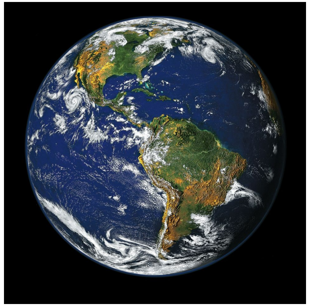
Anomalous magnetic moments of leptons

Vojtěch Pleskot

FZÚ AV ČR seminar, 26. 9. 2024

Dimensions

Earth radius / tennis ball radius = tennis ball radius / atom radius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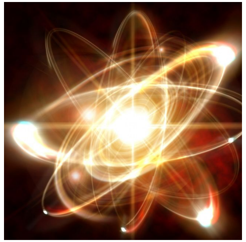

Dimensions

tennis ball radius / atom radius = atom radius / electron radius

(If it has any!
10^{-18} m is the limit of our knowledge…)

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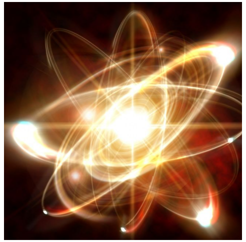

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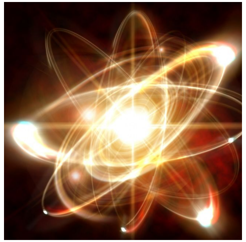

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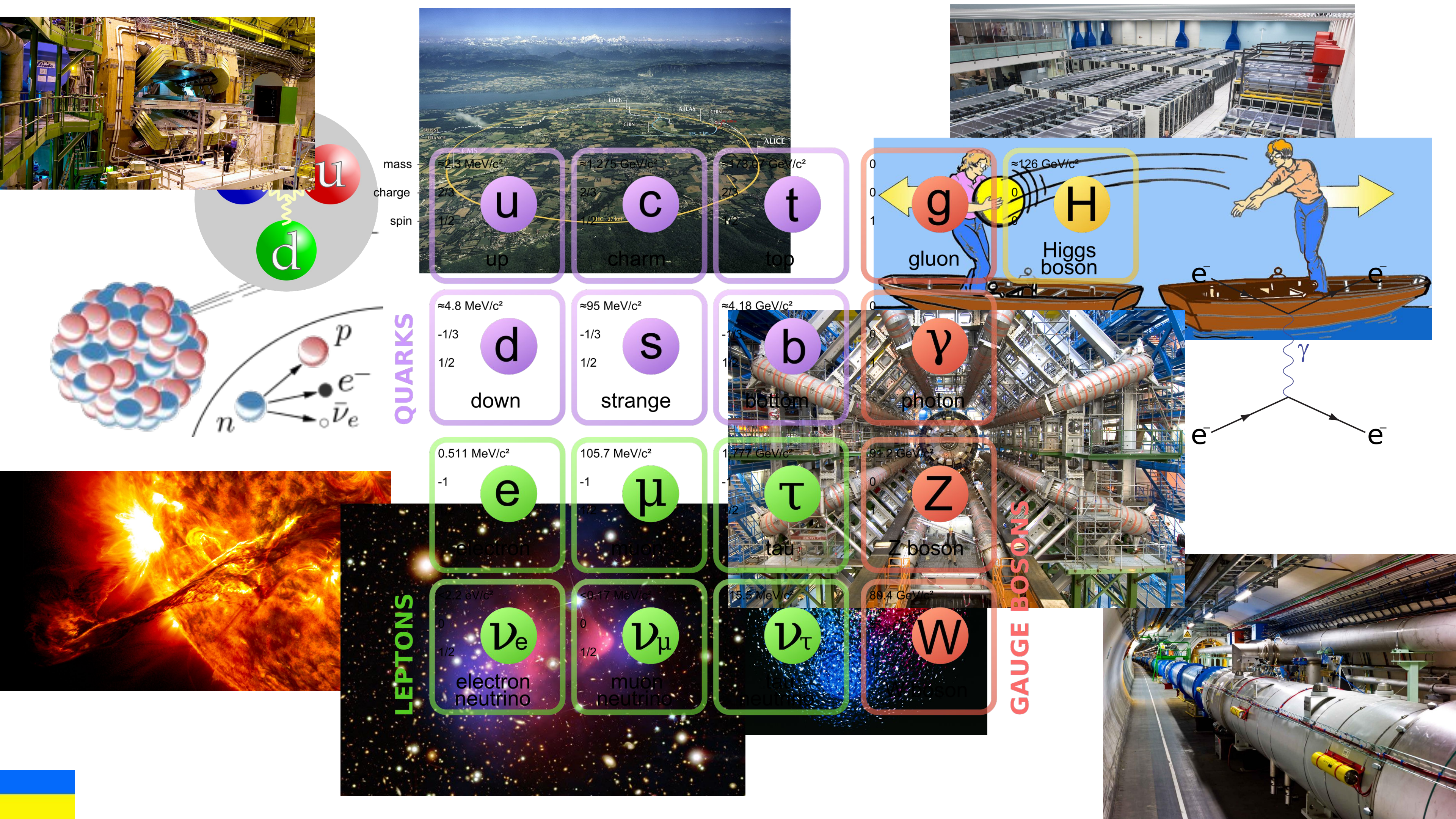
Dimensions

For sure, electron is

smaller than a tennis ball!

Dimensions

For sure, electron is
10 000 000 000 000 000 times
smaller than a tennis ball!


mass
charge
spin
QUARKS
LEPTONS
GAUGE BOSONS
≈2.3 MeV/c²
2/3
1/2
u
up
≈1.275 GeV/c²
2/3
1/2
c
charm
≈173.07 GeV/c²
2/3
1/2
t
top
0
0
1
g
gluon
≈126 GeV/c²
0
0
H
Higgs boson
≈4.8 MeV/c²
-1/3
1/2
d
down
≈95 MeV/c²
-1/3
1/2
s
strange
≈4.18 GeV/c²
b
bottom
0
0
1
γ
photon
0.511 MeV/c²
-1
e
electron
105.7 MeV/c²
-1
μ
muon
1.777 GeV/c²
τ
tau
91.2 GeV/c²
0
1
Z
Z boson
<2.2 eV/c²
0
1/2
ν_e
electron neutrino
<0.17 MeV/c²
0
1/2
ν_μ
muon neutrino
15.5 MeV/c²
ν_τ
80.4 GeV/c²
W
p
e^-
$\bar{\nu}_e$
n
e⁻
e⁻
γ
e⁻
e⁻
ALICE
ATLAS
CERN
LHCb
CMS

Gigantic success of the Standard Model!

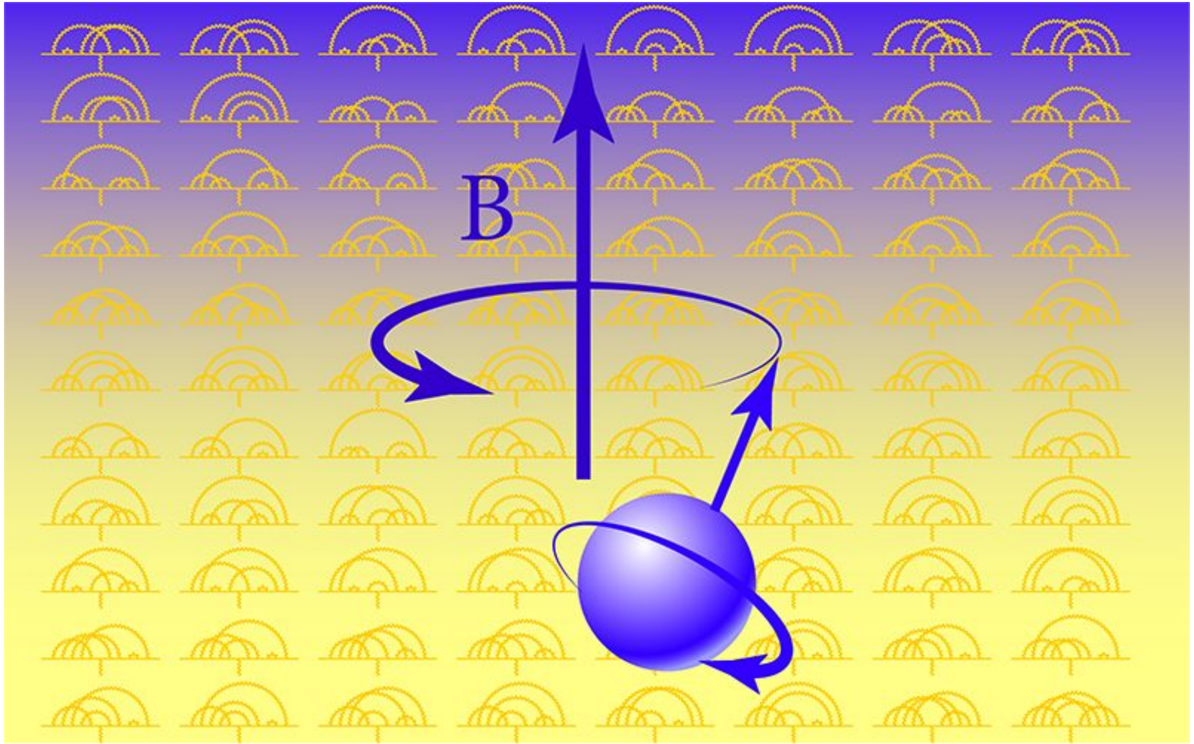
- Electron magnetic moment: 2.00231930436050 ± 0.00000000000019
    - More precise than to measure the Prague - Sydney distance to a human hair breadth!

g - 2

lepton magnetic moments

Magnetic moment of a particle - the consequence of spin

$$\vec{\mu} = g\frac{e}{2m}\vec{S}$$

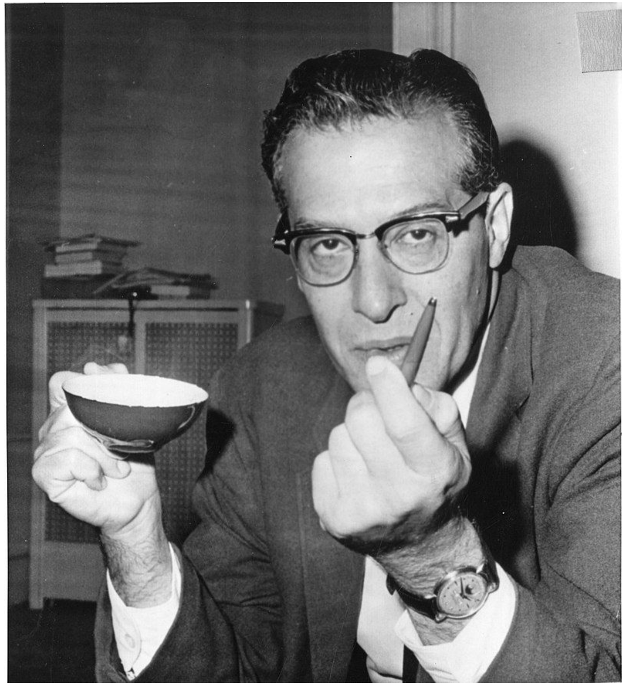


Magnetic moment of leptons

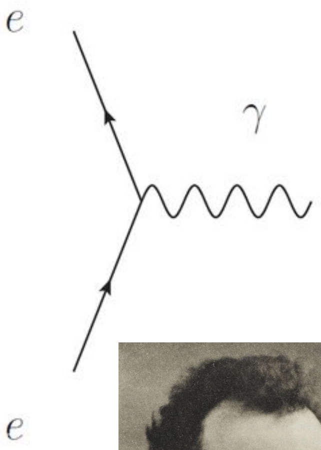
Paul Dirac: g = 2

Julian Schwinger: g = 2.0023

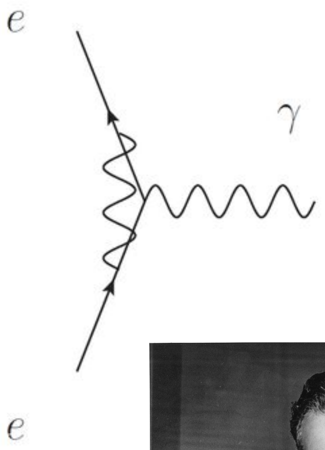
$$\vec{\mu} = {\color{red} g}\frac{e}{2m}\vec{S}$$



Quantum corrections!

