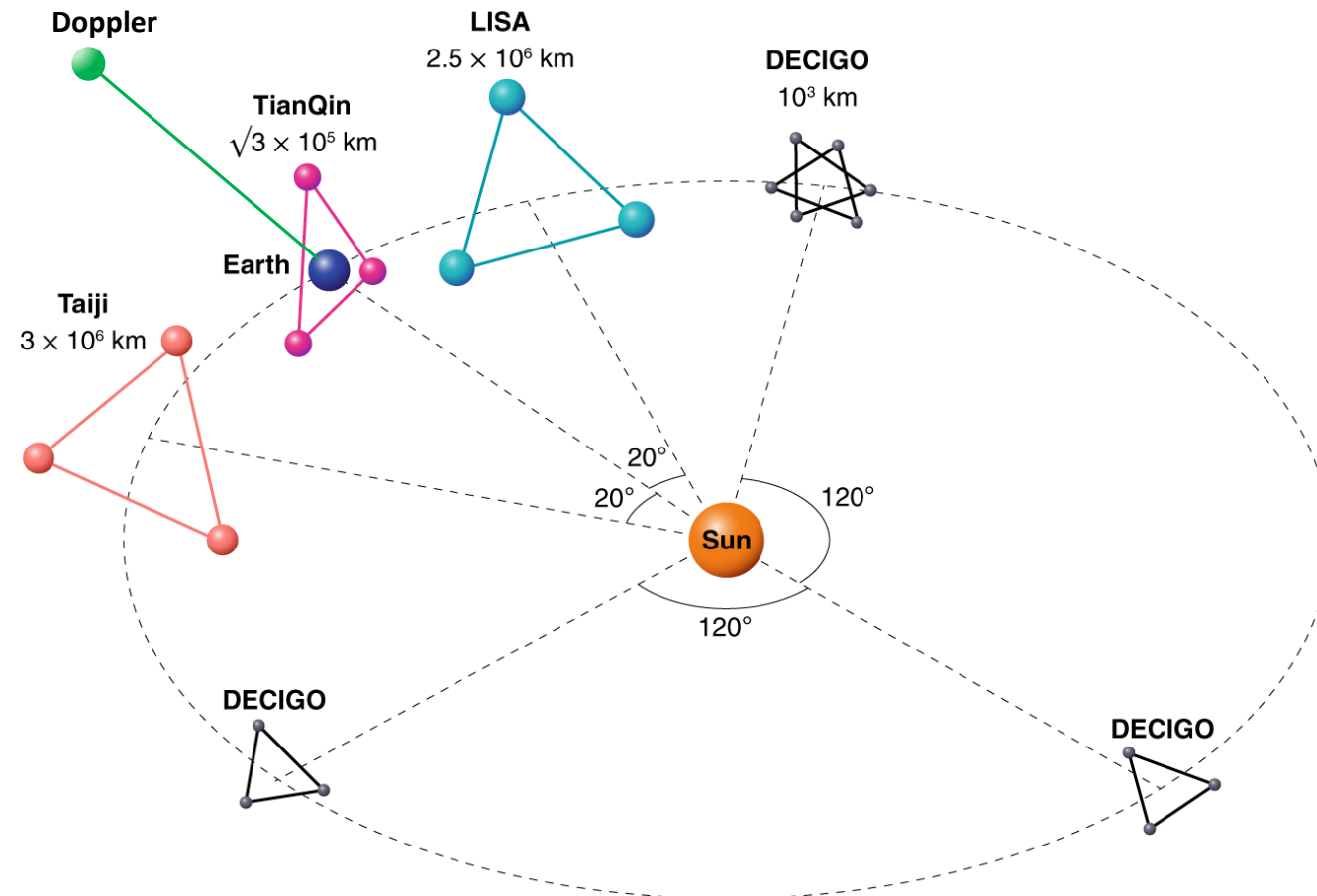
TianQin

Luo, J., Chen, L. S., Duan, H. Z., Gong, Y. G., Hu, S. C., Ji, J. H., et al. (2016). TianQin: A Space-borne gravitational wave detector. *Classical and Quantum Gravity*, 33, 035010.



Other concepts

- **Spacecraft Doppler Tracking** - Davies, R. W., "Issues in Gravitational Wave Detection with Space Missions", in Gravitational Waves and Radiations, Proceedings of the international conference, Université de Paris VII, Paris, France, June 18–22, 1973, Colloques Internationaux du CNRS, 220, pp. 33–45, (CNRS, Paris, France, 1974).
- **Taiji** - Hu, W. R., & Wu, Y. L. (2017). The Taiji program in space for gravitational wave physics and the nature of gravity. National Science Review, 4, 685–686.
- **DECIGO** - Kawamura, S., Ando, M., Seto, N., Sato, S., Nakamura, T., Tsubono, K., et al. (2011). The Japanese space gravitational wave antenna: DECIGO. Classical and Quantum Gravity, 28, 094011.



Challenges

Interferometry with

- Variable arm length
- Single laser path (no coherent beams)
- Overwhelming laser wavelength noise due to unequal arm length -> Time delayed interferometry
- Doppler shifted laser beam
- Mechanical, Thermal, Electronic, Optical noise
- Test mass charging
- High energy cosmic rays
- **Solar wind jitter**
- Timing and Clock Synchronisation
- Laser stability
- Optical impurity
-many other

LISA parameters

- 6 laser links operating on the wavelength

$$\lambda = 1064 \text{ nm}$$

- Variable arm length with average

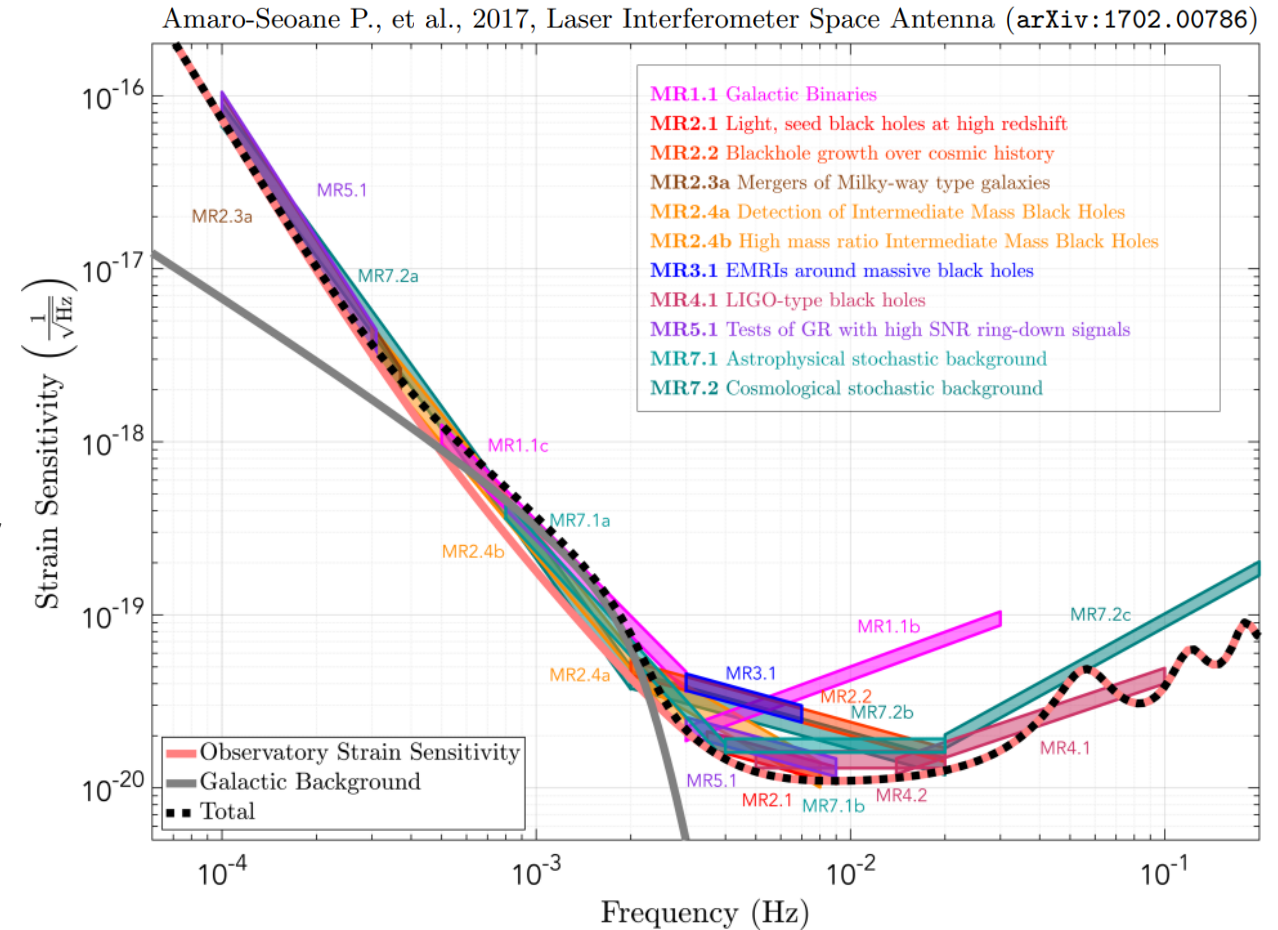
$$L = 2.5 \times 10^6 \text{ km}$$

- Displacement noise linear spectral density

$$\sqrt{S_{\text{IFO}}} \leq 10^{-11} \frac{\text{m}}{\sqrt{\text{Hz}}} \sqrt{1 + \left(\frac{2\text{mHz}}{f}\right)^4}$$

for $10^{-4}\text{Hz} \leq f \leq 10^{-1}\text{Hz}$,

Amaro-Seoane P., et al., 2017, Laser Interferometer Space Antenna (arXiv:1702.00786)



Refractive index of solar wind plasma

Solar wind (SW)

- Composition: electrons \sim protons $>$ alpha $>$ heavier ions
- at 1 AU from the Sun (LISA's and Earth's orbit)
 - average particle density $\sim 10 \text{ cm}^{-3}$ and quasi-neutrality $\rightarrow n_e \approx n_p \approx 5 \text{ cm}^{-3}$
 - at scales of optical wavelengths range $\sim 1 \mu\text{m}$, SW is practically a **collision-less ideal plasma**
 - electron component is characterized by plasma frequency

$$\omega_p^2 = \frac{n_e e^2}{\epsilon_0 m_e}$$

- Which dominates the Sellmeier equation for refractive index n for elmag wave of frequency ω

$$n = \sqrt{1 - \frac{\omega_p^2}{\omega^2}} \doteq 1 - \frac{1}{2} \frac{\omega_p^2}{\omega^2} + \dots$$

