Supergravity at 99% of the speed of light

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FZU Division seminar

How to summarise the results of a research field which is 48 years old?

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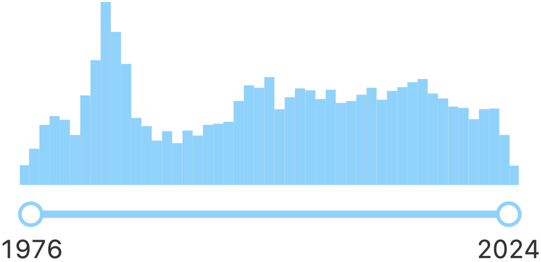

$\sim$ 5000 papers!

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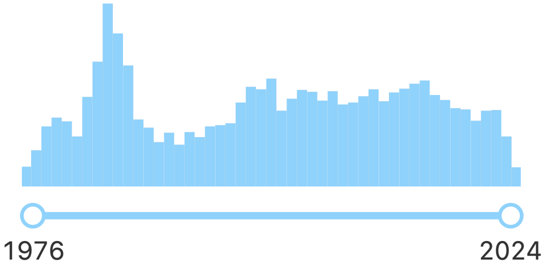

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Disclaimer: Much will be neglected, some will be discussed.

Content

General Relativity as a classical theory of gravity

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- Given the initial conditions, the above system of 10 second order partial differential equations allows to determine the metric $g_{\mu\nu}$, measuring distances and determining the motion of particles in spacetime, in terms of the distribution of matter $T_{\mu\nu}$. Vice versa, the latter can be derived, once the metric is known.

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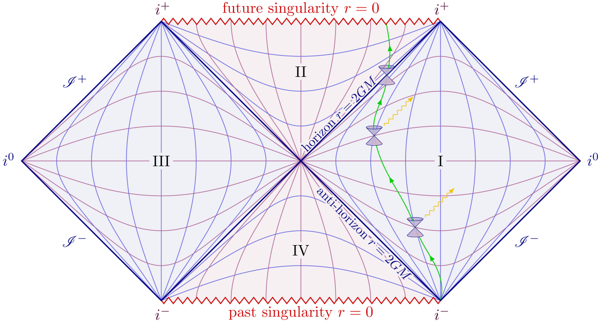
- The causal structure of spacetimes, solutions to the GR equations, can be studied through the so-called Penrose diagrams.

Example: a neutral, spherically symmetric black hole.



The quest for unification and supersymmetry

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- Symmetries have been a powerful ally in unifying electromagnetic, weak and strong interactions in the Standard Model (SM). The latter is a quantum field theory based on the Lie group $SU(3) \times SU(2) \times U(1)$.
- In the early '70s, a novel kind of symmetry, called *supersymmetry*, relating bosonic (integer spin) and fermionic (half-integer spin) particles, was proposed. Schematically

$$\delta|\text{boson}\rangle = |\text{fermion}\rangle\,, \qquad \delta|\text{fermion}\rangle = |\text{boson}\rangle\,.$$

- From an algebraic point of view, supersymmetry is encoded in *superalgebras*, which are $\mathbb{Z}_2$-graded vector spaces with a product satisfying a suitable generalisation of the Jacobi identities.
The simplest example is given by the $\mathcal{N} = 1$, $d = 4$ Poincaré superalgebra

$$[M_{ab}, M_{cd}] = 2(\eta_{c[a} M_{b]d} - \eta_{b[c} M_{d]a}) \,, \qquad [M_{ab}, P_c] = -2\eta_{c[a} P_{b]} \,,$$
$$[P_a, P_b] = 0 \,, \qquad [Q, M_{ab}] = \frac{\gamma_{ab}}{2} Q \,, \qquad \{Q, \bar{Q}\} = 2i\gamma^a P_a \,.$$

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One of the distinguishing features is that supersymmetry closes on diffeomorphisms and, usually, on-shell on the fields.
- Unitary irreducible representations of these algebras provide the so-called *supermultiplets*, collections of fields with the same mass, describing an equal number of bosonic and fermionic degrees of freedom.
Example: the Wess-Zumino massless supermultiplet [Wess, Zumino '74]

$$(\underbrace{\phi}_{\text{Complex scalar}}, \underbrace{\xi^\alpha}_{\text{Majorana spinor}})\,, \qquad 2=2 \qquad \text{(on-shell)}.$$

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- In view of these considerations, at the end of the '70s, combining GR with supersymmetry seemed a promising attempt towards the unification of gravity with the other interactions. Moreover, supersymmetry was thought to be capable of curing the diverging nature of GR.