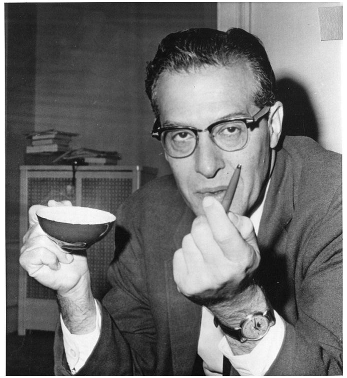


$+$



$+\cdots$

$$\alpha \approx \frac{1}{137}$$



Anomalous magnetic moment

$$a = \frac{g-2}{2}$$

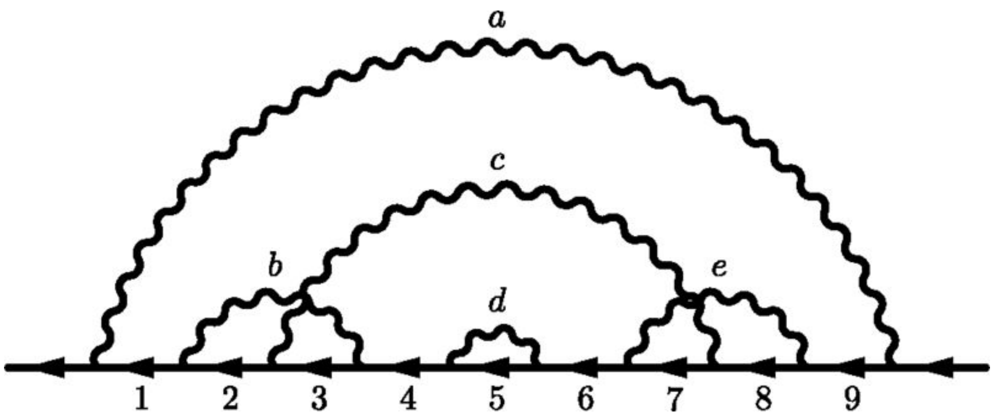
$$= \frac{\alpha}{2\pi} + \cdots$$

$$a_e$$

electron

anomalous magnetic moment

$$a_e^{\text{theory}} = 0.00115965218025\,2\,(95)$$



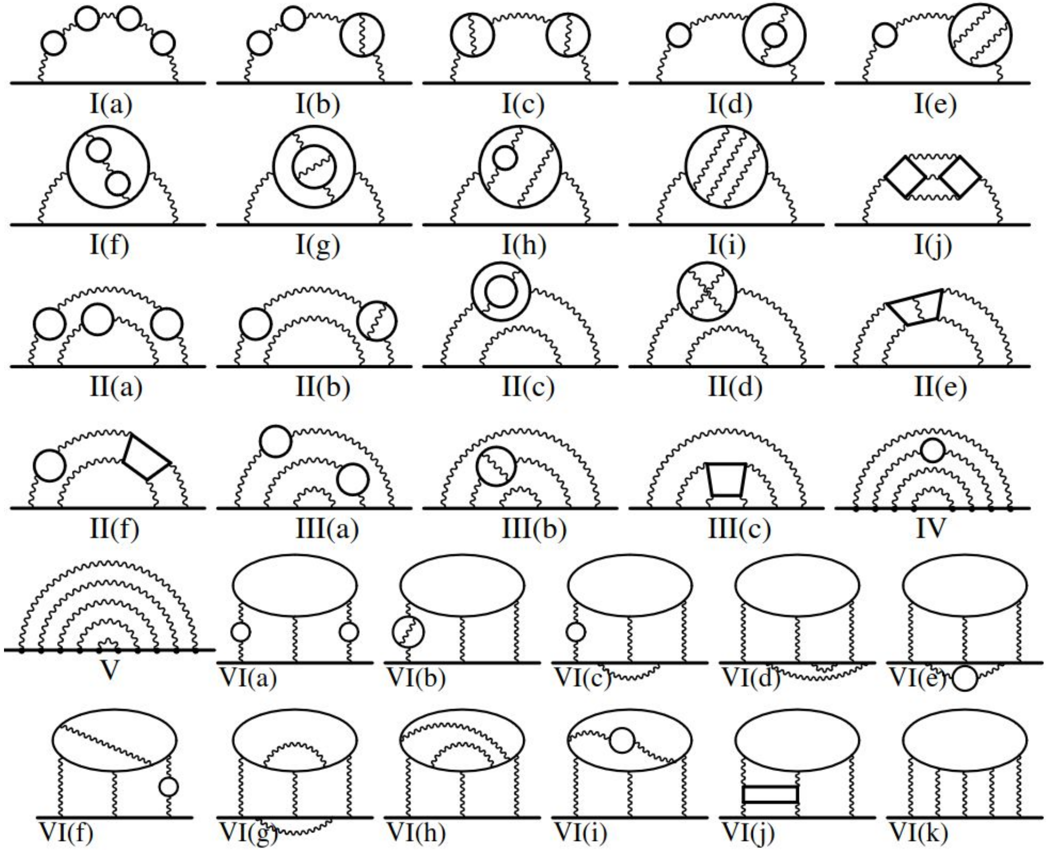
Aoyama, Kinoshita, Nio, 2018

- Not just one but five loops!
- 12672 diagrams!
- Precision: 10^{-13}

$$\frac{g}{2} = 1 + C_2\left(\frac{\alpha}{\pi}\right) + C_4\left(\frac{\alpha}{\pi}\right)^2 + C_6\left(\frac{\alpha}{\pi}\right)^3 + C_8\left(\frac{\alpha}{\pi}\right)^4 + C_{10}\left(\frac{\alpha}{\pi}\right)^5 + \ldots + a_{\mu\tau} + a_{\text{hadronic}} + a_{\text{weak}}.$$

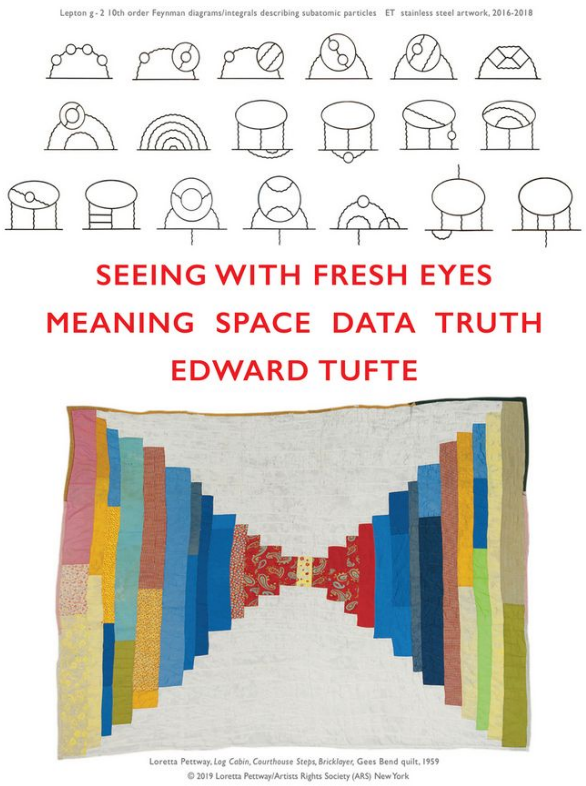
$$\boxed{\alpha \approx \frac{1}{137}}$$

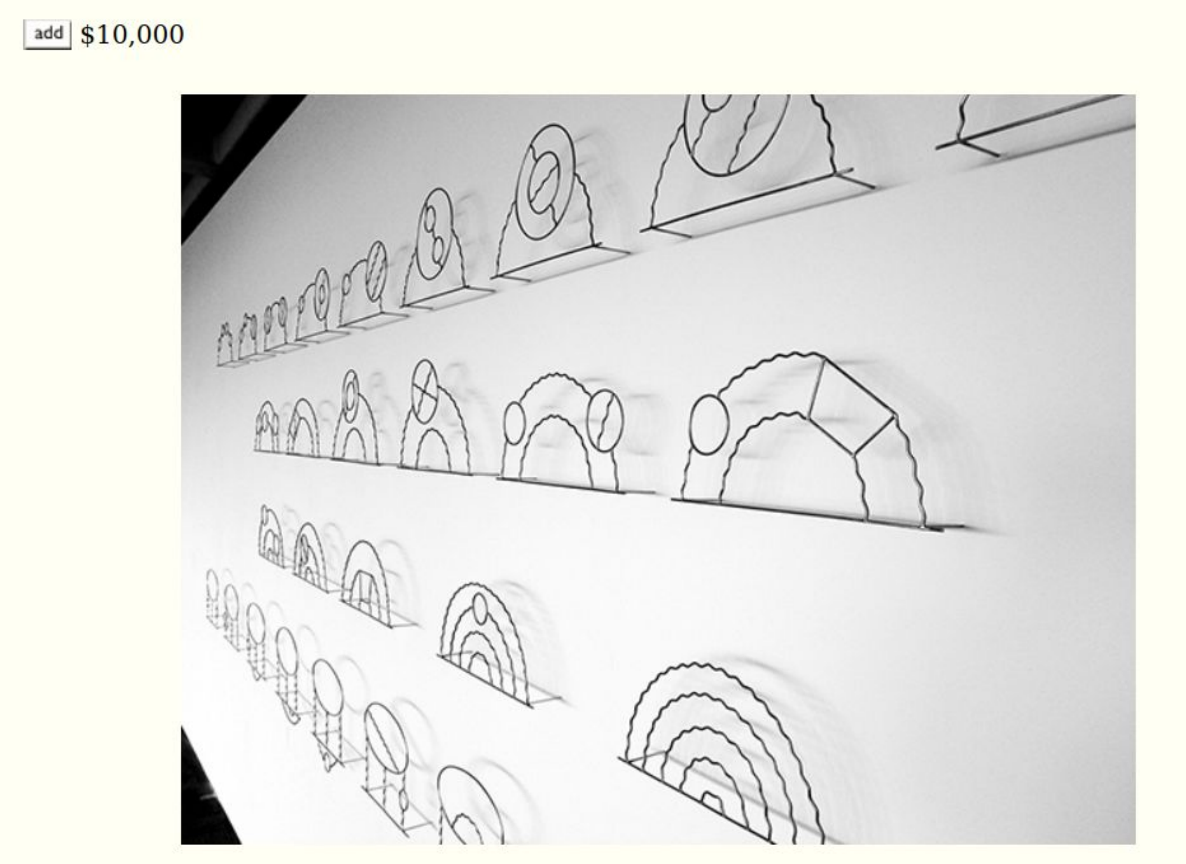
Pure beauty!



and 12000 similar diagrams…

Even arts got inspired!