For optical elmag wave $\Delta n \sim 10^{-21}$

Estimation of the background for LISA from the solar wind plasma effect

- Effective displacement:

$$h_{\text{SW}}(t) = \frac{1}{2} \frac{n_e(t) e^2}{\epsilon_0 m_e \omega^2} L$$

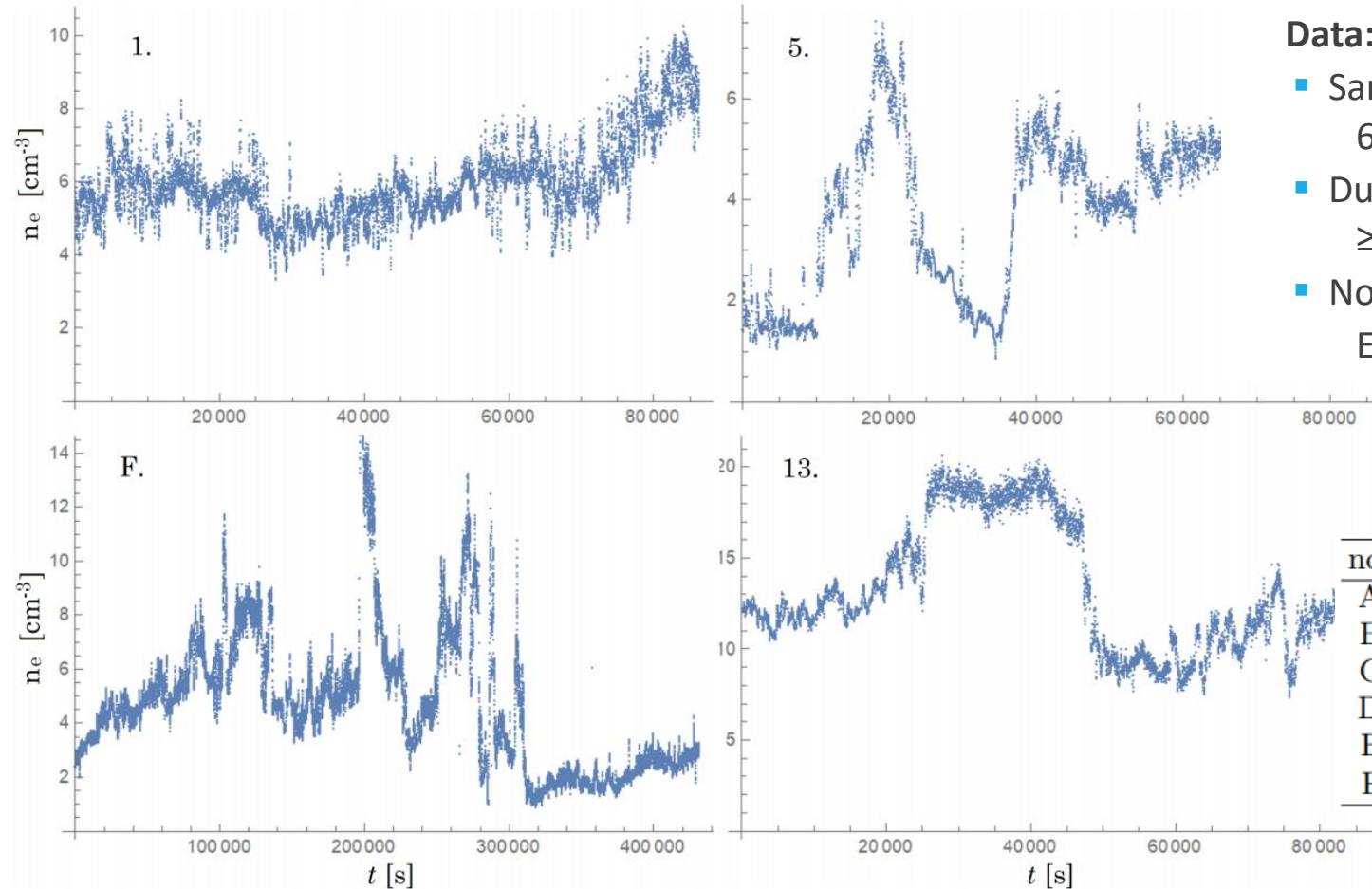
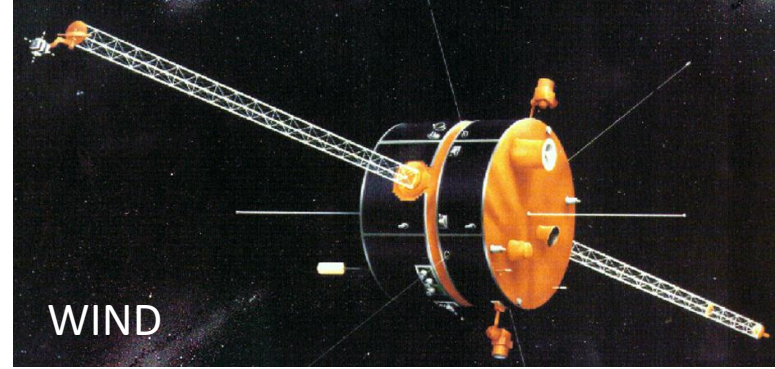
- Effective strain:

$$h_{\text{SW}} / L$$

- Linear spectral density:

$$\sqrt{S_{\text{SW}}(f)} \quad \text{to be compared with LISA's required } \sqrt{S_{\text{IFO}}}$$

Solar wind data



Data:

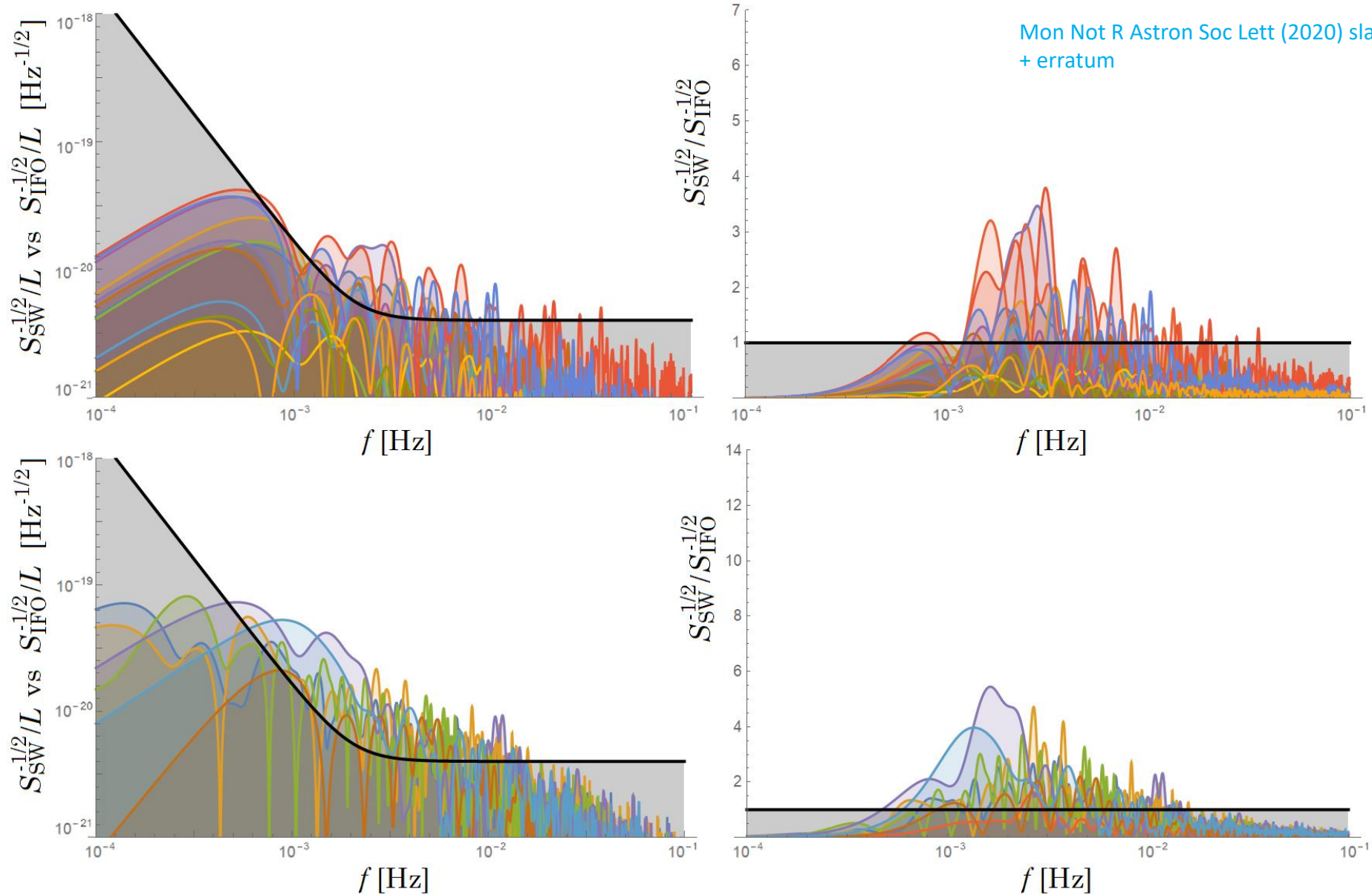
- Sampling frequency
6 s – 12 s
- Duration ≥ 24 h
 $\geq 10 \cdot (0.1 \text{ mHz})^{-1}$
- No interference with
Earth's magnetosphere

no.	initial date	length	\bar{n}_e	n_e^{\max}	n_e^{\min}	[cm^{-3}]
1.	1997/07/11	24 h	5.9	10.0	3.0	
2.	1997/08/11	24 h	6.4	10.5	3.5	
3.	1997/09/11	24 h	4.4	7.3	2.7	
4.	1997/10/11	24 h	7.6	13.5	4.0	
5.	1997/12/11	24 h	4.0	7.2	0.8	
6.	1998/01/11	24 h	7.4	10.3	4.6	
8.	1998/03/11	24 h	3.4	4.5	2.6	
9.	1998/04/11	24 h	5.6	9.2	2.2	
10.	1998/05/11	24 h	3.3	4.4	2.3	
11.	1998/06/11	24 h	6.6	13.5	0.1	
12.	1998/07/11	24 h	5.3	11.4	1.4	
13.	1998/09/11	24 h	13.1	20.6	7.4	
14.	1998/10/11	24 h	4.0	6.0	2.4	

no. event	initial date	length	\bar{n}_e	n_e^{\max}	n_e^{\min}	[cm^{-3}]
A. ISTEP-SEC	1997/11/04	24 h	6.1	12.0	4.0	
B. ISTEP-SEC CME	1998/01/21	24 h	8.0	22.0	3.5	
C. ISTEP-SEC CME	1998/04/15	24 h	4.2	7.2	3.1	
D. ISTEP-SEC CME-Geostorm	1998/04/29	96 h	5.4	15.0	1.2	
E. ISTEP-SEC Solar-GeoStorm	1998/09/23	96 h	5.5	15.0	0.5	
F. ISTEP-SEC Solar-GeoStorm	1998/08/24	120 h	4.7	14.5	1.0	