Linearised gravity and renormalisation

- The quantisation of GR, described by the action

$$S = -\frac{1}{2\kappa^2}\int d^4x\sqrt{-g}R\,, \qquad \kappa = \sqrt{8\pi G}\,,$$

around a flat background is hurdled by the fact that the coupling constant κ has negative mass dimension ($[\kappa] = M^{-2}$ in natural units), making the theory *non-renormalisable* by power counting.

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- Divergences of loop diagrams, if present, cannot be absorbed by renormalisation of the Einstein-Hilbert action, as they necessarily require higher derivatives.
- While the S-matrix is finite at one-loop, it diverges at two loops [Goroff, Sagnotti '85]. The behaviour with bosonic matter is even worse.

D=4, N=1 pure supergravity

- The first supergravity theory was constructed in 1976 and described the *graviton supermultiplet* alone, which is composed of

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The field ψ_μ^α is the supersymmetric partner of the graviton.
As a spinorial 1-form, in principle it has two components: $\mathbf{1}\otimes\frac{\mathbf{1}}{\mathbf{2}}=\frac{\mathbf{3}}{\mathbf{2}}\oplus\frac{\mathbf{1}}{\mathbf{2}}$.
However, the gravitino by definition satisfies

$$\gamma^\mu\psi_\mu=0\,,$$

selecting only the $\frac{\mathbf{3}}{\mathbf{2}}$ component.
The $\frac{\mathbf{1}}{\mathbf{2}}$ component has been selected in "*unconventional*" supersymmetry models [Alvarez, Valenzuela, Zanelli '11].

- The action encoding the dynamics of these fields is

$$S = \int d^4x \left(-\frac{1}{2\kappa^2}\sqrt{-g}R + \epsilon^{\mu\nu\rho\sigma}\bar{\psi}_\mu \gamma_5 \gamma_\nu D_\rho \psi_\sigma \right).$$

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$$\delta g_{\mu\nu} = -2i\kappa\bar{\epsilon}\gamma_{(\mu}\psi_{\nu)}\,, \quad \delta\psi^\alpha_\mu = \frac{1}{\kappa}D_\mu \epsilon^\alpha \implies \delta S = 0\,.$$

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Local supersymmetry $\implies$ Supergravity

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- How many on-shell degrees of freedom (dofs)?
  The graviton has two dofs, so we expect the same number for the gravitino. One finds

$$\mathrm{dofs}(\psi) = (d-3) 2^{[d/2]-1} \overset{d=4}{=} 2 \,.$$

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- The gravitational supermultiplet can be coupled with other supermultiplets yielding *matter coupled supergravities*.

Is supergravity free of divergences?

- $\mathcal{N} = 1$ pure supergravity is a divergent theory: it diverges at 3 loops [Deser, Kay, Stelle '77] and the inclusion of matter only makes the behaviour worse. Example: the coupling with a Maxwell supermultiplet (spin $\mathbf{1}, \frac{\mathbf{1}}{\mathbf{2}}$) leads to divergences at 1 loop [Van Nieuwenhuizen, Vermaseren '76].

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- If adding a "single" Q^{α} improved the behaviour of gravity, why not more? This leads to the so-called *Extended supergravities*.
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  One could try and take $\mathcal{N}$ copies of these generators, $Q^{\alpha I}$: supermultiplets would then collect more and more particles with higher and higher spin.
- However we know that the coupling of a finite number of particles with spin/helicity higher than 2 is inconsistent.
  $\implies$ There is a maximum amount of supersymmetry: $\mathcal{N} = 8$.