Decomposing the theory

Volkov arXiv:1909.08015

$$a_e^{\text{theory}} = 0.001159652181547(6)(12)(229)$$

QED (6)

Hadronic (12)

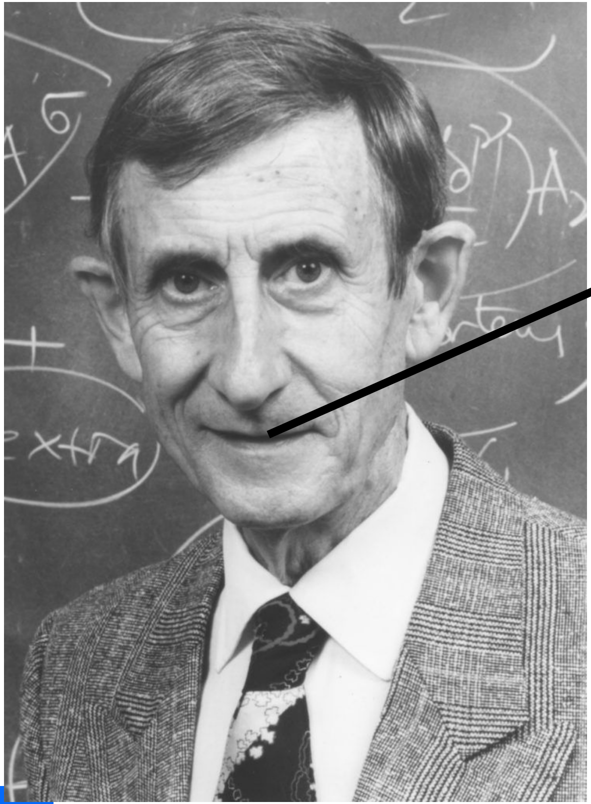
α(Cs) measurement unc. (229)

N.B.: Weak contribution unc. is negligible

N.B.: The newest α(Rb) measurement yields the unc. 95 instead of 229

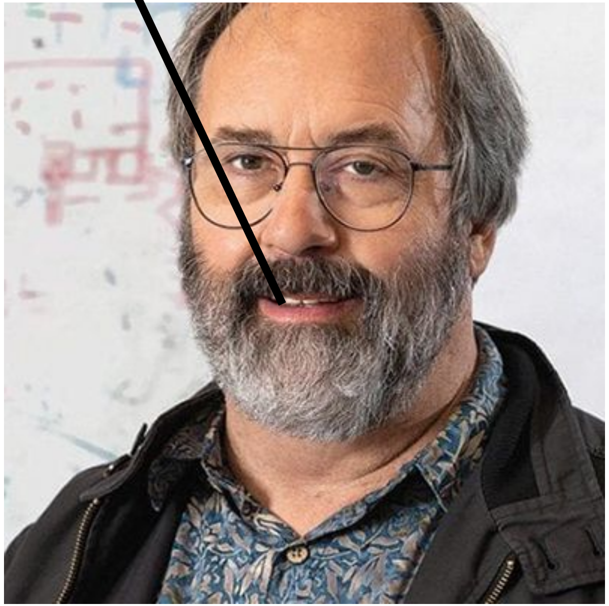
$$a_e^{\mathrm{exp}} = 0.00115965218059\,(130)$$

Freeman Dyson

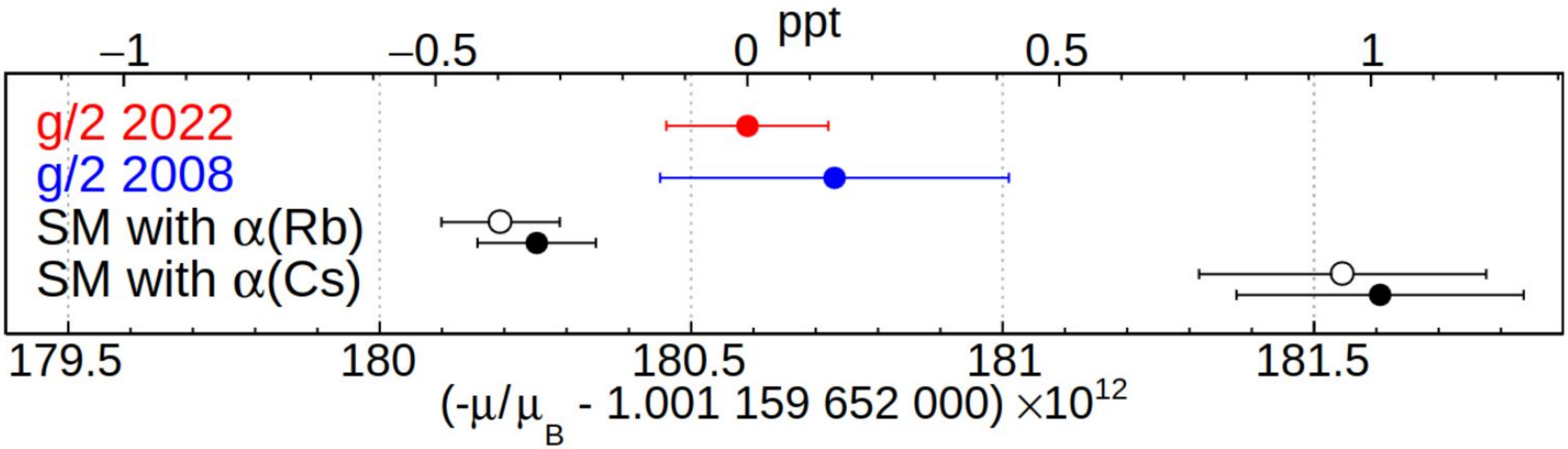


Wow! But we were cooking up just something before we get to invent a better theory…!