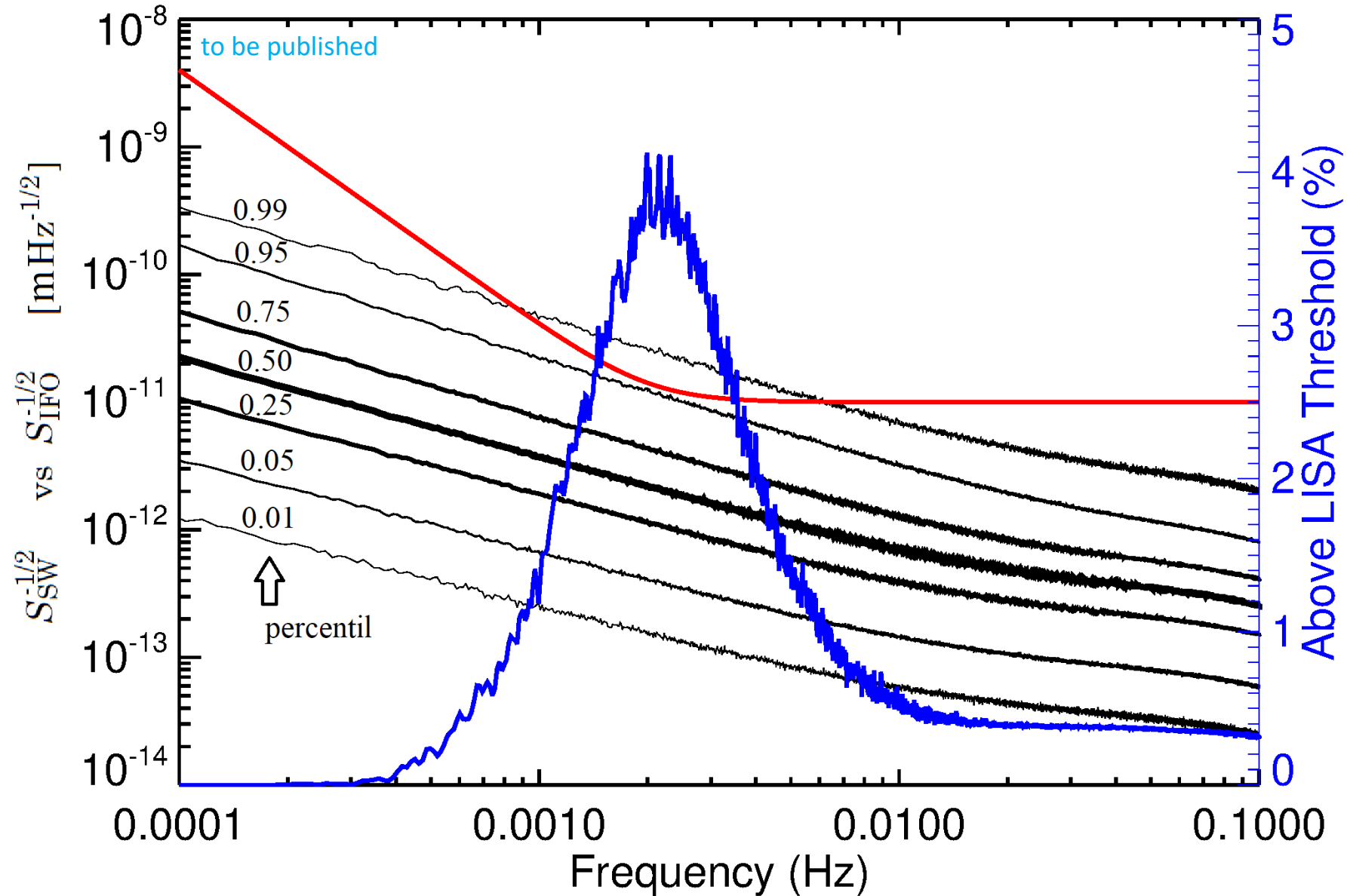
Preliminary results

Mon Not R Astron Soc Lett (2020) slaa155 1745-39254
+ erratum



More detailed results

Credit: František Němec



- WIND data
- 2004 – 2019
- 24h window
- shifted by 1h
- 76210 spectra

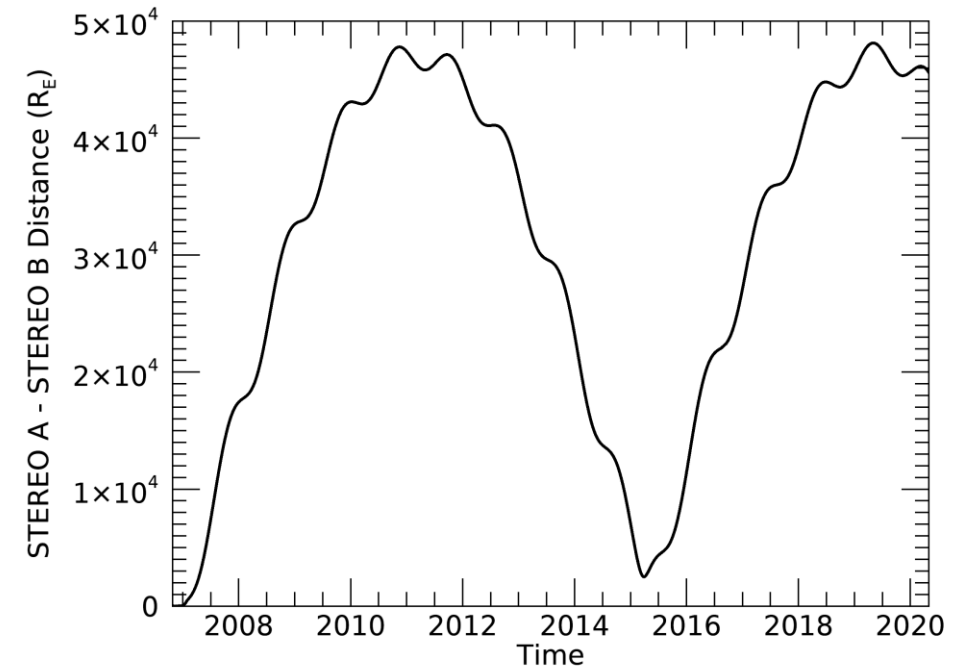
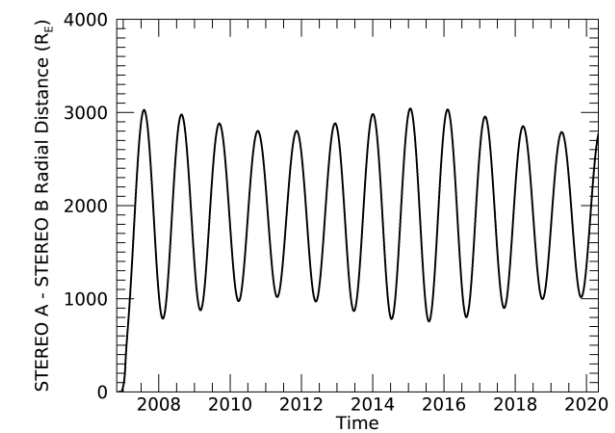
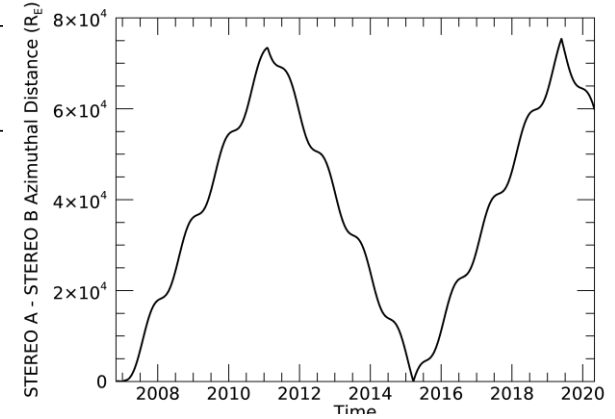
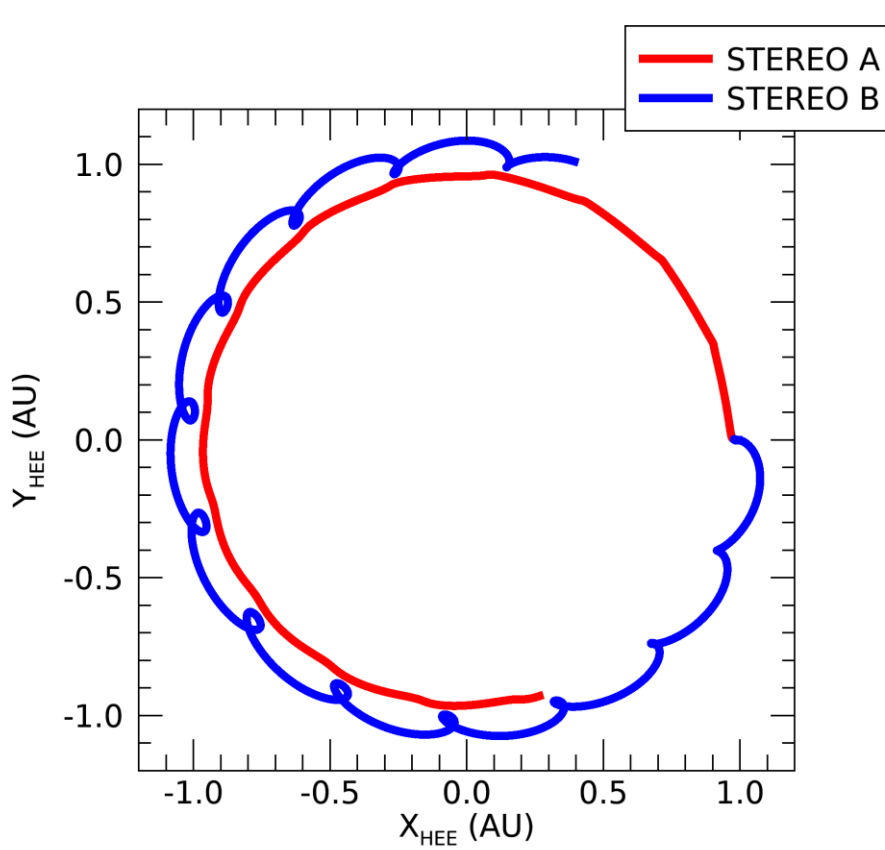
- interpolated data resampled to 10 Hz

Discussion of the result

- The result is applicable to any space-based interferometer of the LISA's type
- The effect will be observable only at the full scale LISA, no „Pathfinder“ mission would discover it
- Averaging-out effect
- Single arm analysis