- The upshot is that Extended supergravities are not finite for $\mathcal{N} < 7$.
  The fate of the *maximal* $\mathcal{N} = 8$ supergravity (which coincides with $\mathcal{N} = 7$), describing the gravitational supermultiplet

$$(g_{\mu\nu}, \psi_\mu^{\alpha I}, A_\mu^{[IJ]}, \lambda^{\alpha[IJK]}, \phi^{[IJKL]}) \qquad 2 + 56 + 70 = 128 = 16 + 112\,,$$

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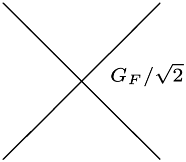
  However there is no clear proof of its finiteness.
- If (most probably) all four-dimensional supergravities are divergent, do we give up on the idea of merging supersymmetry and gravity?

Diverging theories are not the worst

- The old Fermi theory described the weak interactions as a four fermion vertex with coupling constant $[G_F] = M^{-2}$, making it non-renormalisable.



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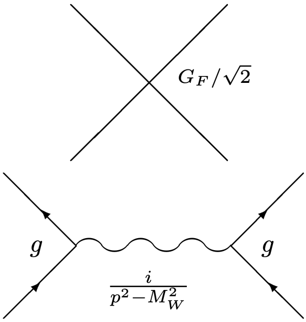

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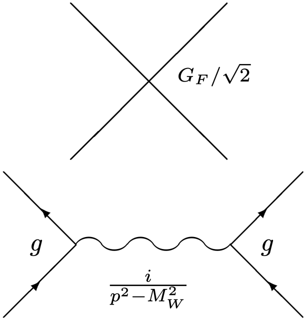

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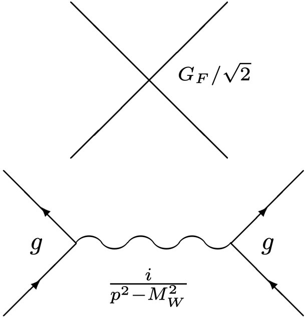

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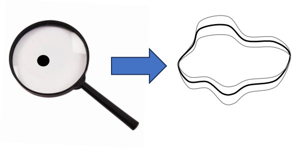
  $\implies$ We can trust the Fermi theory results in this regime.
- Is there a more fundamental quantum theory of gravity for which supergravity is a good approximation in the regime $p \ll M_P$?

String theory in three slides

- String theory assumes that ordinary particles correspond to certain vibrational modes of very small strings.



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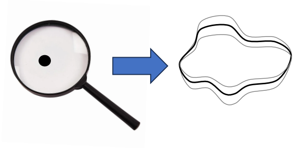
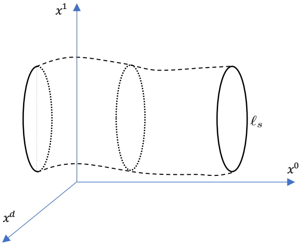


- These objects move and oscillate in a certain background space.



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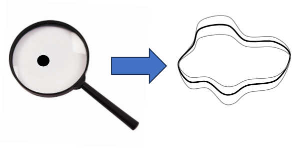

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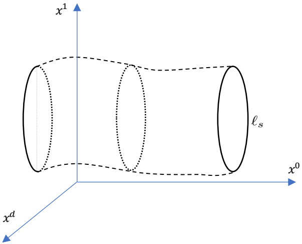

- They can be of two types: *closed* strings and *open* strings.

- The *spectrum* of closed strings (= the set of possible vibrational modes) includes a graviton and and therefore gravity, among an infinite number of other particles.

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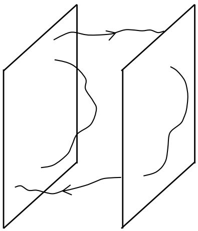

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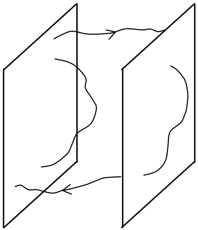

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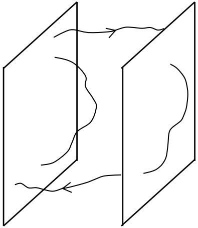

- The theory necessarily includes supersymmetry, in order to be able to describe particles with half-integer spin.
- The background space in which the strings move is not arbitrary. Consistency of the theory requires such space to be ten-dimensional.