Gerald Gabrielse



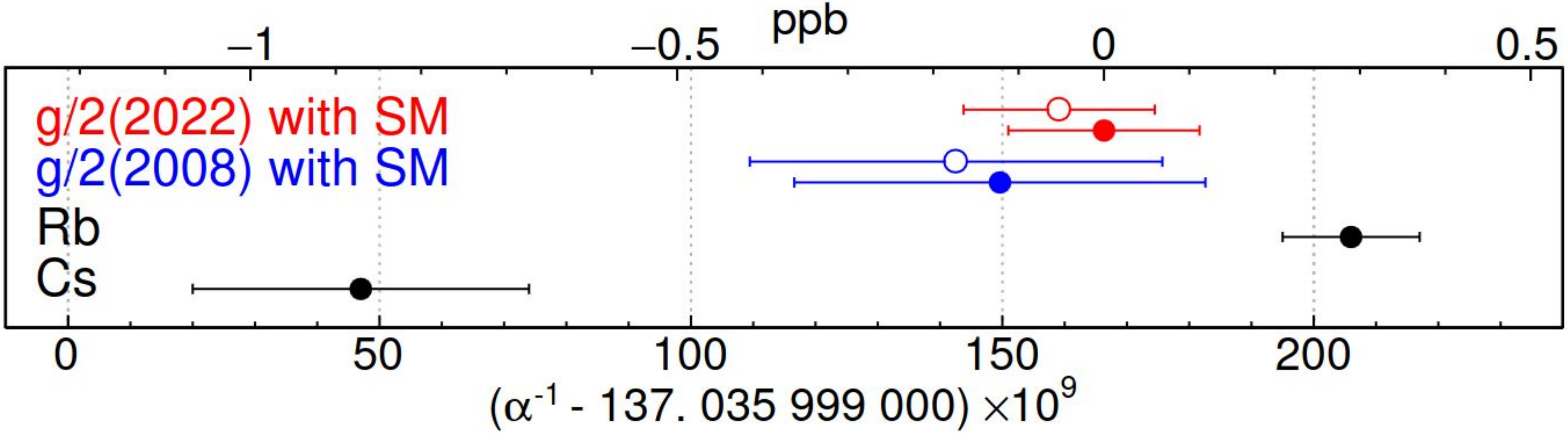
Measurement



$$-\frac{\mu}{\mu_{\rm B}} = \frac{g}{2} = a + 1$$

- SM precision: 10^{-13} …but there's the α measurement discrepancy…

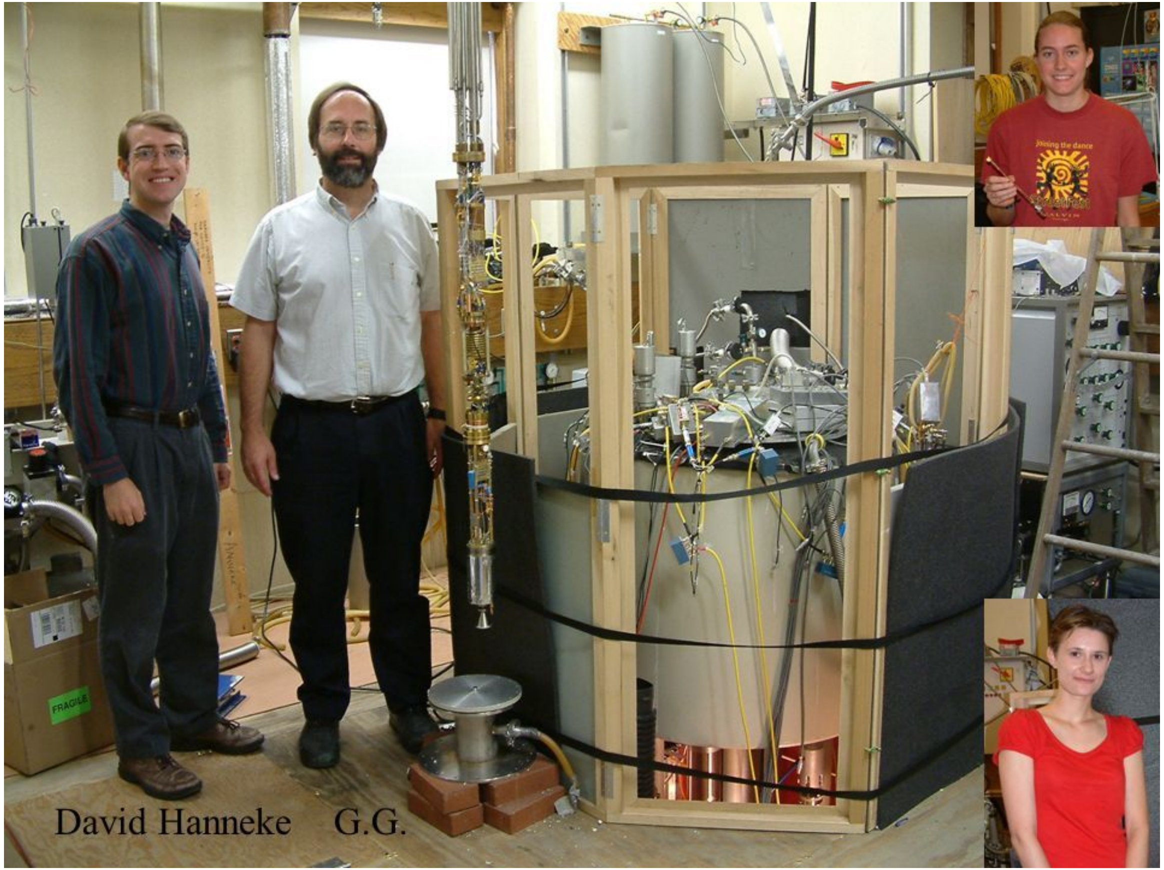
α measurement



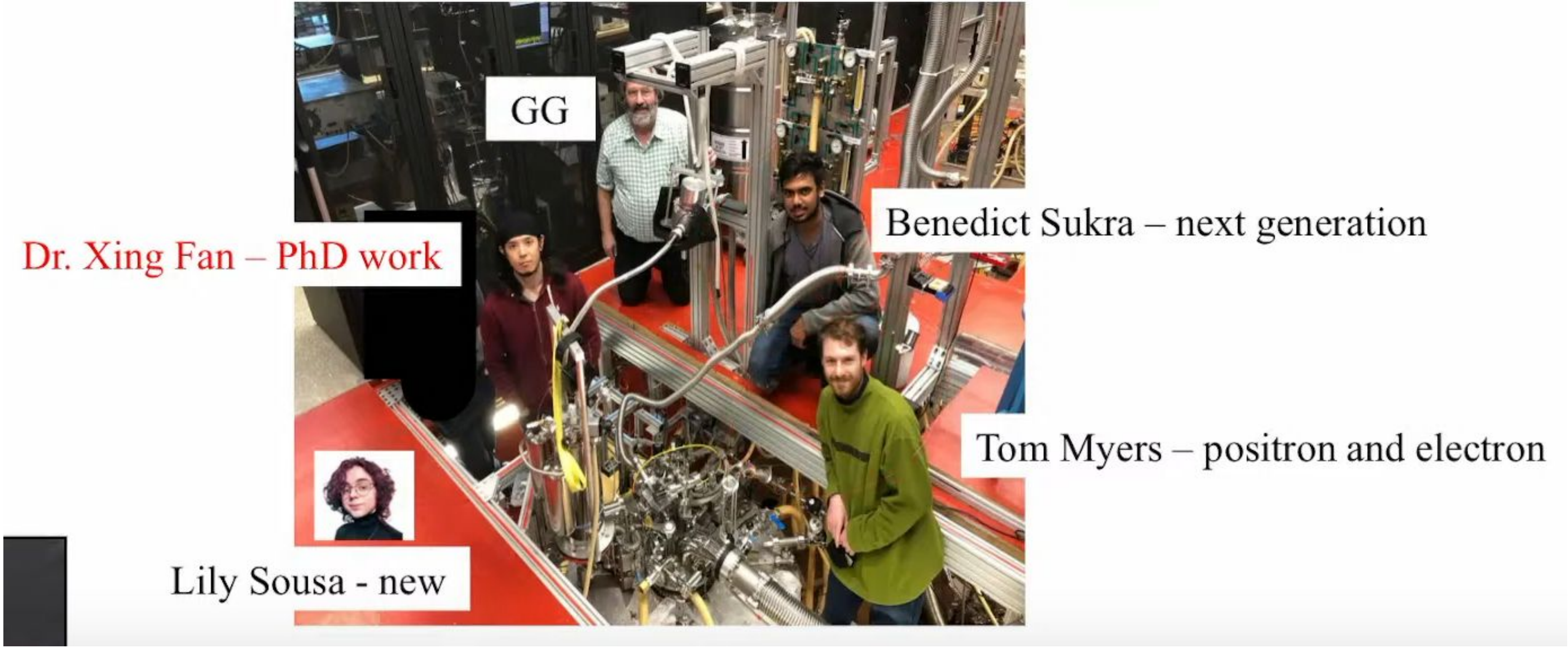
- a_e measurement together with the SM prediction can be used as the α measurement!
    - Precision very comparable to the most precise α(Rb) measurement!

Gerald Gabrielse, USA

Gerald Gabrielse, tabletop experiment

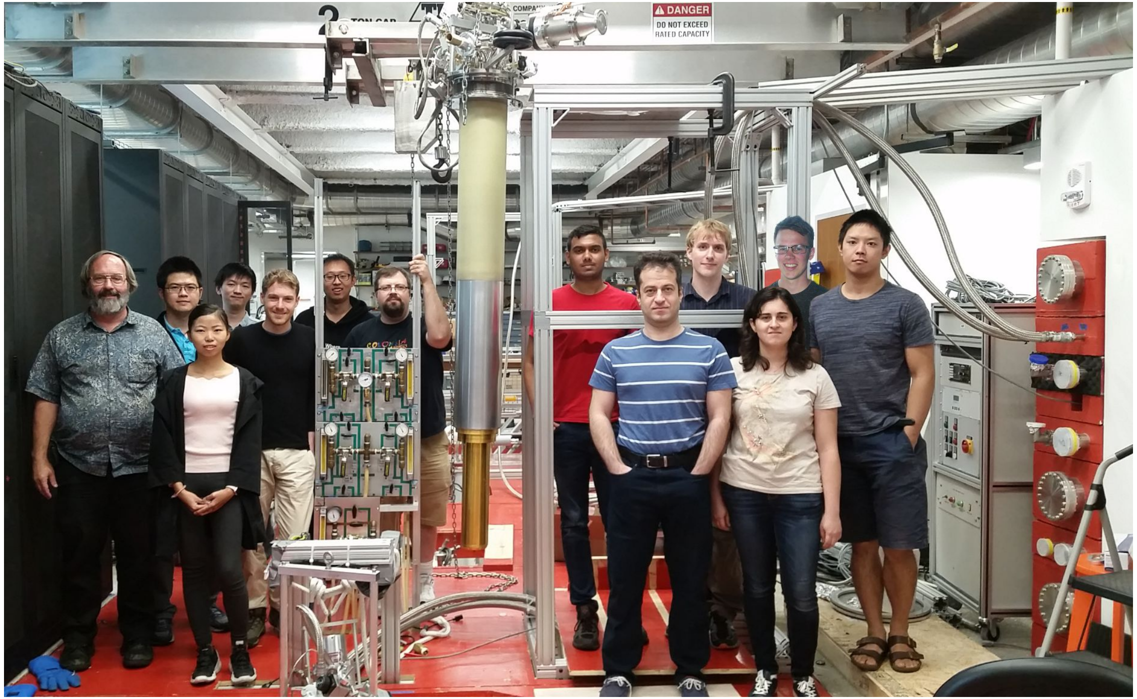


Gerald Gabrielse with his a_e group

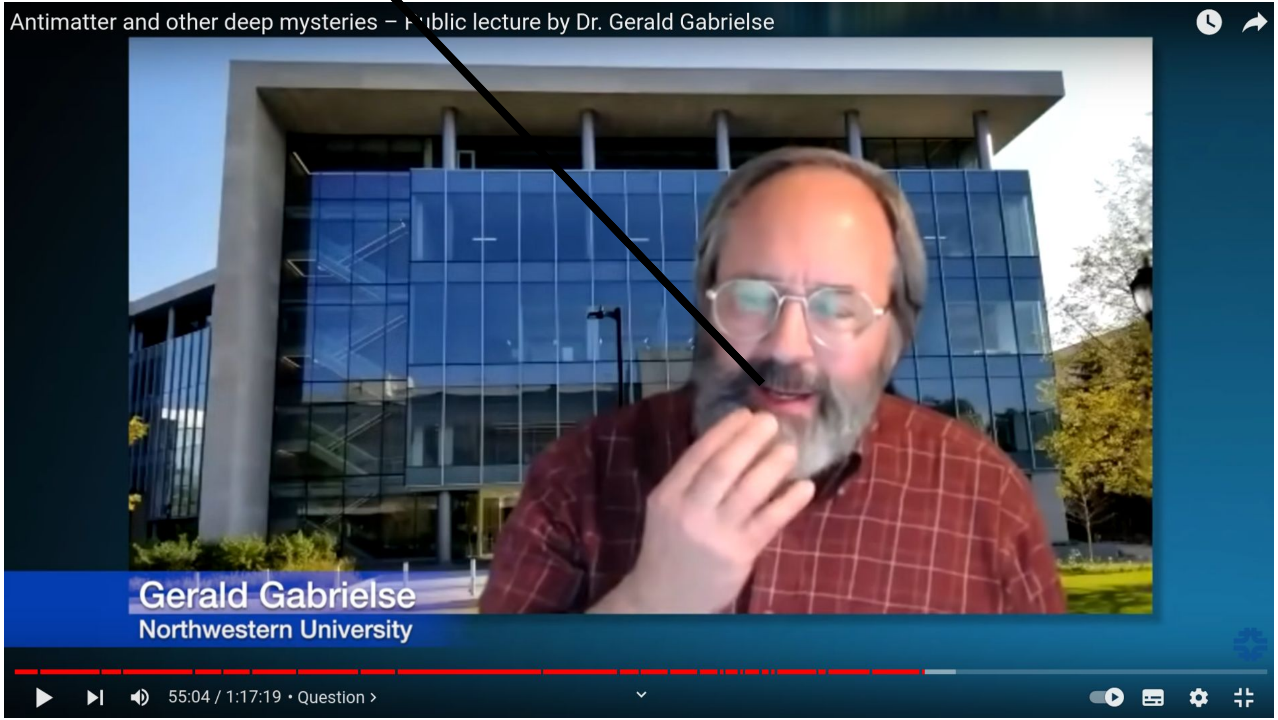


Gerald Gabrielse with his whole group

G.G.: “...good students… especially if they are tenacious, and very skilled and don’t break more than $2000 worth of apparatus per week…”

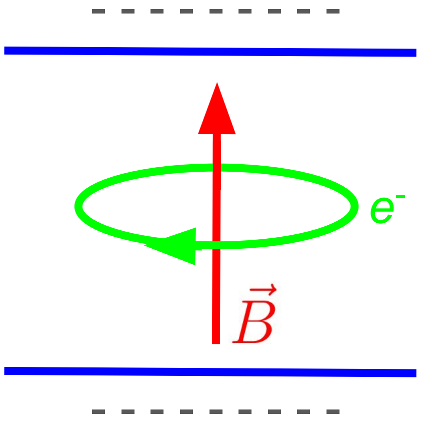


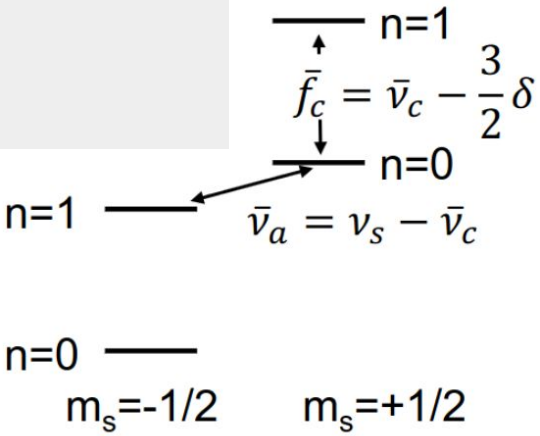
Just to make sure that our apparatus is turned on, we have to make more precise measurements than most physicists make in their whole career…



Principle of the a_e measurement

- one-electron quantum cyclotron
    - Penning trap
    - cooled to 50 mK, the electron can stay in the ground state ~forever





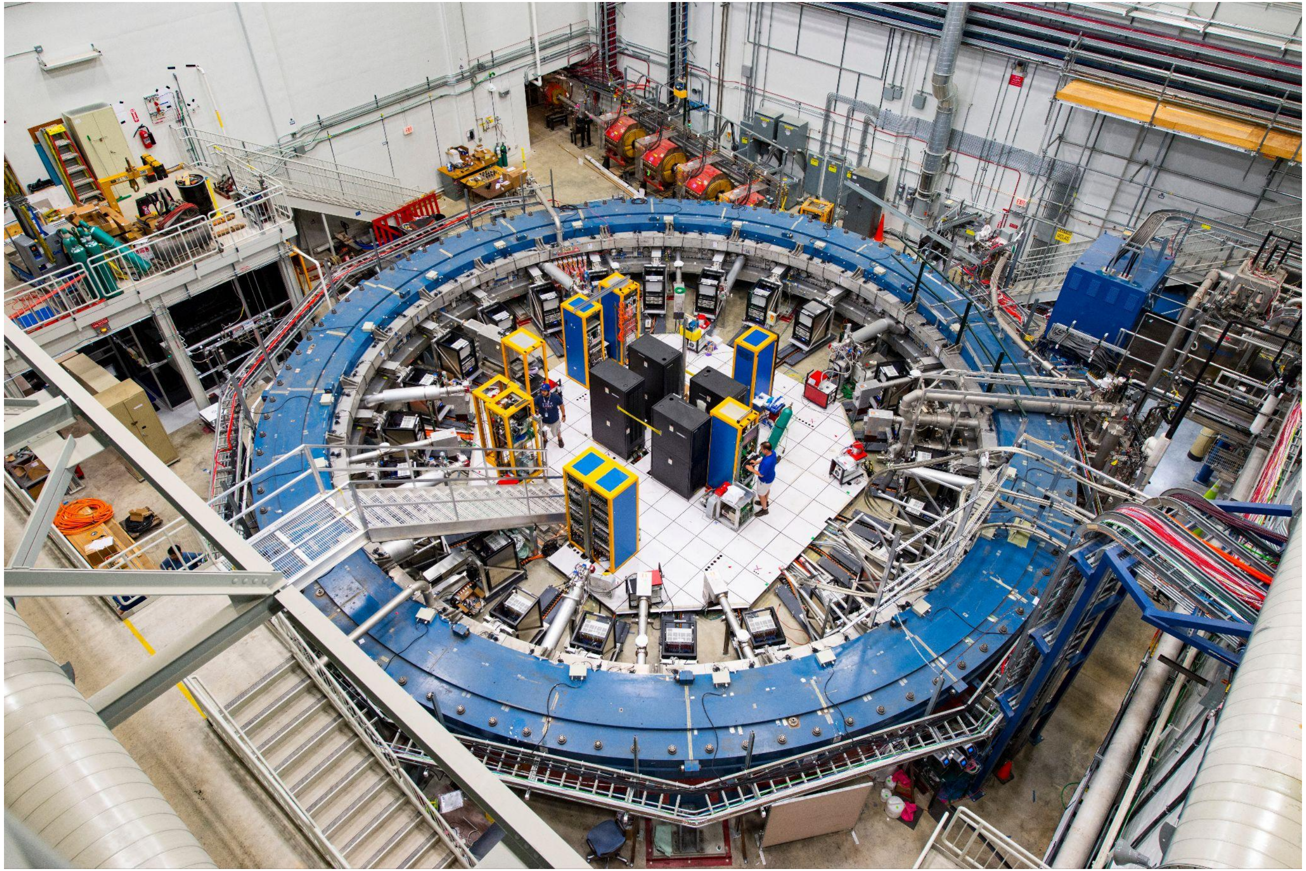
$$a_e = \frac{\bar{\nu}_a}{\bar{\nu}_c}$$