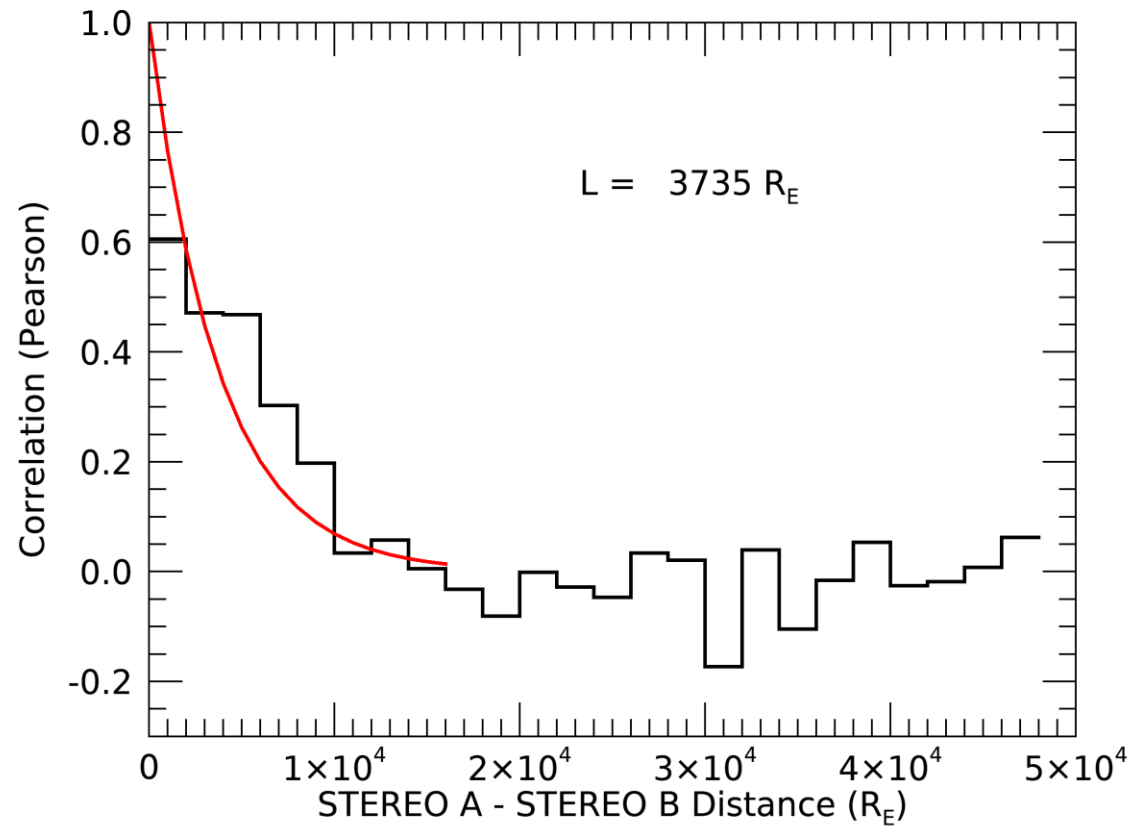


Hint from STEREO - trajectories



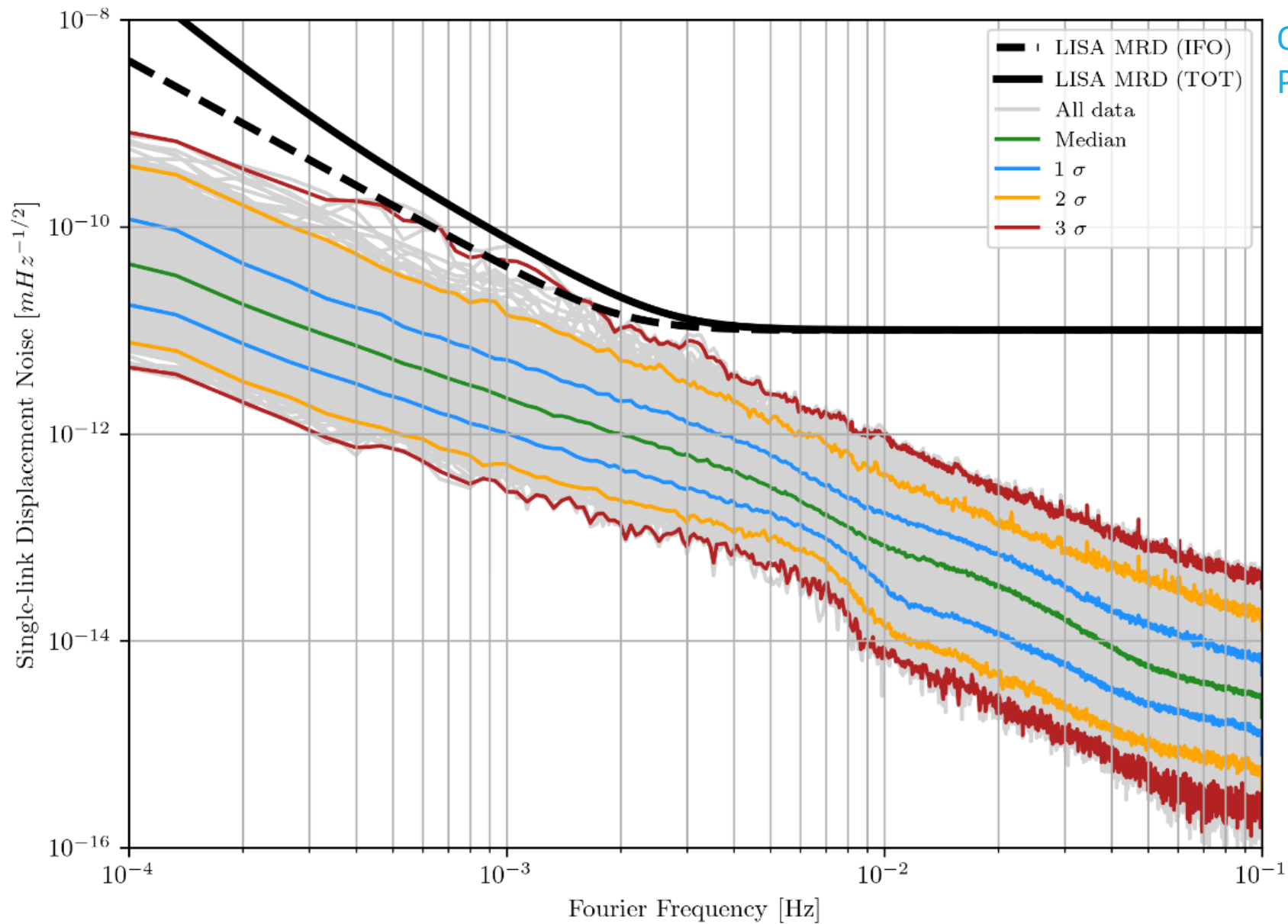
Hint from STEREO - correlations

Credit: František Němec



Correlation length:
 $L = 24 \text{ mil. km}$

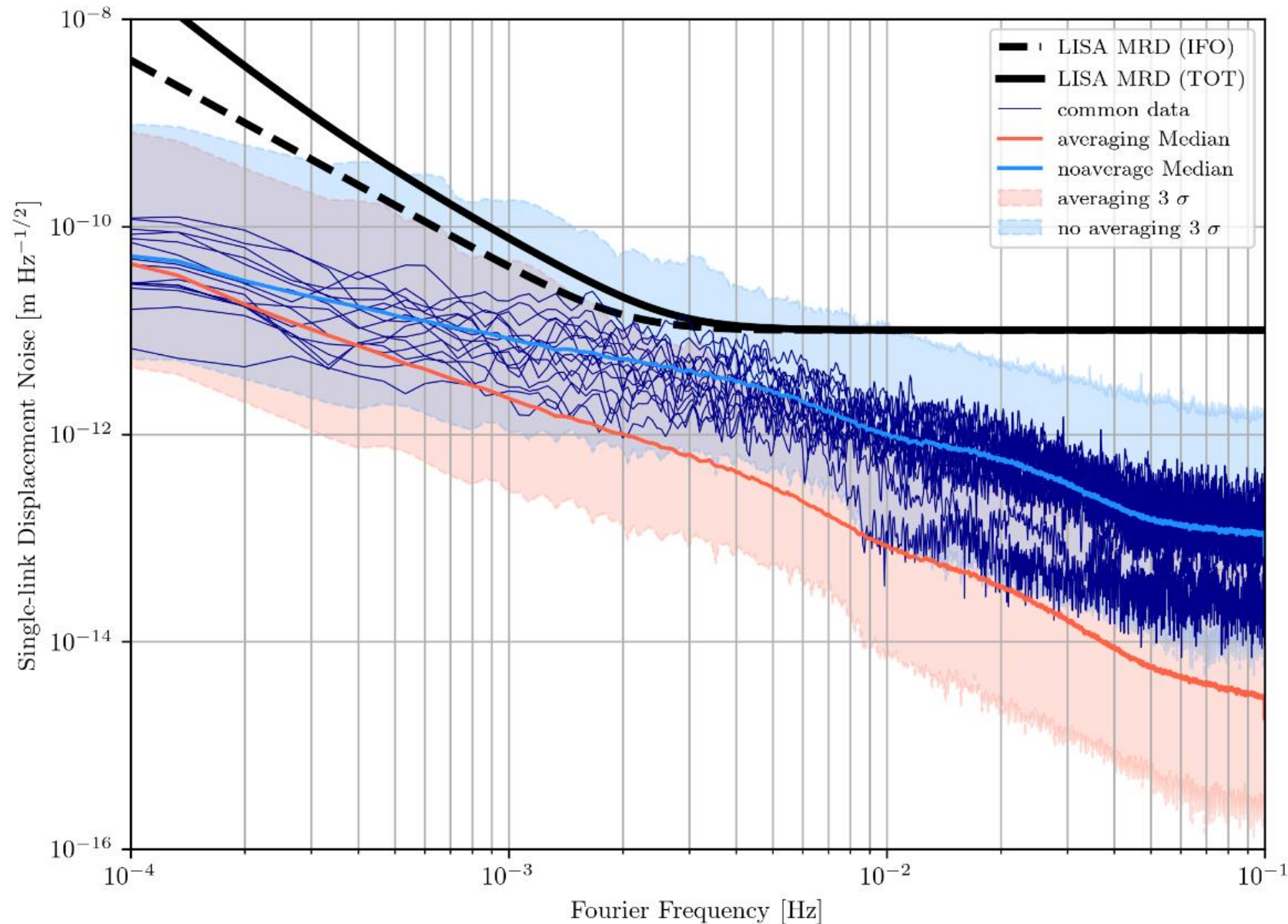
LISA collaboration cross check



O. Jennrich, et al.,
Phys.Rev.D 104 (2021) 6, 062003

- Single arm analysis

LISA collaboration cross check

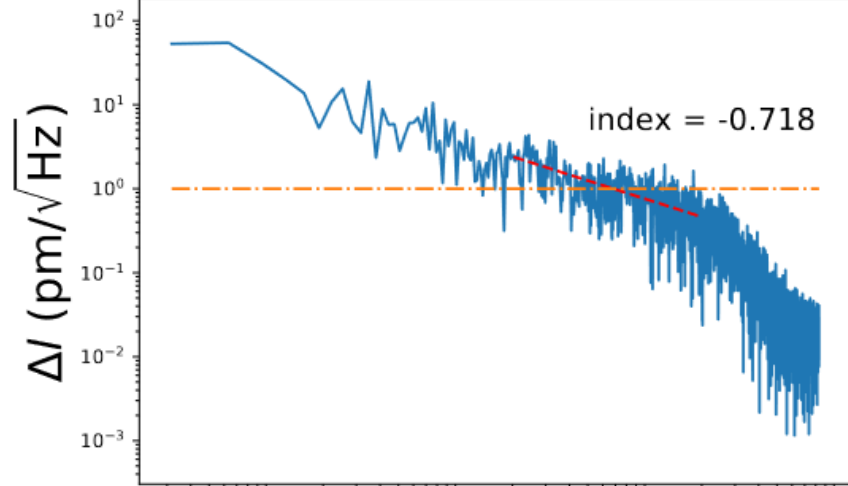


O. Jennrich, et al.,
[Phys.Rev.D 104 \(2021\) 6, 062003](#)

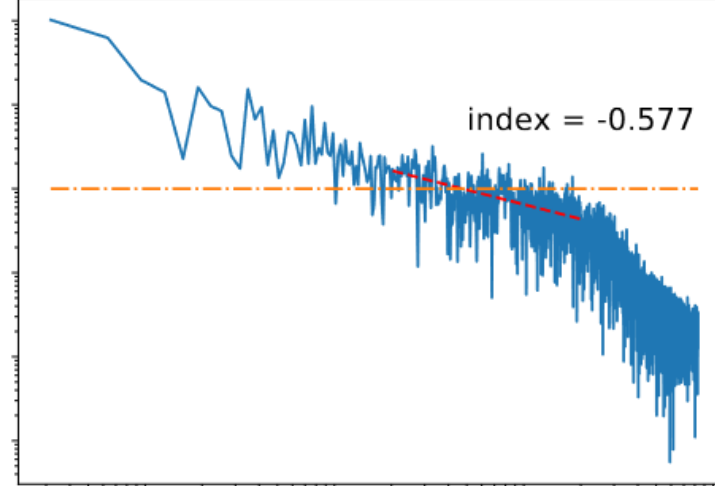
- Averaging due to the spatial variations of the SW electron density results in suppression of the effect for LISA
- Kolomogorov spectrum of electron density variations