- Four dimensional physics can be recovered by assuming six of the ten spacetime dimensions to be very small (compact) $\Longrightarrow$ *Compactification.*

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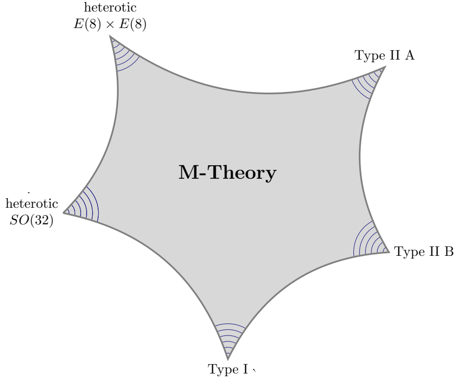
- There are five formulations of string theory, each with a different spectrum and symmetries. These are all connected by *dualities* and can be seen as different limits of a single 11-dimensional *Membrane/Mother/Mystery* theory.



Supergravity as an effective field theory

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Supergravity as an effective field theory

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  The consistency of string theory itself at the quantum level (=absence of conformal anomaly) imposes constraints on the background space.

  $\Longrightarrow$ It requires the background in which strings propagate to satisfy the equations of motion of a ten-dimensional supergravity theory!
- More precisely, supergravity provides a low energy ($\ell_s \to 0$) description of the massless particles in the string spectrum. Stringy corrections introduce higher derivatives and massive fields.
  (See [Sen, Zwiebach '24] for a clean derivation of linearised gravity from the string field theory action.)

Example: type IIA supergravity

- The massless fields in the type IIA string spectrum compose a supermultiplet given by

$$(\phi, B_{\mu\nu}, g_{\mu\nu}, A_\mu, C_{\mu\nu\rho}, \lambda^{+\alpha}, \lambda^{-\alpha}, \psi_\mu^{+\alpha}, \psi_\mu^{-\alpha}) \qquad \Longrightarrow \qquad \mathcal{N} = 2$$
$$1 + 28 + 35 + 8 + 56 = 8 + 8 + 56 + 56 = 128.$$

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- The supermultiplet is described by the action [Giani, Pernici '84]

$$
\begin{aligned}
S = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-g} \Big(& R - \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - \frac{1}{12} e^{-\phi} H_{\mu\nu\rho} H^{\mu\nu\rho} \\
& + \frac{1}{144^2} \epsilon^{\mu_0 \mu_1 \dots \mu_9} F_{\mu_0\mu_1\mu_2\mu_3} \left(3 F_{\mu_4\mu_5\mu_6\mu_7} B_{\mu_8\mu_9} - 8 H_{\mu_4\mu_5\mu_6} C_{\mu_7\mu_8\mu_9}\right) \\
& - \frac{1}{48} e^{\phi/2} G_{\mu\nu\rho\sigma} F^{\mu\nu\rho\sigma} - \frac{1}{4} e^{3/2\phi} F_{\mu\nu} F^{\mu\nu} \Big) + \text{fermions}\,,
\end{aligned}
$$

where $F^{(2)} = dA^{(1)}$, $H^{(3)} = dB^{(2)}$ and $F^{(4)} = dC^{(3)} + F^{(2)} \wedge B^{(2)}$.

M-theory and 11d Supergravity

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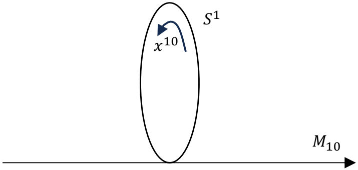
- Type IIA supergravity is recovered through compactification of one of the 11 dimensions along a circle.