$$a_{\mu}$$

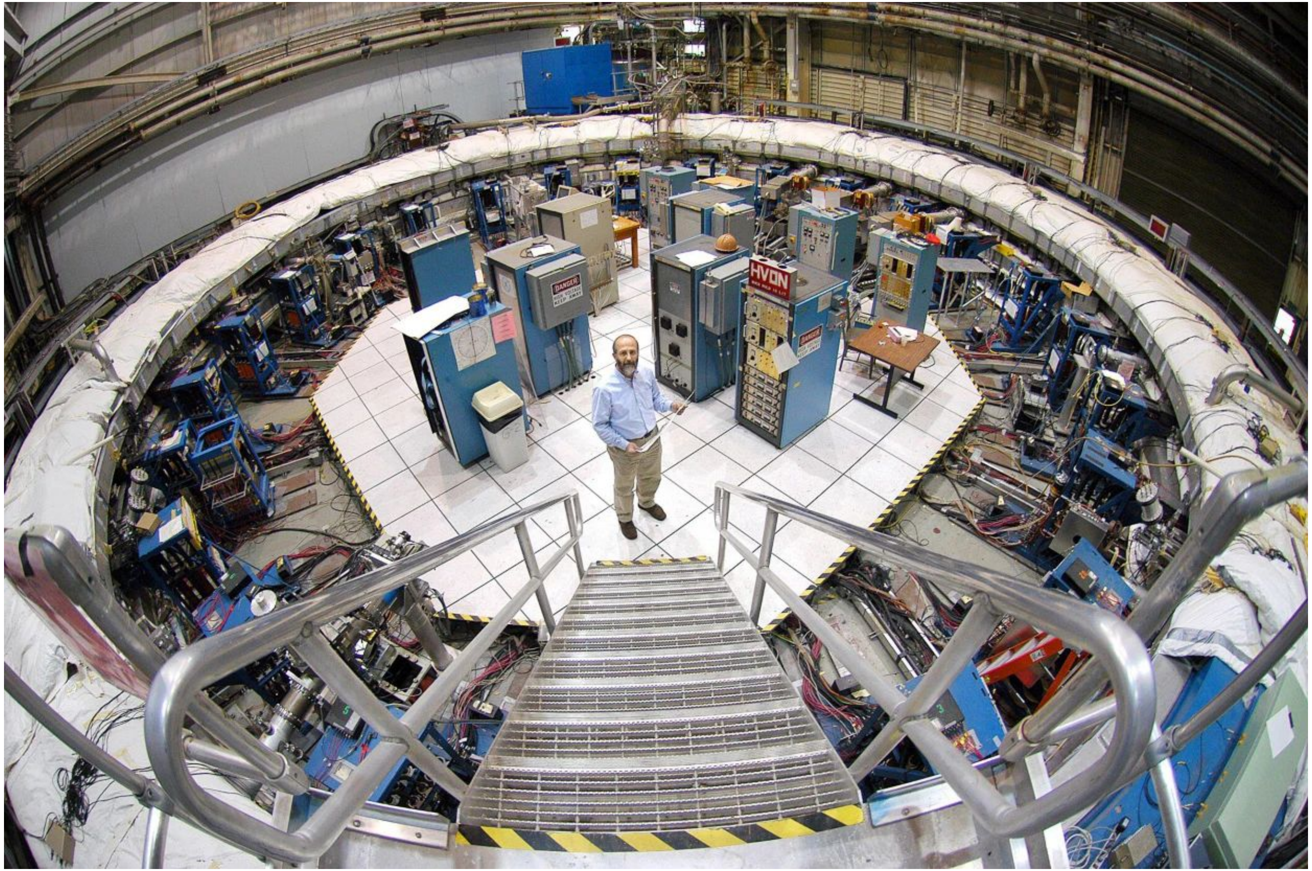
muon

anomalous magnetic moment

Muon g - 2 experiment



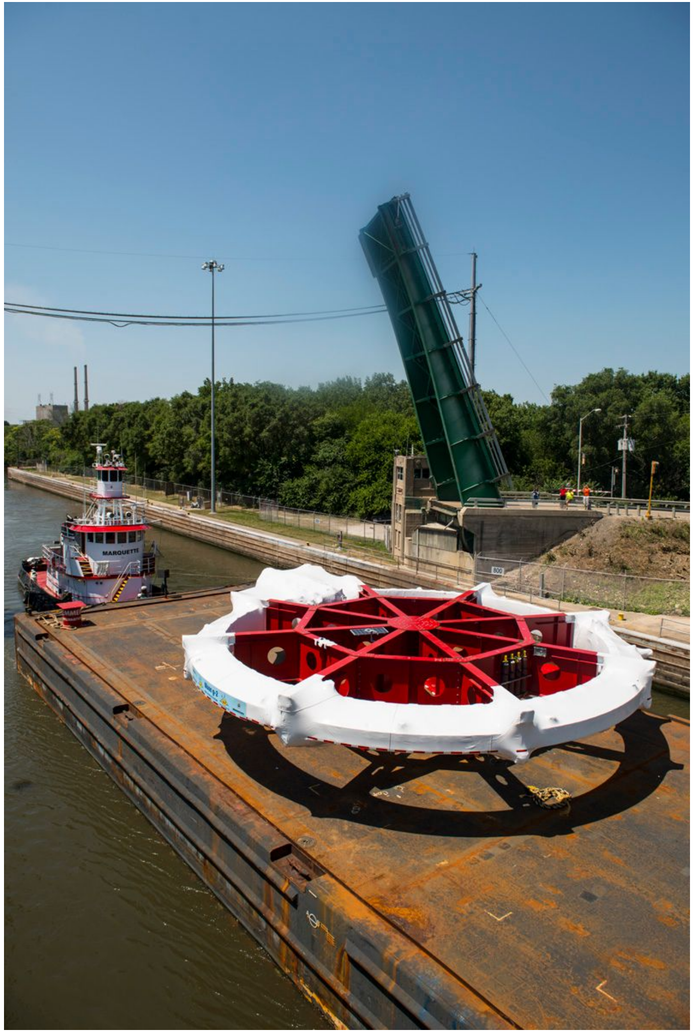
Muon g - 2 experiment



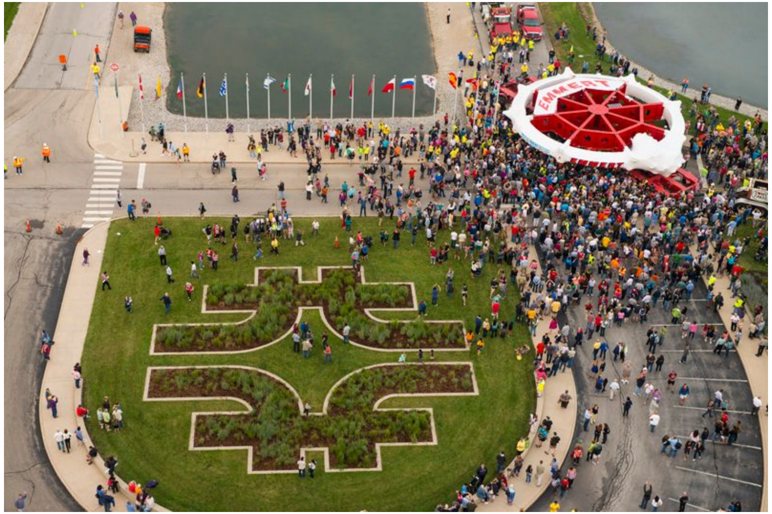

Fermilab
Brookhaven
OVERSIZE LOAD
Big Move
OVERSIZE LOAD

Muon g-2 experiment

- Superconducting magnet
  - transported from BNL

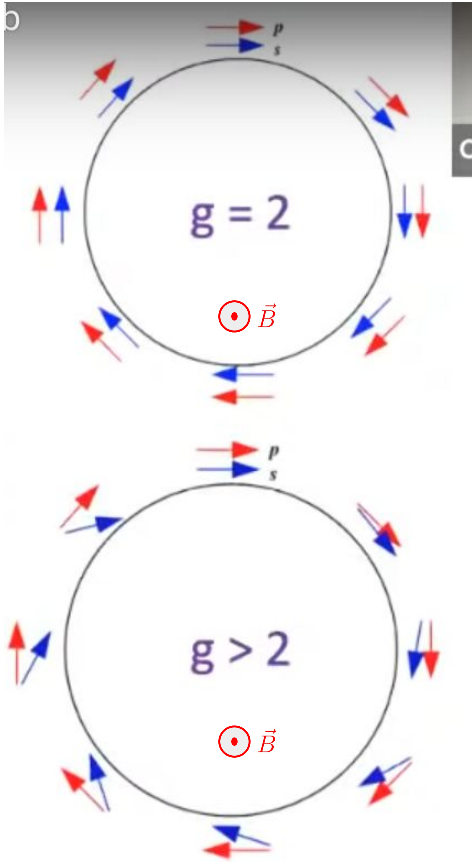


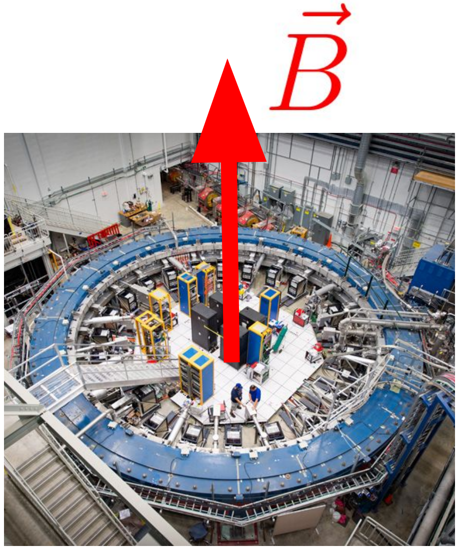
Fermilab



Muon g - 2 principle

- Muon circulates in $\vec{B}$ with the cyclotron frequency
- At injection, muon spin is aligned with the muon momentum
- The spin precedes around $\vec{B}$ with a frequency similar but not identical to the cyclotron frequency
- High-energy decay positrons emitted in the spin direction predominantly