TianQin collaboration analysis

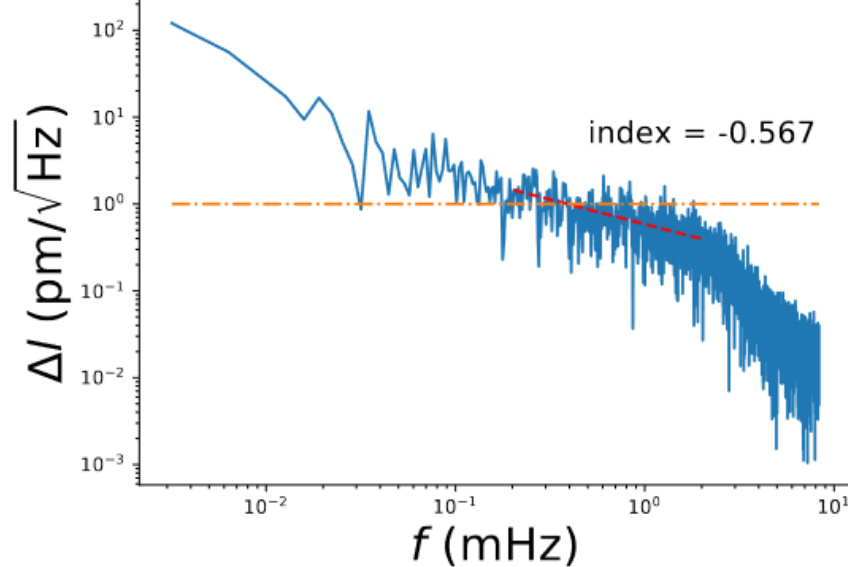
Single arm analysis $\varphi_S = 0^\circ$



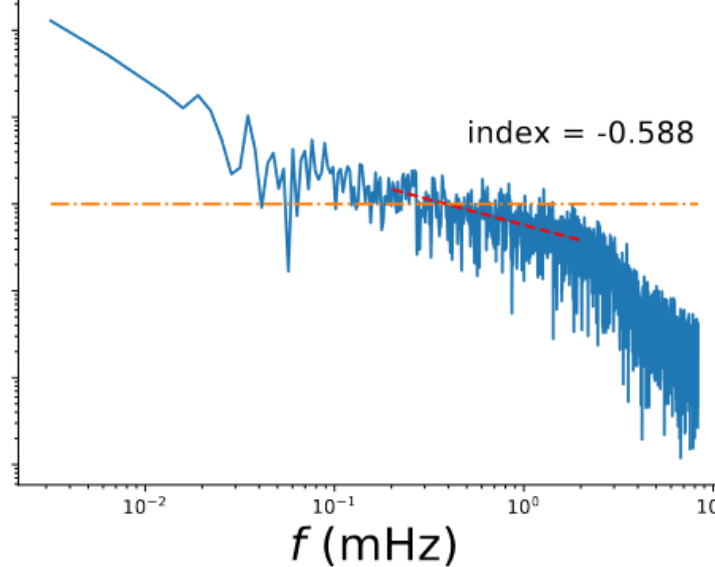
$\varphi_S = 30^\circ$



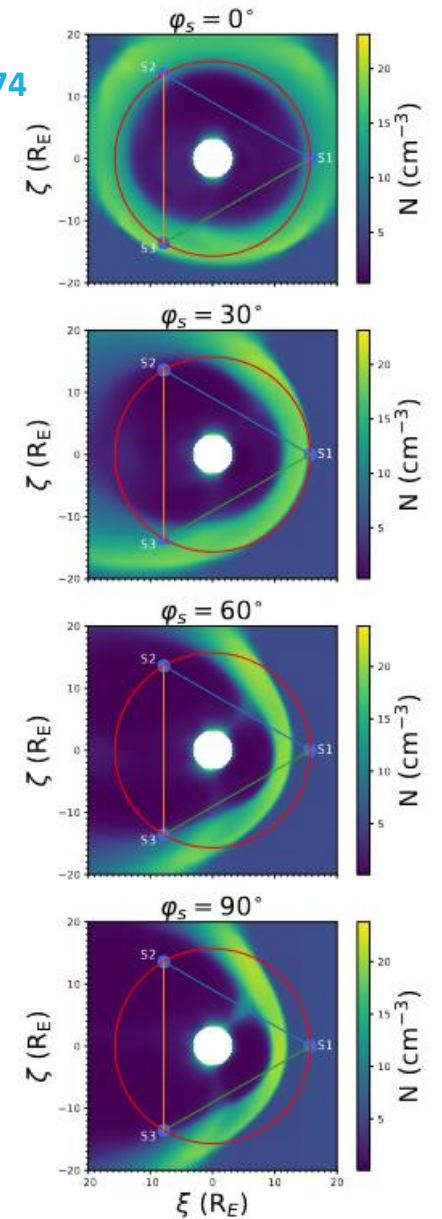
$\varphi_S = 60^\circ$



$\varphi_S = 90^\circ$



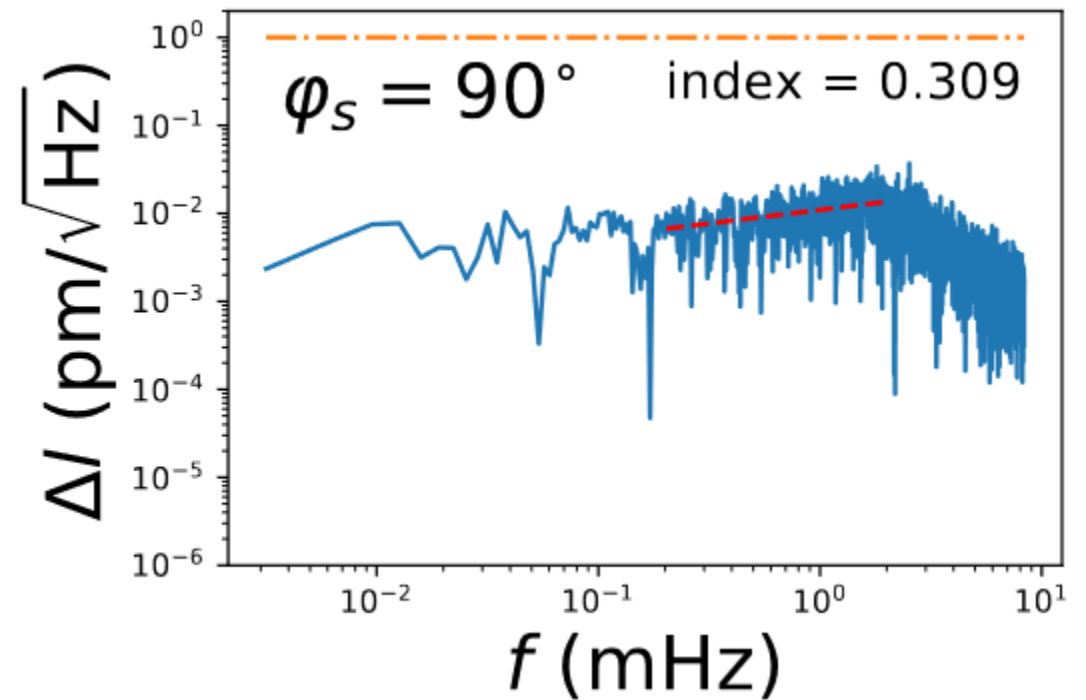
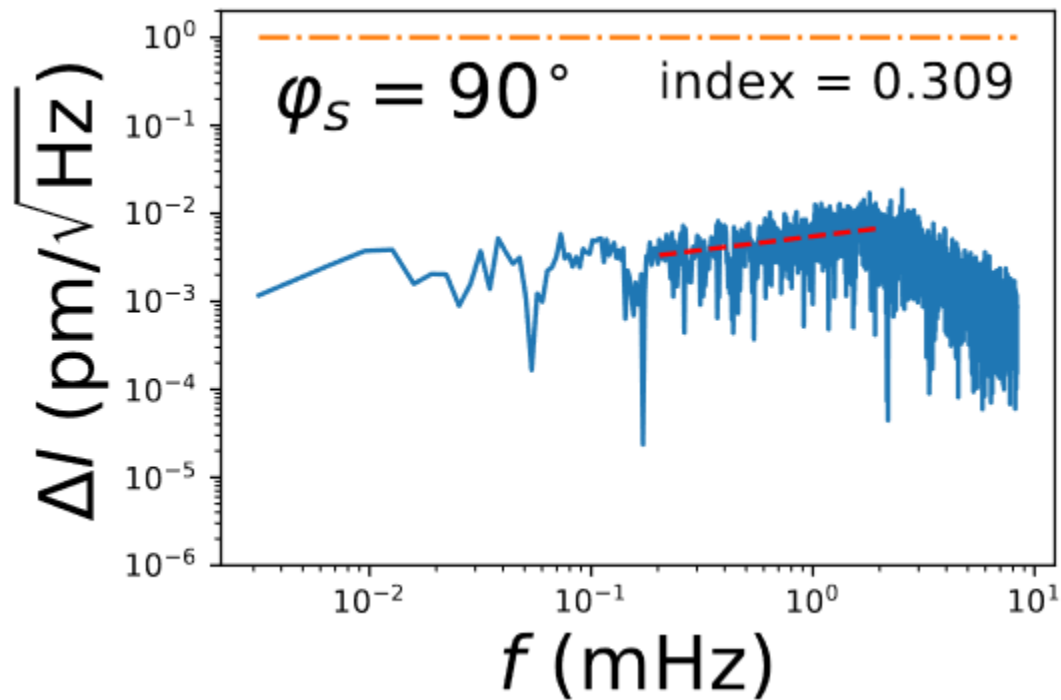
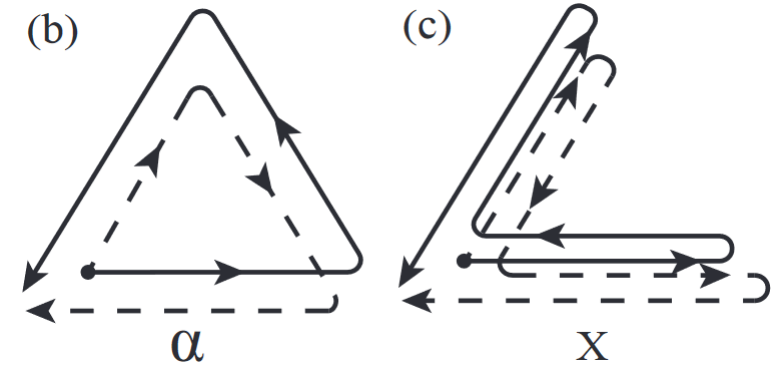
W. Su, et al.,
arXiv:2102.10574
(2021)



TianQin collaboration analysis

W. Su, et al., arXiv:2102.10574 (2021)

- Full constellation simulation



Mitigation strategies exist

1. Measure the solar wind density along the LISA's arms
 - **What accuracy is needed?**
 - **How many detectors?**

2. Deploy second laser system of different wavelength

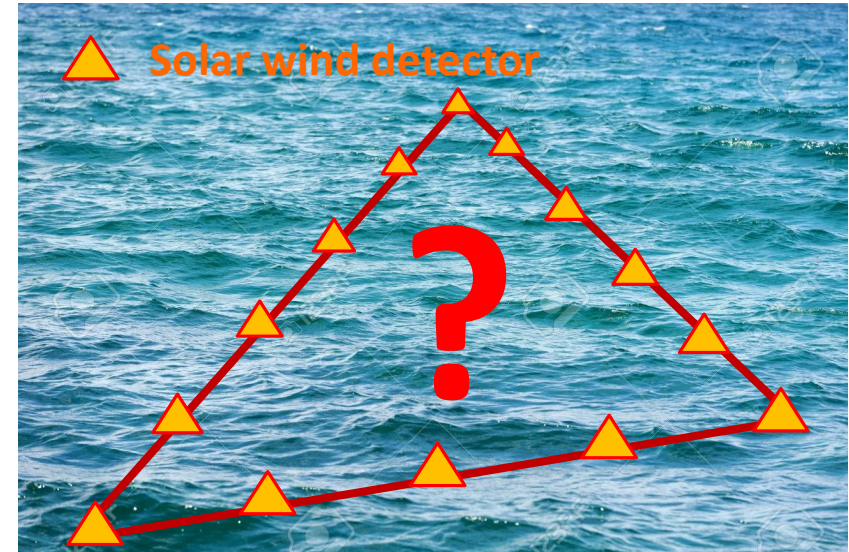
$$\lambda' = r\lambda$$

and subtract their signals

$$\Delta h(t) = r^2 h(t) - h'(t)$$

- **What is a systematic bias introduced?**

J. W. Armstrong, "Low-Frequency Gravitational Wave Searches Using Spacecraft Doppler Tracking", *Living Rev. Relativity*, 9 (2006)