- The action reads [Cremmer, Julia, Scherk '78]

$$S = \frac{1}{2\kappa_{11}^2} \int d^{11}x \sqrt{-g} \Big(R - \bar{\psi}_\mu \gamma^{\mu\nu\rho} D_\nu \psi_\rho - \frac{1}{24} G_{\mu\nu\rho\sigma} G^{\mu\nu\rho\sigma}$$
$$+ \frac{\sqrt{2}}{192} (\bar{\psi}_\mu \gamma^{\mu\nu\rho\sigma\tau\eta} \psi_\nu + 12 \bar{\psi}^\rho \gamma^{\sigma\tau} \psi^\eta)(2 G_{\rho\sigma\tau\eta} - \frac{3\sqrt{2}}{2} \kappa \bar{\psi}_{[\rho} \gamma_{\sigma\tau} \psi_{\eta]})$$
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- Example of a *bosonic* solution: the M2-brane.
  We expect a two dimensional extended object as a solution to this supergravity, since we have a 3-form in the theory. Such solution preserves a $ISO(1,2) \times SO(8)$ symmetry together with half of the original supersymmetry. Explicitly it reads

$$
\begin{aligned}
g &= \left(1 + \frac{q}{r^6}\right)^{-\frac{2}{3}} \eta_{\mu\nu} dx^\mu dx^\nu + \left(1 + \frac{q}{r^6}\right)^{\frac{1}{3}} \delta_{ij} dx^i dx^j , \\
G_{i\mu\nu\rho} &= \epsilon_{\mu\nu\rho} \partial_i \left(1 + \frac{q}{r^6}\right)^{-1} .
\end{aligned}
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The AdS/CFT correspondence "era"

- Being closely related to string theory, it comes with no surprise that supergravity plays a central role in the AdS/CFT correspondence, one of today's most active research areas, even after more than 20 years from its inception.

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- The AdS/CFT correspondence is a conjectured strong/weak coupling duality between theories of gravity on anti-de Sitter (AdS) backgrounds in d+1 dimensions and conformal field theories (CFTs) in d dimensions.
- The dual theories are related in such a way that (through the so-called *holographic dictionary* [Gubser, Klebanov and Polyakov '98]) the identification of the respective path integrals is possible

$$Z^{\mathrm{Gravity}}[\Phi_0] = Z^{\mathrm{CFT}}[J \simeq \Phi_0]\,.$$

This identification in turn allows the computation of any physical quantity (n-point function) of a theory from the knowledge of the other.
See [Zaffaroni '00] for an example.

The role of supergravity

- In its strong formulation, the gravity theory is taken to be string theory itself. If we consider the low energy limit on the gravity side (supergravity), we get strong coupling on the CFT side.
  This opens a window on the non-perturbative regime of gauge theories!

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  This opens a window on the non-perturbative regime of gauge theories!
- This gravity/gauge duality was first proposed in [Maldacena '97], where an equivalence between type IIB supergravity on $AdS_5 \times S_5$ to $\mathcal{N} = 4$ super Yang-Mills, in the large colour limit was discussed. The correspondence between the two theories at all regimes was only conjectured.

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$\Longrightarrow$ Supergravity was one of the key ingredients for the development of this conjectured duality, motivating the continued interest by the physics community in such a topic.

The construction of supergravity models

- We have seen that supergravity, despite being non-renormalisable and most probably divergent, has a vital role in relation to string theory. For this reason it will probably enjoy/suffer the same fate.
- However... How are supergravity theories constructed exactly?

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- Let us illustrate the idea with an example: the Wess-Zumino model.

$$\begin{aligned}\text{Lagrangian}: \quad & L = -\partial_\mu\phi\partial^\mu\phi^* - (\bar{\xi}_R \partial\!\!\!/ \xi_L + \bar{\xi}_L \partial\!\!\!/ \xi_R)\,, \\ \text{Susy}: \quad & \delta_\epsilon\phi = \bar{\epsilon}_L\xi_L\,, \quad \delta_\epsilon\phi^* = \bar{\epsilon}_R\xi_R\,, \\ & \delta_\epsilon\xi_L = \frac{1}{2}\partial\!\!\!/\phi\epsilon_R\,, \quad \delta_\epsilon\xi_R = \frac{1}{2}\partial\!\!\!/\phi^*\epsilon_L\,.\end{aligned}$$