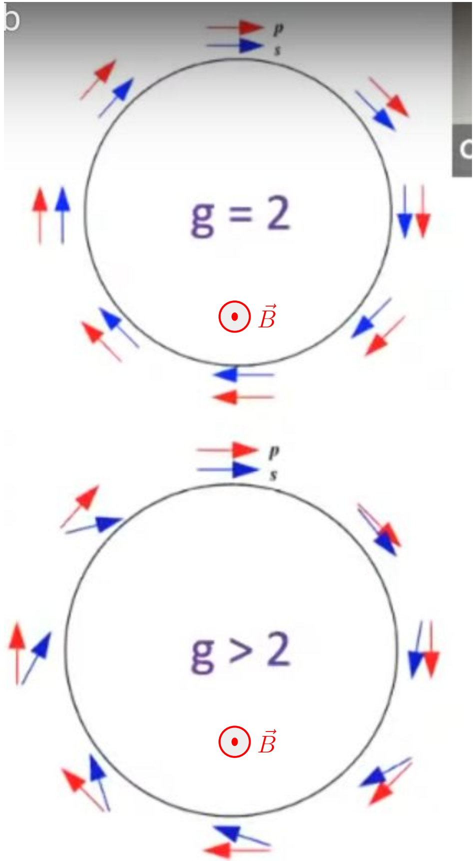
Muon g - 2 principle

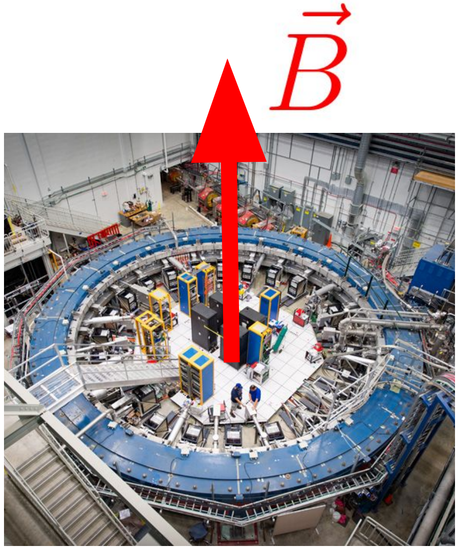
$$\vec{\omega}_a \equiv \vec{\omega}_s - \vec{\omega}_c = -a_\mu \frac{q\vec{B}}{m_\mu}$$

Cyclotron f

Spin precession f

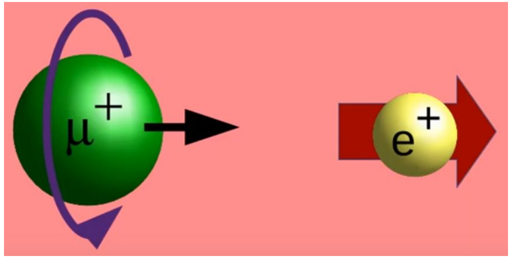
Anomalous f

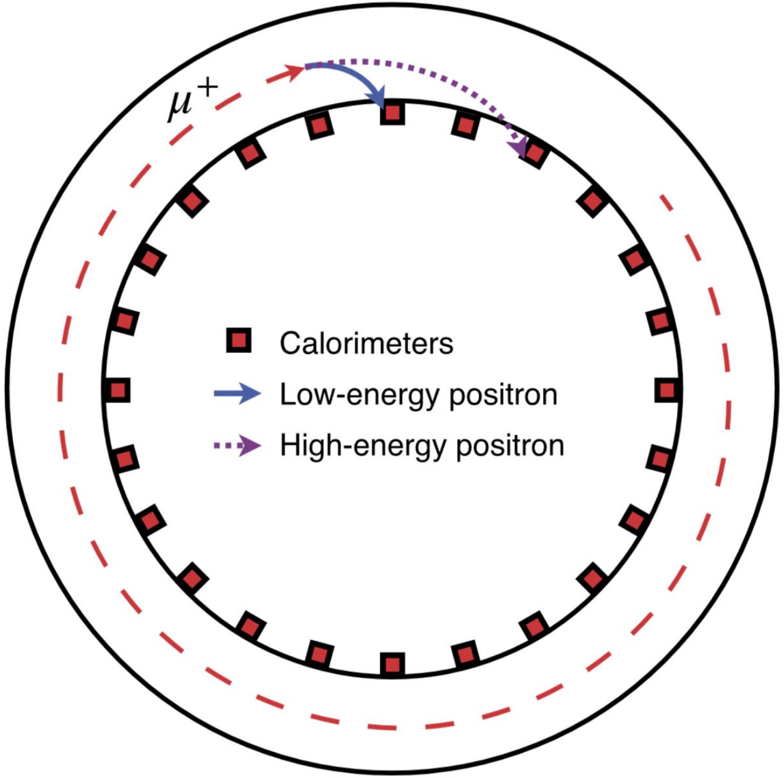




Just count positrons with the highest energy

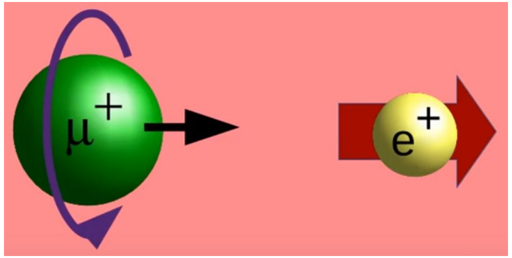
- High energy positrons emitted predominantly in the spin direction

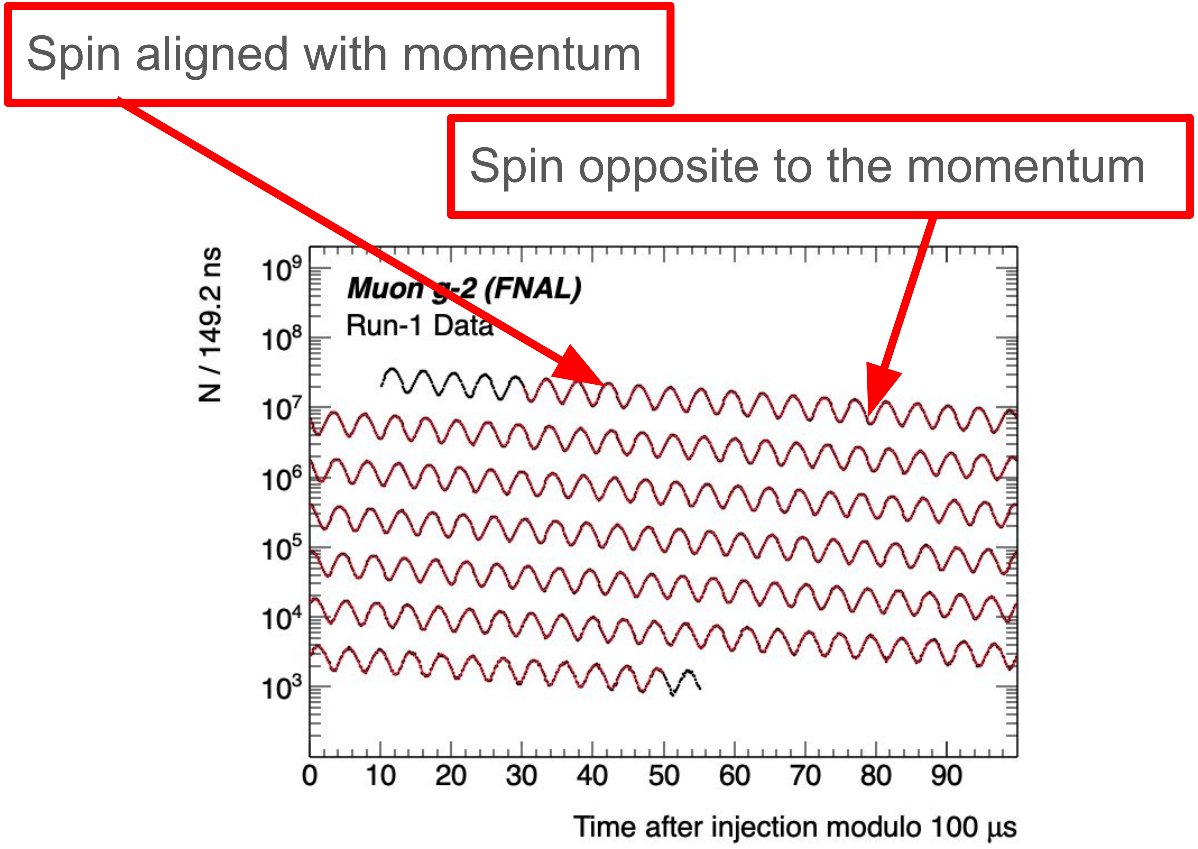




Just count positrons with the highest energy

- High energy positrons emitted predominantly in the spin direction





Wow numbers!

$$r = 7.112 \text{ m}$$

$$\left|\vec{B}\right| = 1.47 \text{ T}$$

$$\gamma \approx 29.3$$

$$v \approx 99.94\ \%\ c$$

$$\gamma\tau \approx 64.4\ \mu\text{s}$$

$$T_{\text{c}} = 149.2\text{ns}$$

$$1 \text{ fill} = 1 \text{ bunch of length } 120 \text{ ns} \times c$$

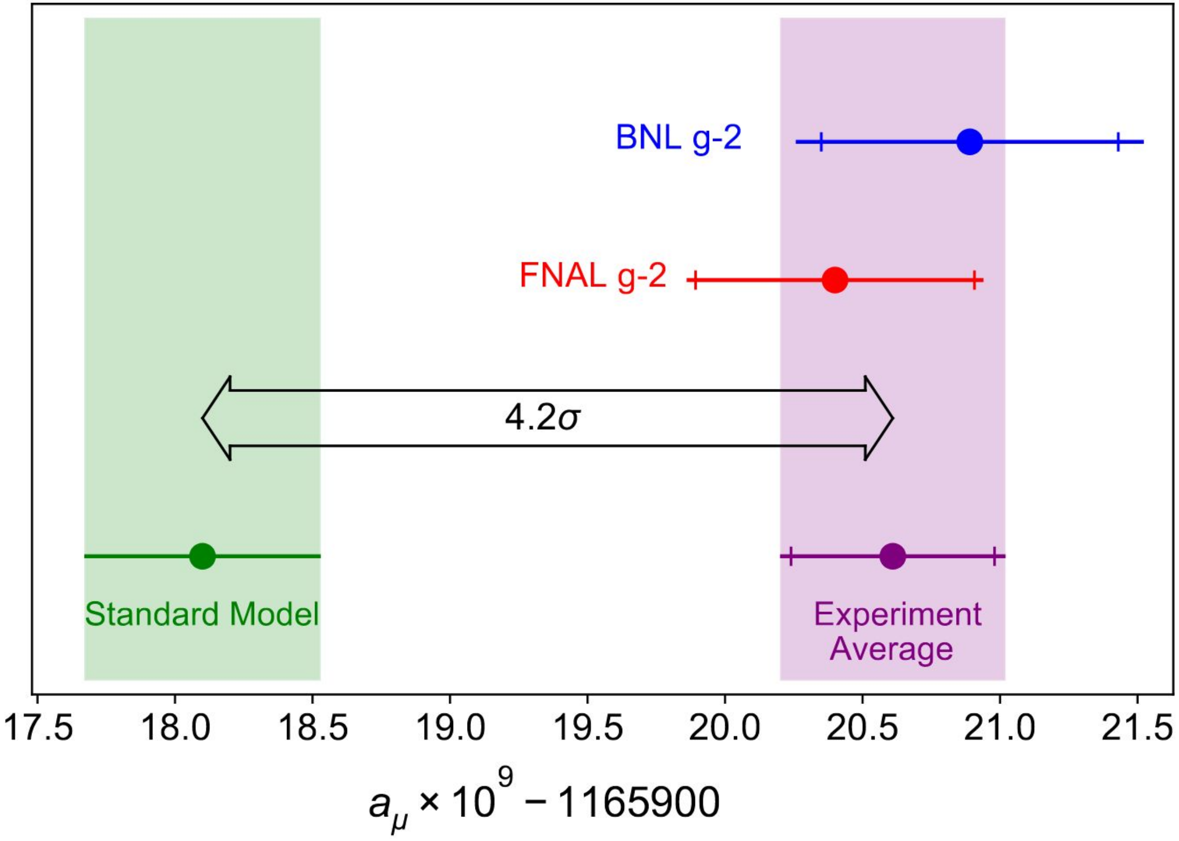
$$1 \text{ fill lasts } 700\ \mu\text{s}$$