Conclusions

Cross check with LISA collaboration – has been made

- Presentation Sept 1-3 2020, LISA Symposium
- The effect was considered in the past
 - [K. Danzmann and LISA Science Team, Advances in Space Research 32, 1233 \(2003\)](#) – „The infrared light has a frequency of 3×10^{14} Hz which renders it immune from refraction caused by the charged particles (plasma) which permeate interplanetary space.“
 - [J. W. Armstrong, "Low-Frequency Gravitational Wave Searches Using Spacecraft Doppler Tracking", Living Rev. Relativity, 9 \(2006\)](#)
- The error of factor ~ 1.4 in the LSD was detected and removed
 - [ERRATUM done to Mon Not R Astron Soc Lett \(2020\) slaa155 1745-39254](#)
- Feedback in community:
 - [O. Jennrich, et al., Phys.Rev.D 104 \(2021\) 6, 062003](#) (LISA collaboration)
„For second-generation space missions with increased displacement sensitivity, this may be a noise source that requires mitigation of some kind “
„The authors would like to thank Adam Smetana for his paper and presentation at the 2020 LISA Symposium on this same topic which inspired this analysis as well as for some helpful discussions during and after the LISA Symposium. “
 - [W. Su, et al., arXiv:2102.10574 \(2021\)](#) (TianQin collaboration)
 - [Lu, L.-F., Su, W., Zhang, X., et al. 2021, JGR: Space Physics, 126, e2020JA028579](#) (TianQin collaboration)

Collaboration with Charles University

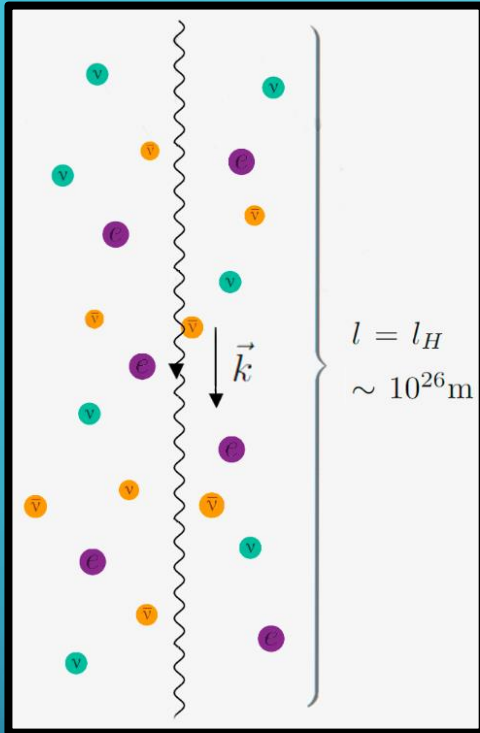
- Correlation length estimates based on real data (STEREO, L1 missions)
- More complex analysis of WIND data has been performed

LISA as a solar wind probe

- 3D modelling of solar events + Simulation of the 3-arm LISA response

Optical properties of neutrino media

BIREFRINGENCE OF COSMIC NEUTRINO BACKGROUND



The P-violating polarization function in vacuum

$$\Pi_P(k^2, K) = \frac{\sqrt{2}G_F\alpha}{3\pi} \frac{k^2}{m_e^2} (n_\nu - n_{\bar{\nu}})K, \quad k^2 \ll m_e^2 \ll M_W^2$$

Mohanty et al, Phys. Rev. D 58, 093007 (1998)

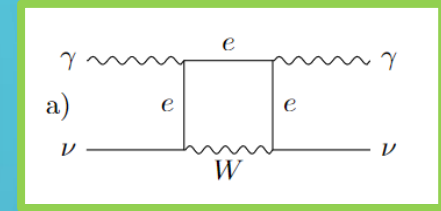
The presence of electron component of plasma for intergalactic plasma $n_e \sim 3 \times 10^{-4} \text{ cm}^{-3}$

$$k^2 = \omega_p^2 \neq 0$$

Recently improved thermal approach:

$$7\pi/(2\alpha_{em}) \sim 10^3 \text{ enhancement [Dvornikov21]}$$

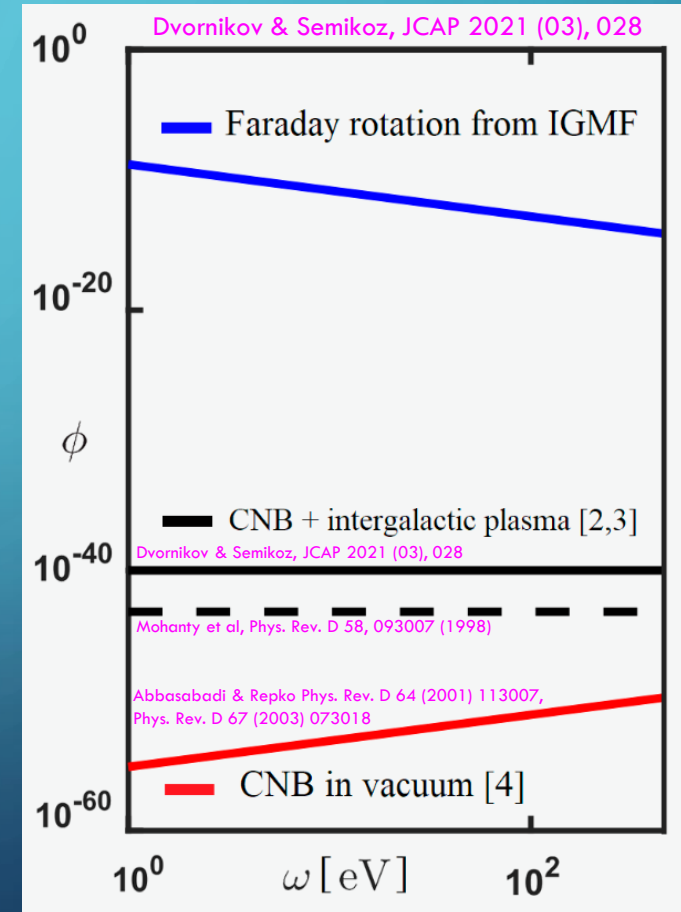
Dvornikov & Semikoz, JCAP 2021 (03), 028



numerical estimate for CNB over l_H

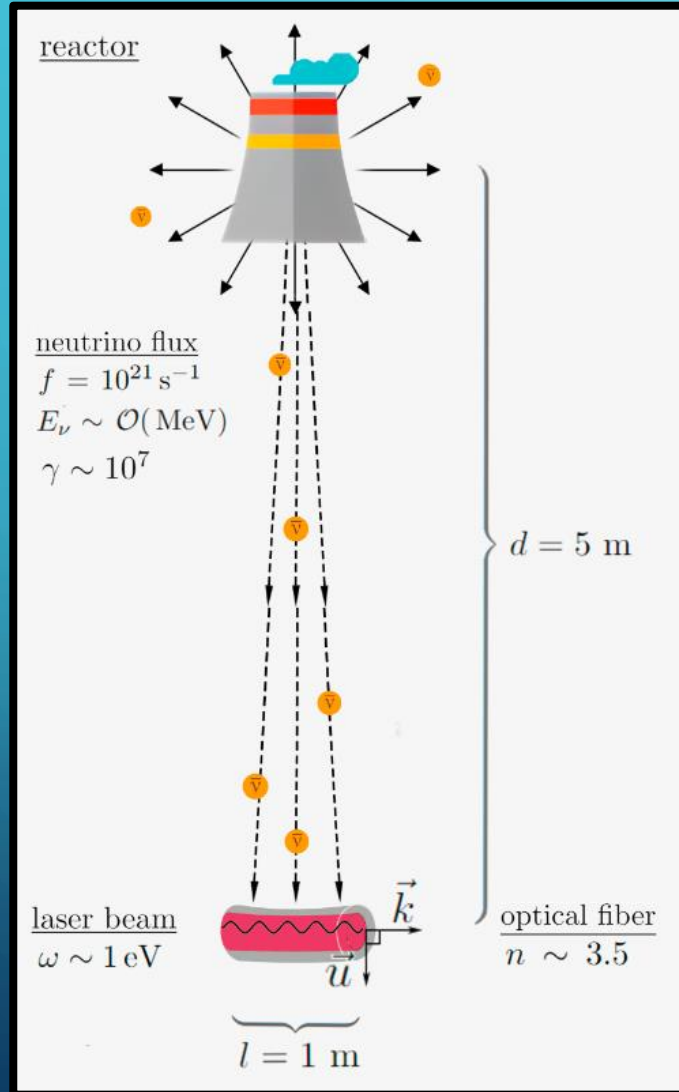
$$\phi \sim 10^{-40} \text{ rad}$$

It demonstrates that slowing down photons by letting them pass through the additional refractive component of a medium increases their exposure to the neutrinos, enhancing the neutrino effect.



OPTICAL EFFECTS OF NEUTRINO MEDIUM IN LABORATORY

A thought experiment:



our result for general angular setting

$$\frac{\phi}{l} \simeq \frac{\Pi_P}{2\omega n} = \frac{G_F \alpha}{3\sqrt{2}\pi} \frac{\omega^2 (1 - n^2)}{m_e^2} \gamma (n_\nu - n_{\bar{\nu}}) \left| 1 - \frac{\beta}{n} \cos \theta_{ku} \right|$$

numerical estimate
for the thought experiment

$$\phi \sim 4.6 \times 10^{-39} \text{ rad}$$

for comparison: aLIGO sensitivity

$$\Delta n \sim 10^{-23}$$

VARIATION OF INDEX OF REFRACTION FROM NEUTRINO MEDIUM IN LABORATORY

definition:

$$\Delta n \approx \frac{\Pi_T}{2\omega^2}$$

our result for general angular setting
(perfect uniformity approximation)

$$\Delta n \simeq -\frac{\mu_\nu^2 \gamma n_{\bar{\nu}}}{E_\nu} (1 - n^2)^2 \frac{\sin^2 \theta_{ku}}{(1 - n\beta \cos \theta_{ku})^2 - \Gamma}$$

$$\Gamma = \frac{\omega^2 (1 - n^2)^2}{4E_\nu^2} \sim 10^{-12}$$

numerical estimate for the thought experiment

$$\Delta n \sim 1.2 \times 10^{-65} \Theta(\theta_{ku}).$$

possible resonance

$$\cos \theta_{ku} \sim 1/(n\beta)$$

ultra-relativistic regime

possible resonance

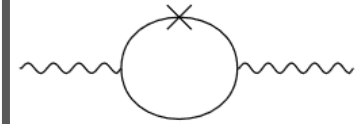
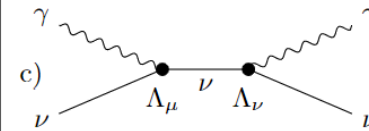
$$\Gamma \sim (1 - n\beta \cos \theta_{ku})^2$$

non-relativistic regime

for comparison: aLIGO sensitivity

$$\Delta n \sim 10^{-23}$$

details of calculation



matter part of the polarization tensor:

$$\Pi_{\mu\nu}^{(\Lambda)'}(k) = i \int \frac{d^4 p}{(2\pi)^4} \text{Tr}\{S'(p, u) [T_{\mu\nu}(p, k) + T_{\nu\mu}(p, -k)]\}$$

neutrino medium insertion:

$$S'(p, u) = (\not{p} + m_\nu) 2\pi i \delta(p^2 - m_\nu^2) \times [\theta(p \cdot u) f(p \cdot u) + \theta(-p \cdot u) \bar{f}(-p \cdot u)]$$

tensor structure of the scattering amplitude:

$$T_{\mu\nu}(p, k) = \frac{\Lambda_\mu (\not{p} + \not{k} + m_\nu) \Lambda_\nu}{(p + k)^2 - m_\nu^2}$$

to make rough estimate we take a perfect uniformity approximation of anti-neutrino flux:

$$f = 0, \quad \bar{f} = (2\pi)^3 n_{\bar{\nu}} \delta^{(3)}(\mathbf{p})$$

DISCUSSION OF THE RESULTS

neutron magnetic moment?
dark matter?