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$$\begin{aligned} \text{Lagrangian :} \qquad & L = -\partial_\mu \phi \partial^\mu \phi^* - (\bar{\xi}_R \not{\partial} \xi_L + \bar{\xi}_L \not{\partial} \xi_R)\,, \\ \text{Susy :} \qquad & \delta_\epsilon \phi = \bar{\epsilon}_L \xi_L\,, \quad \delta_\epsilon \phi^* = \bar{\epsilon}_R \xi_R\,, \\ & \delta_\epsilon \xi_L = \frac{1}{2} \not{\partial} \phi \epsilon_R\,, \quad \delta_\epsilon \xi_R = \frac{1}{2} \not{\partial} \phi^* \epsilon_L\,. \end{aligned}$$

- The variation of the lagrangian is

$$\delta_\epsilon L = \partial_\mu (\bar{\epsilon}_L) j^\mu_L + \partial_\mu K^\mu + \text{h.c.}\,, \qquad j^\mu_L = -\not{\partial} \phi^* \gamma^\mu \xi_L\,, \quad \partial_\mu j^\mu_L|_{\text{on-shell}} = 0\,.$$

When the symmetry is global ($\epsilon_{L,R}$ constants) the lagrangian is invariant, up to a total derivative.

- If $\epsilon_{L,R} = \epsilon_{L,R}(x)$, then the lagrangian is not invariant: we introduce new 'gauge fields' $\psi_{\mu L,R}$ and

$$L_1 = -\frac{1}{\kappa}\bar{\psi}_{\mu L} j_L^{\mu} + \mathrm{h.c.}\,, \qquad \delta_\epsilon \psi_{\mu L,R} = \kappa \partial_\mu \epsilon_{L,R}\,.$$

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- Ok, but now we have to take the $\delta_\epsilon j^\mu_{L,R}$ into account, right?

$$-\bar{\psi}_{\mu L}\,\delta_\epsilon j_L^\mu = \bar{\psi}_{\mu L}\gamma_\nu \epsilon_R T^{\mu\nu} + \dots\,, \qquad T^{\mu\nu} = \partial^{(\mu}\phi\partial^{\nu)}\phi^* - \frac{1}{2}\eta^{\mu\nu}\partial_\rho\phi\partial^\rho\phi^*\,.$$

Then we see that we can introduce a new term in the lagrangian

$$L_2 \sim -g_{\mu\nu} T^{\mu\nu}\,, \qquad \delta_\epsilon g_{\mu\nu} \sim \frac{1}{\kappa}\left(\bar{\psi}_{\mu L}\gamma_\nu\epsilon_R + \text{h.c.}\right)\,.$$

- This shows that local supersymmetry requires the introduction of a metric and a gravitino!
  We also see that this procedure is not over and would require additional steps (introducing kinetic terms for the new fields, covariantising the supersymmetric transformation rules, ...)

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- The obtained *supermanifold* is described locally by coordinates $z^M = (x^\mu, \theta^\alpha)$, where θ^α are Grassmann variables, satisfying $\theta^\alpha \theta^\beta = -\theta^\beta \theta^\alpha$. Fields are now promoted to *superfields*

$$\Phi(x, \theta) = \phi(x) + \xi_\alpha(x)\theta^\alpha + O(\theta^2)\,.$$

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- The precise mathematics behind supermanifolds and superfields is a bit more subtle, but here we are interested in key ideas.

The rheonomic approach

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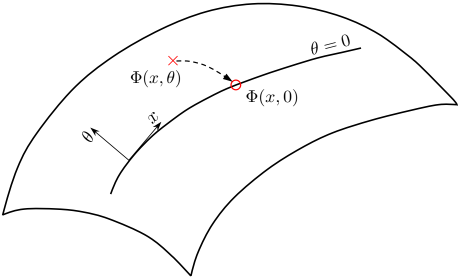

The *rheonomic* constraint:
All physical information should be contained in the slice $\theta = 0$:

$$\Phi(x, \theta) \propto \Phi(x, 0)\,.$$

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- The rheonomic approach is a versatile formalism, which can be applied in various spacetime dimensions and for different supermultiplets. Moreover, it has strong ties both with supergeometry and superalgebras/strong homotopy Lie algebras [Cremonini, Grassi, N., Ravera '22]
- This explains the longevity of such approach, which has been used even recently in [Dall'Agata, Liatsos, N., Trigiante '23] and [Andrianopoli, N., Trigiante, Zanelli '24].

Thank you for your attention!