$$1 \text{ fill contains } 5000 \text{ muons}$$

$$16 \text{ fills per } 1.4 \text{ s}$$

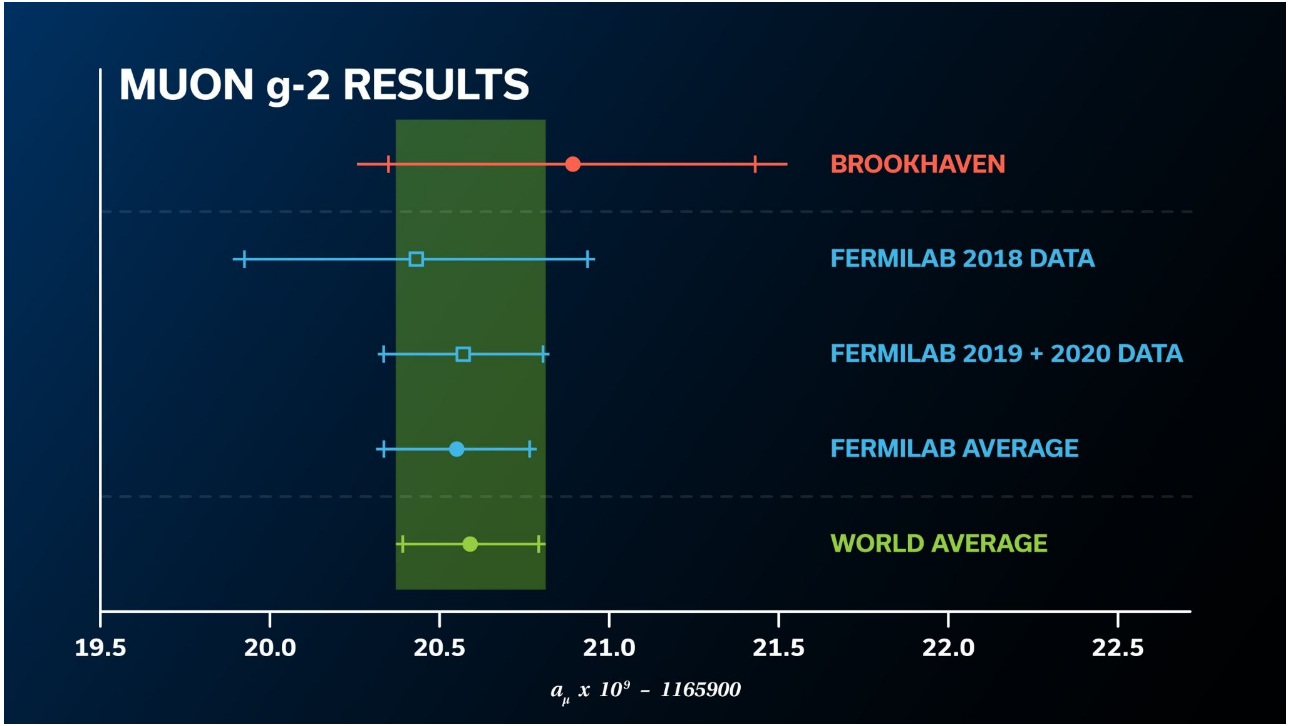
2021 announcement

- Run 1 data
- Accuracy of 10^{-10}!
- 4.2σ = 1:40000!
- a_μ 40000 times more sensitive to new heavy particles than a_e
    - $(m_{e,\mu}/M_{new})^2$ dependency

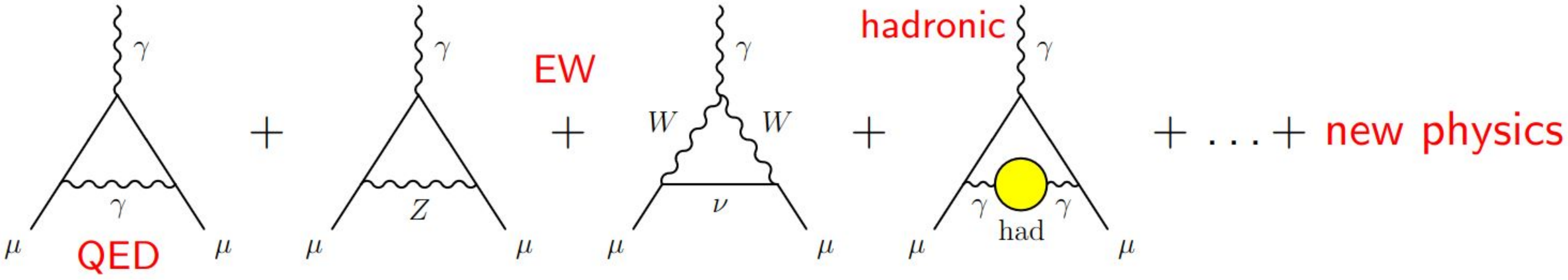


2023 announcement

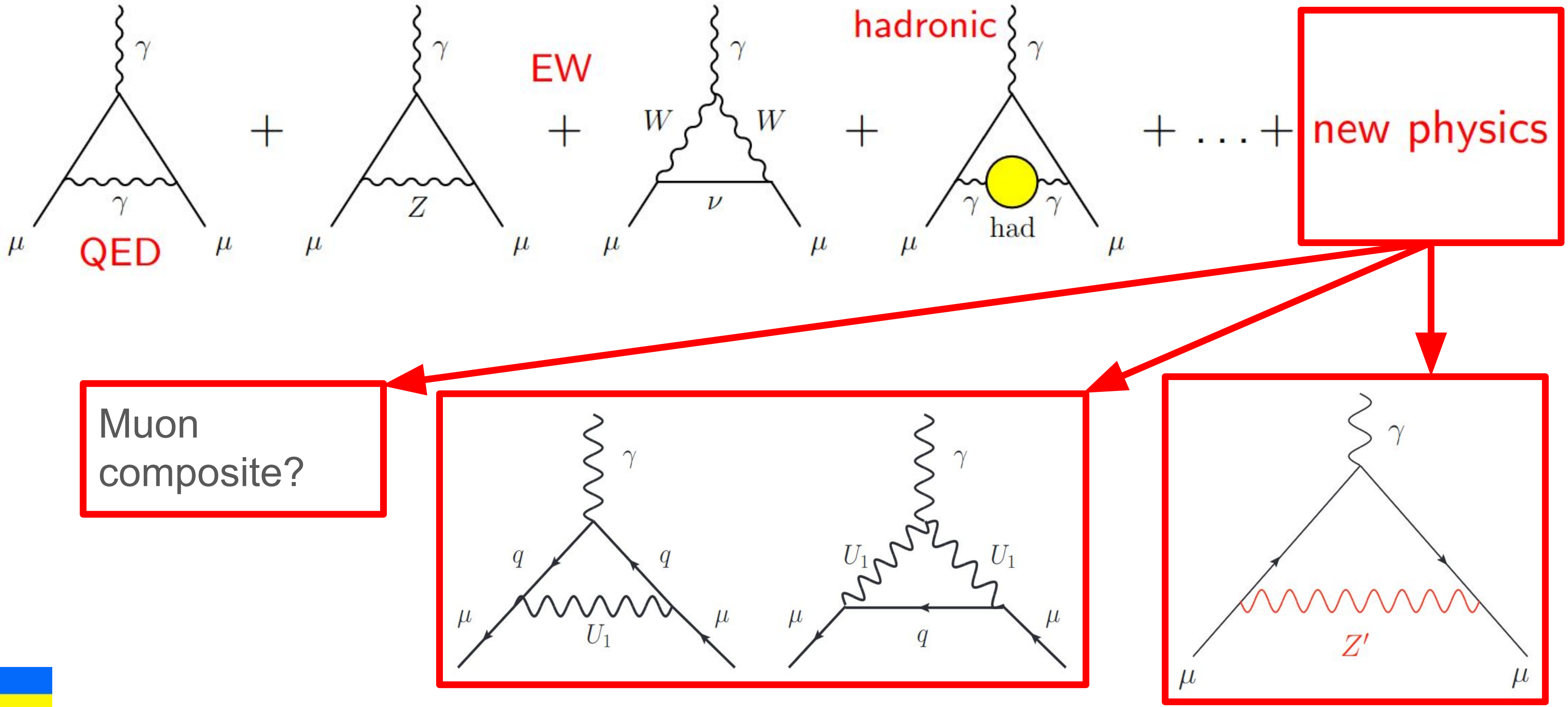
- Run 2 + Run 3
- Twice higher accuracy than the Run 1 result!
- 5.1σ from theory!
    - 1:3,000,000