our result for neutrinos

$$\Delta n \simeq -\frac{\mu_\nu^2 \gamma n_{\bar{\nu}}}{E_\nu} (1 - n^2)^2 \frac{\sin^2 \theta_{ku}}{(1 - n\beta \cos \theta_{ku})^2 - \Gamma} \quad \Gamma = \frac{\omega^2 (1 - n^2)^2}{4E_\nu^2} \sim 10^{-12}$$

numerical estimate for the thought experiment

$$\Delta n \sim 1.2 \times 10^{-65} \Theta(\theta_{ku}).$$

neutrons

$$\mu_n \sim 10^{16} \mu_\nu$$

→ greatest accessible non-relativistic neutron flux

$$\sim 10^{15} \text{ n/s/cm}^2$$

→ numerical estimate

$$\Delta n \sim 10^{-31}$$

dark matter

$$\mu_{\text{DM}} < 10^{14} \mu_\nu$$

asymmetric dipolar DM
scenario

→ future work ???

Optical properties of axionic DM

Axion

Peccei-Quinn mechanism for solving the strong CP problem of QCD.

Axion is the pseudo-Goldstone boson of the spontaneously broken $U(1)_{PQ}$ symmetry.

It leads to axion-gluon coupling:

$$\mathcal{L} = \left(\frac{a}{f_a} - \bar{\theta} \right) \frac{\alpha_s}{8\pi} G^{\mu\nu a} \tilde{G}_{\mu\nu}^a$$

that effectively via **instantons** provides the periodic effective potential for the axion:

$$V_{\text{eff}} \sim \cos \left(\theta + \xi \frac{\langle a \rangle}{f_a} \right)$$

the minimization effectively eliminates the CP -violating parameter θ

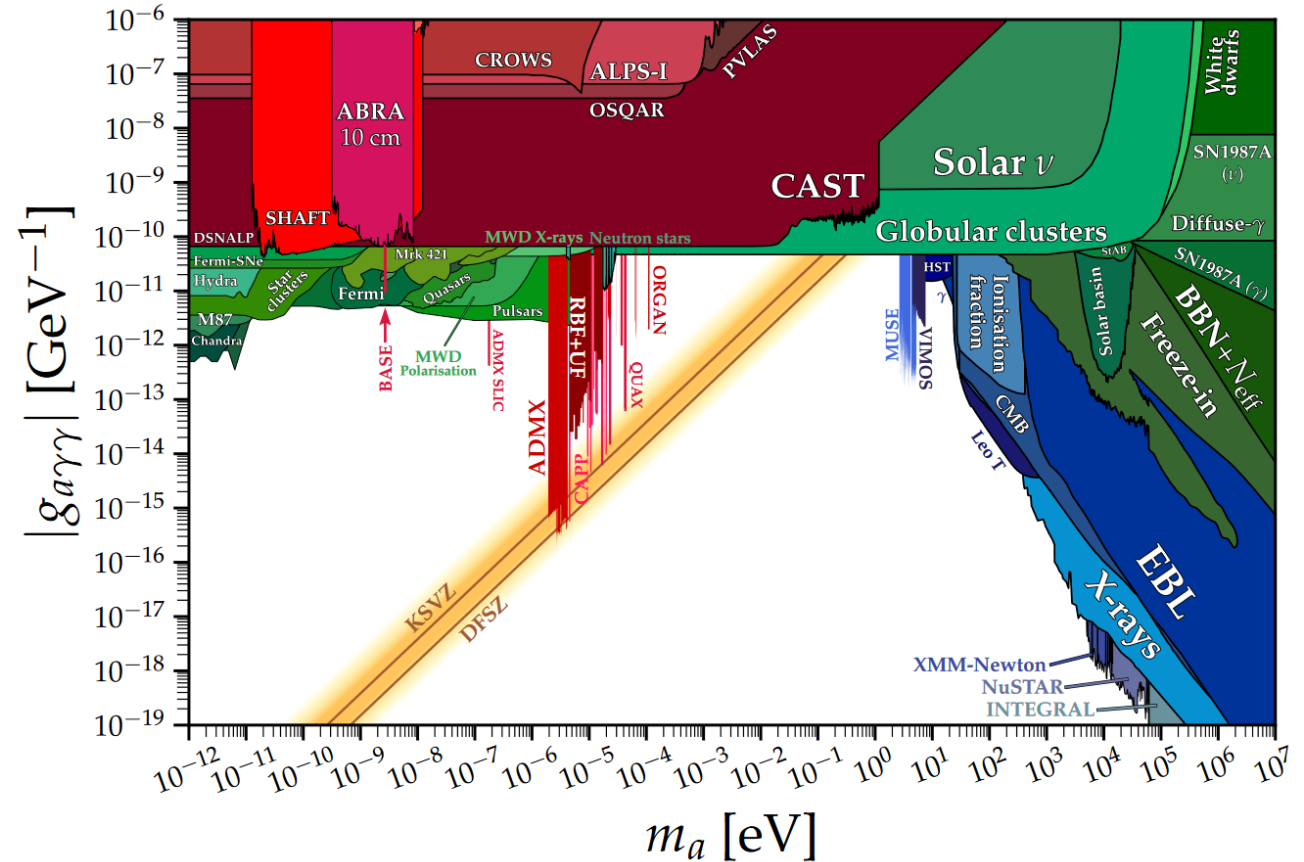
$$\bar{\theta}_{\text{eff}} \equiv \langle a \rangle / f_a - \bar{\theta}$$

Properties of the QCD axion:

mass $m_a = 5.691(51)\mu\text{eV}(10^{12}\text{GeV}/f_a)$

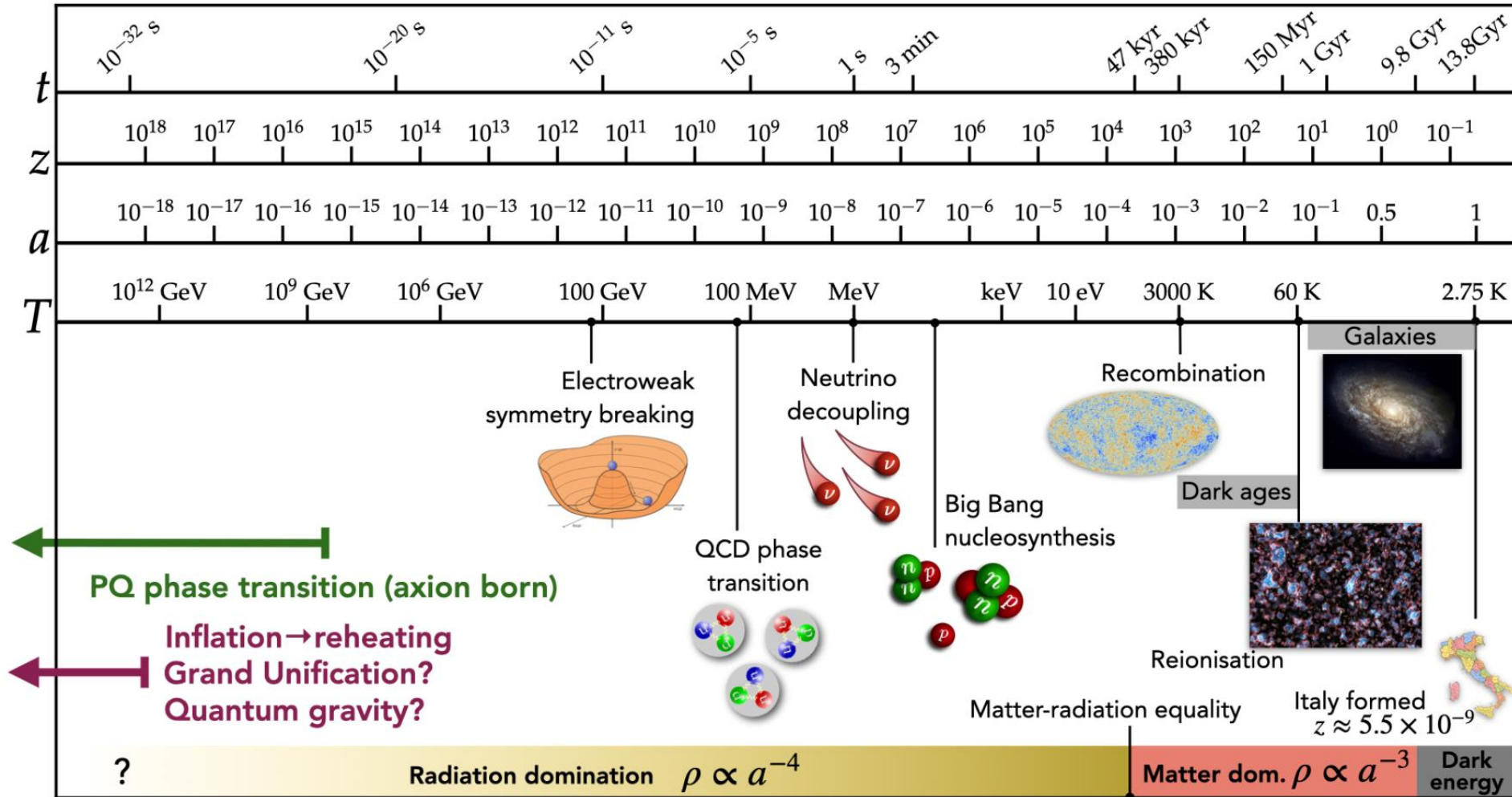
electromagnetic interaction

$$\mathcal{L}_{a\gamma\gamma} = -\frac{g_{a\gamma\gamma}}{4} a F_{\mu\nu} \tilde{F}^{\mu\nu}$$



Axion dark matter

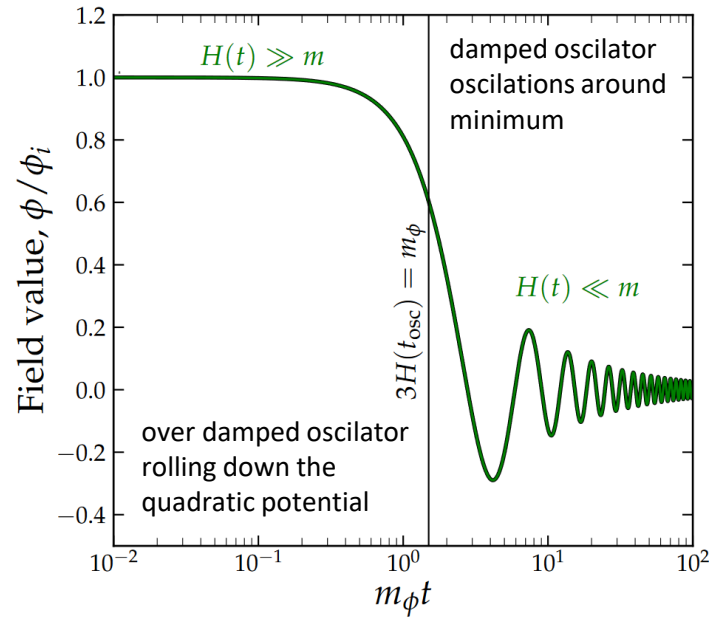
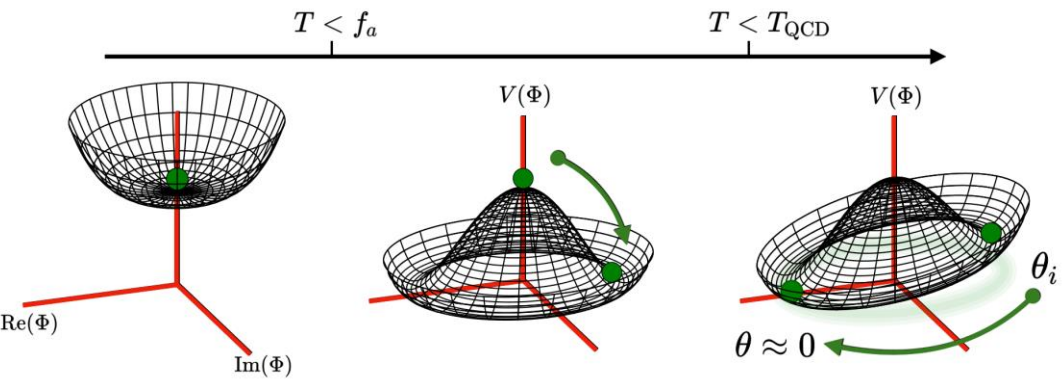
C.A.J. O'Hare, PoS COSMICWISPers (2024) 040



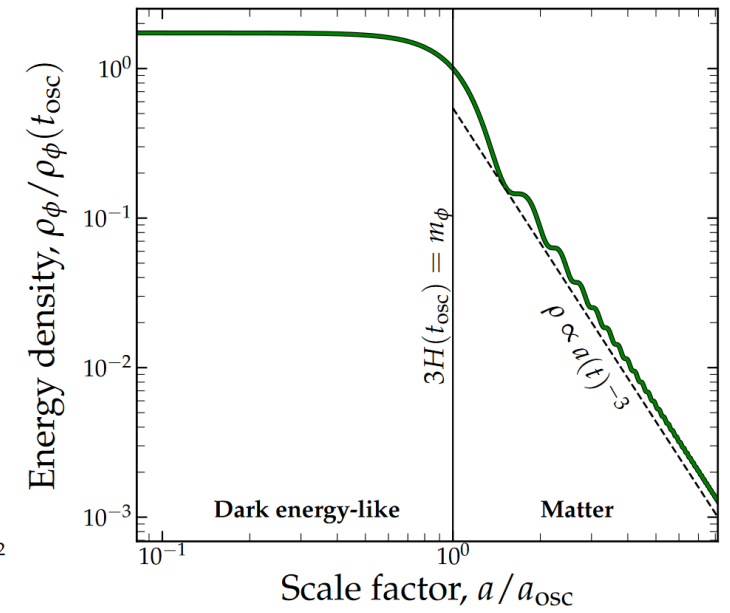
Axion dark matter

Misalignment mechanism

$$V(\Phi) = V_{\text{PQ}}(|\Phi|) + V_{\text{QCD}}(\theta) = \frac{\lambda}{8} (|\Phi|^2 - f_a^2)^2 + \chi(T)(1 - \cos \theta)$$



C.A.J. O'Hare, PoS COSMICWISPers (2024) 040



The present background axion field

$$a(t) = a_0 \cos(mt + \delta_\tau(t))$$

Birefringence of the axion dark matter

I. Obata, T. Fujita, Y. Michimura, Phys.Rev.Lett. 121 (2018) 16, 161301

Photon-axion interaction in the Coulomb gauge:

$$\frac{g_{a\gamma}}{4} a F_{\mu\nu} \tilde{F}^{\mu\nu} = g_{a\gamma} \dot{a} A_i \epsilon_{ijk} \partial_j A_k + (\text{total derivative})$$

Photon equation of motion with external classical axion field:

$$\ddot{A}_i - \nabla^2 A_i + g_{a\gamma} \dot{a} \epsilon_{ijk} \partial_j A_k = 0$$

$$\ddot{A}_k^\pm + \omega_\pm^2 A_k^\pm = 0$$

where

$$\omega_\pm^2 \equiv k^2 \left(1 \pm \frac{g_{a\gamma} a_0 m}{k} \sin(mt + \delta_\tau) \right)$$

Resulting misalignment in phase velocities of two circular polarizations is:

$$\delta c \simeq \frac{g_{a\gamma} a_0 m}{k} \sin(mt + \delta_\tau) \equiv \delta c_0 \sin(mt + \delta_\tau)$$

where

$$\delta c_0 \simeq 3 \times 10^{-24} \left(\frac{g_{a\gamma}}{10^{-12} \text{ GeV}^{-1}} \right)$$

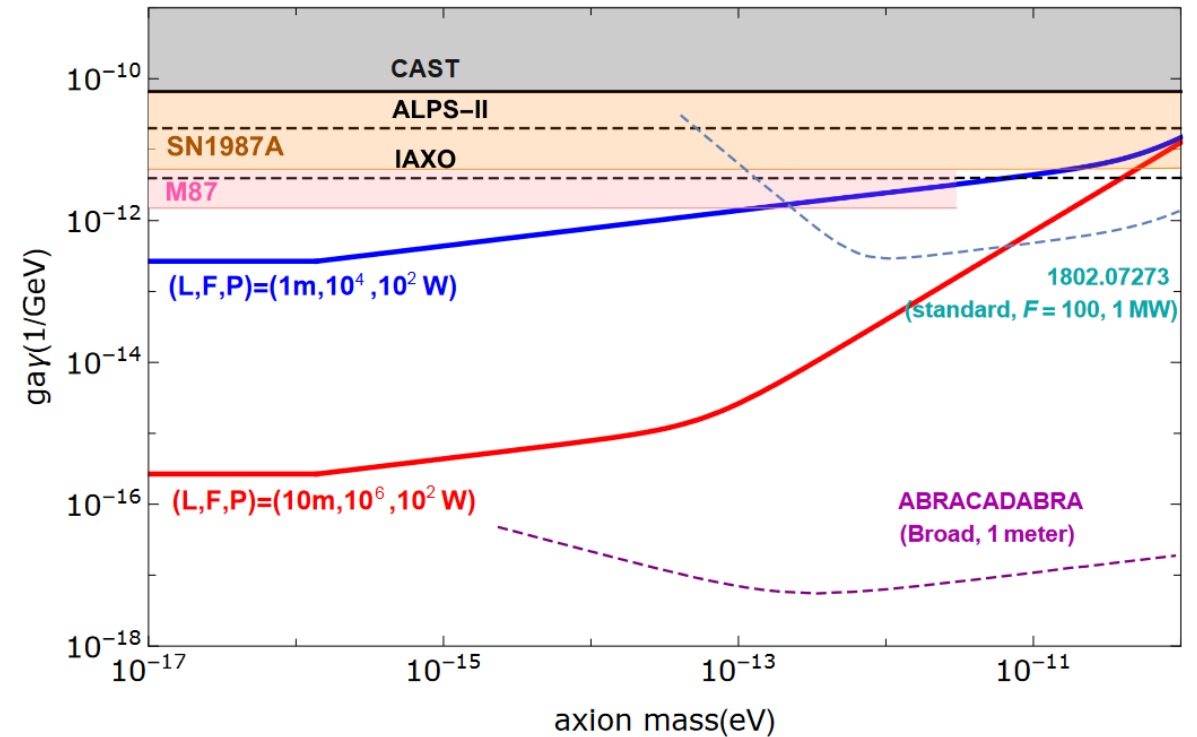
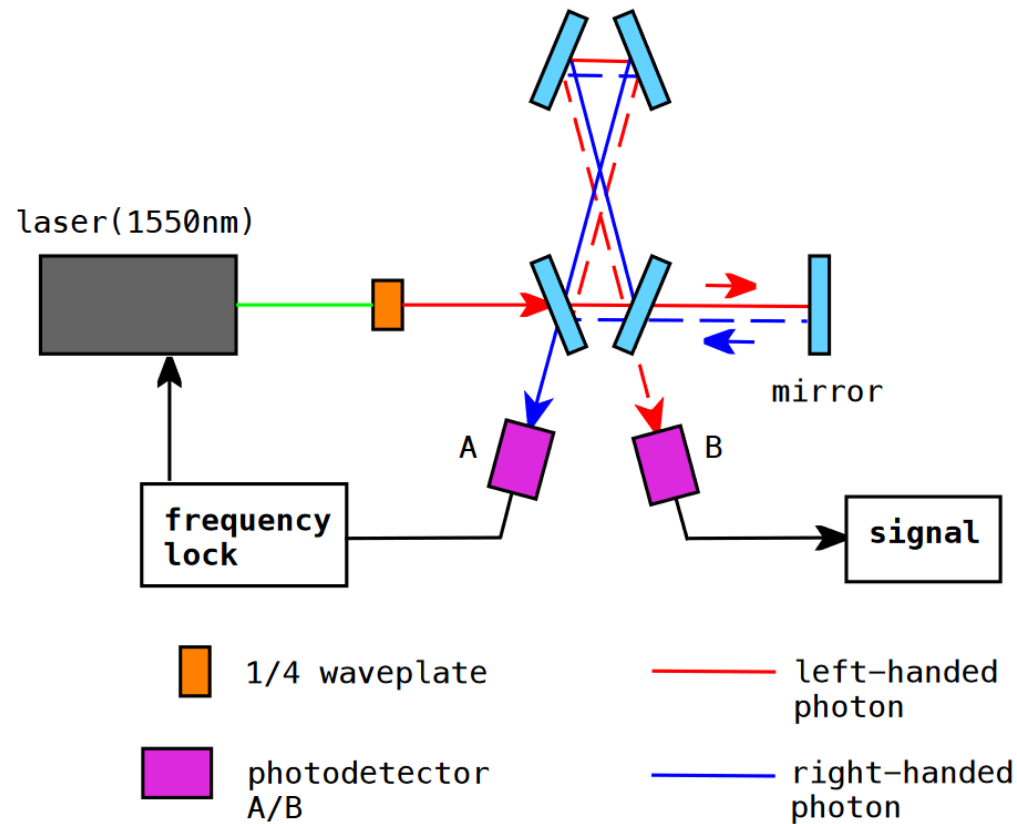
for

$$\lambda = 2\pi/k = 1550 \text{ nm}$$

$$\rho_a = m^2 a_0^2 / 2 \simeq 0.3 \text{ GeV/cm}^3$$

Optical Ring Cavity Search for Axion Dark Matter

I. Obata, T. Fujita, Y. Michimura, Phys.Rev.Lett. 121 (2018) 16, 161301



Thank you for your attention