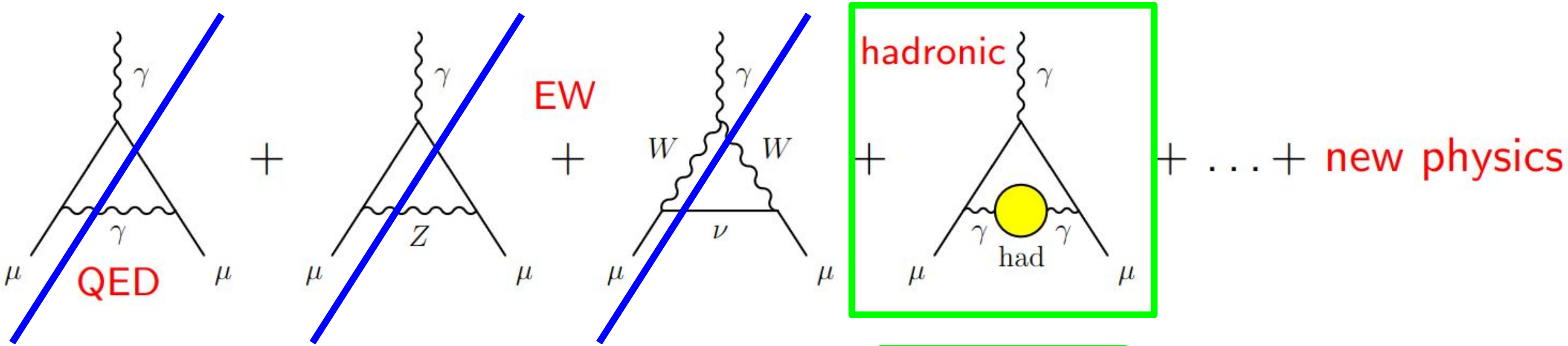
Theory prediction



Theory prediction



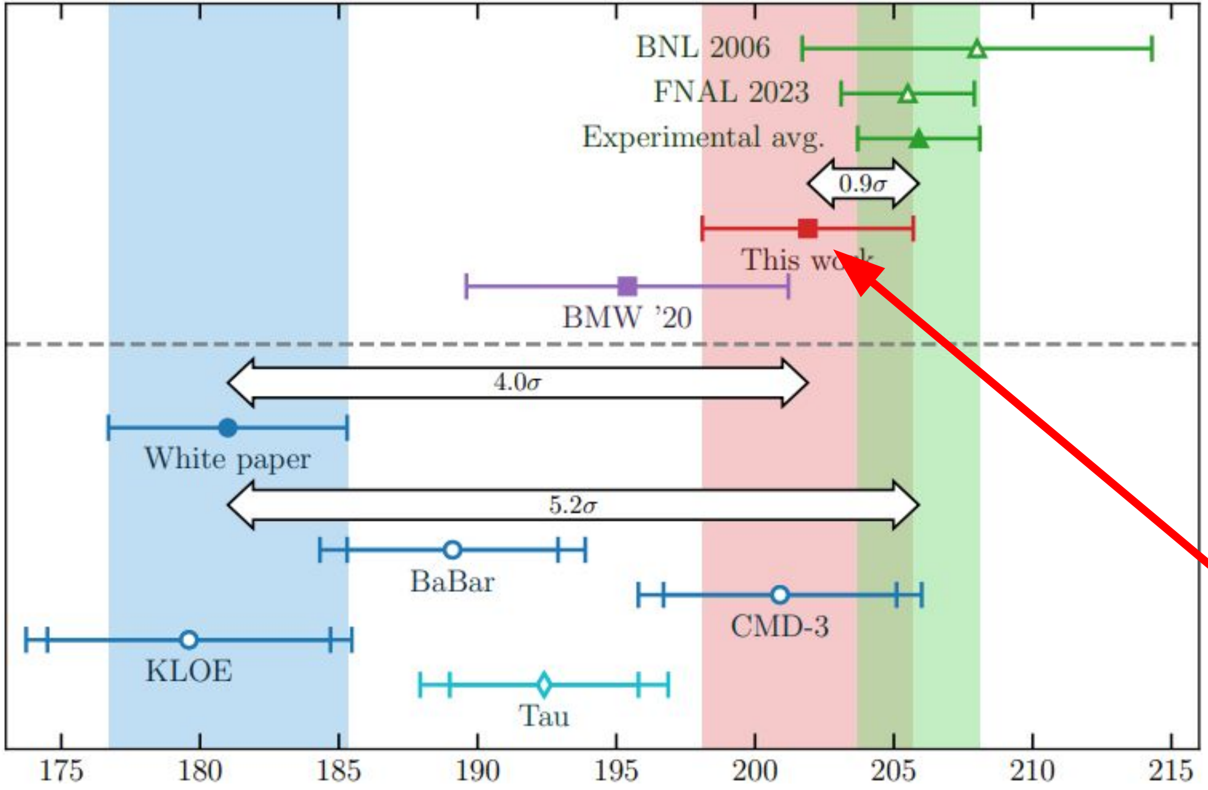
Theory prediction uncertainty

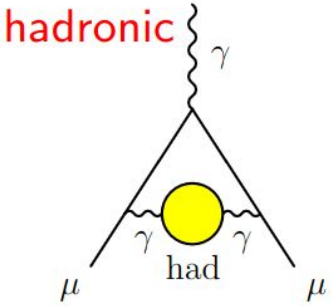


Negligible unc. on a_μ

Forms the unc. on a_μ

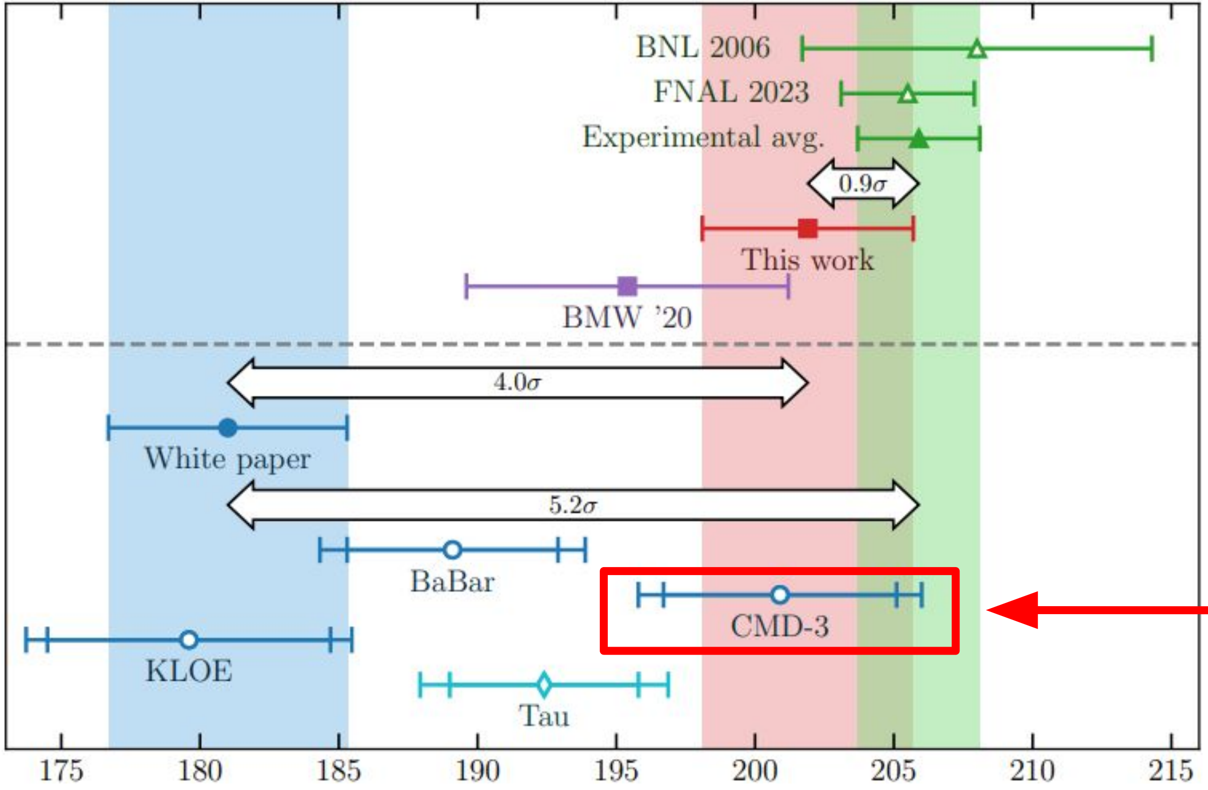
Lattice QCD…

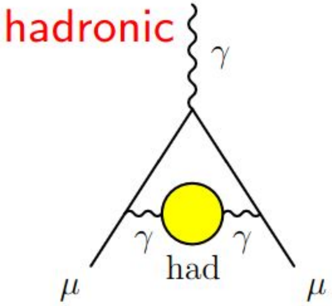




Presented at ICHEP 2024 in Prague!

Lattice QCD…

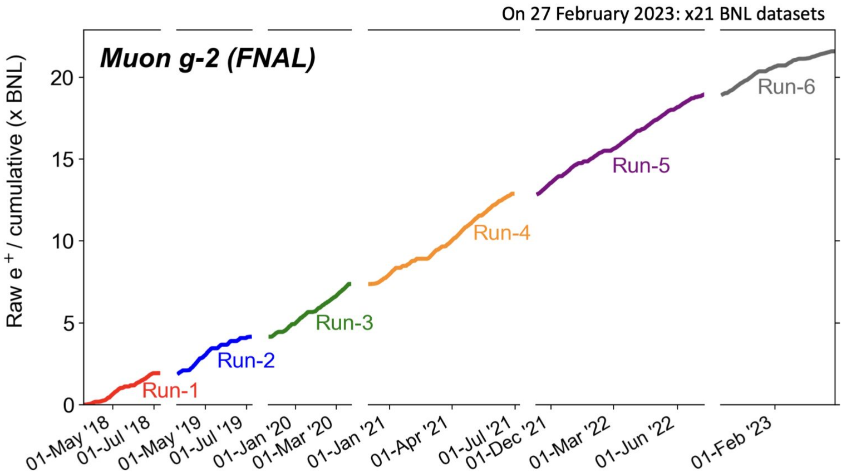




Main contribution to σ (e^+e^- -> hadrons)

Muon g-2: to be continued…

- Muon g-2: stopped last year
- Final result to be announced in 2025: twice smaller uncertainty!



- New theory predictions in a couple of years!
- To be further continued in:
    - J-PARC, PSI, Fermilab

$$a_\tau$$

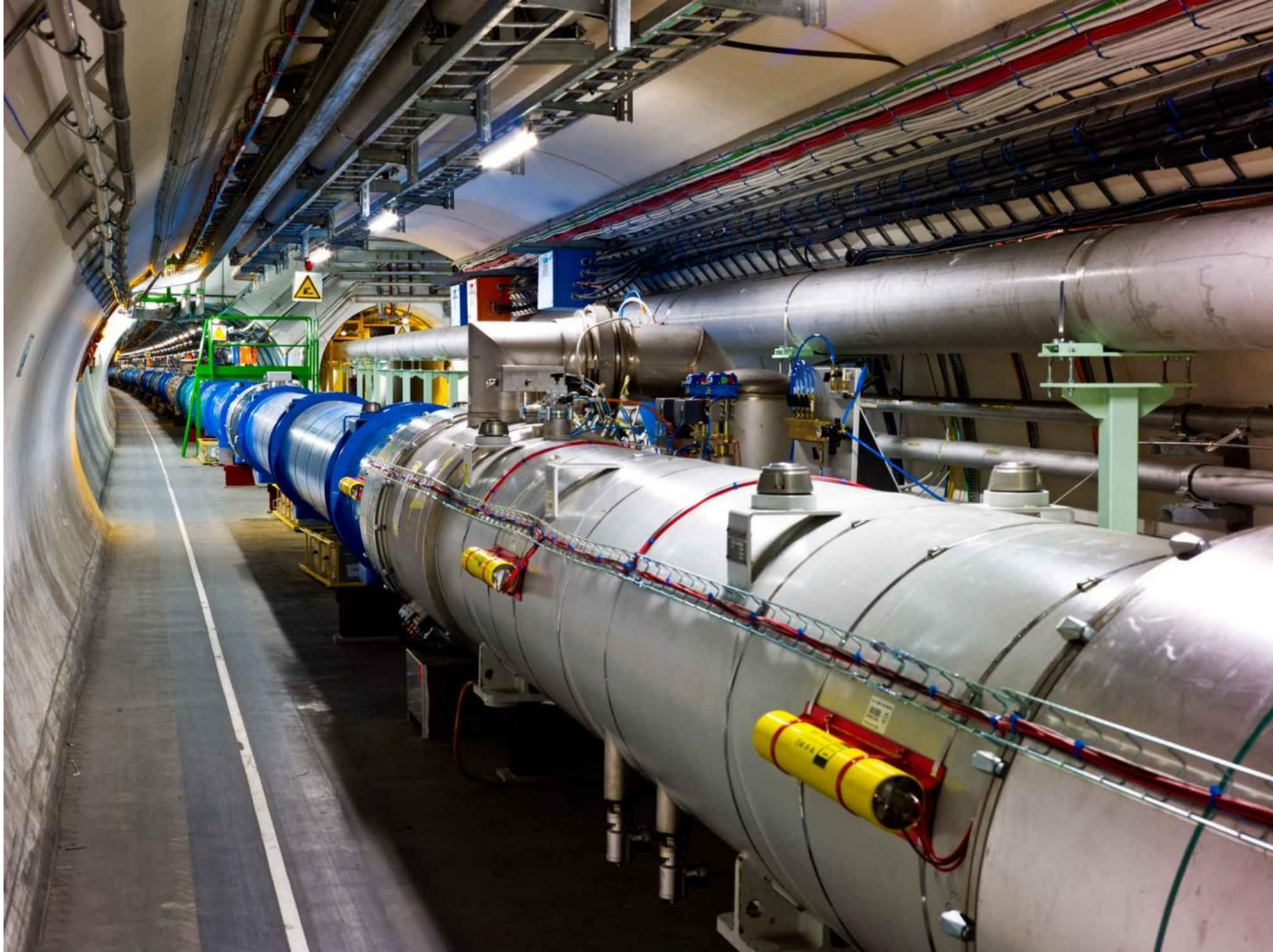
tau

anomalous magnetic moment

Tau g-2

Large Hadron Collider (LHC)

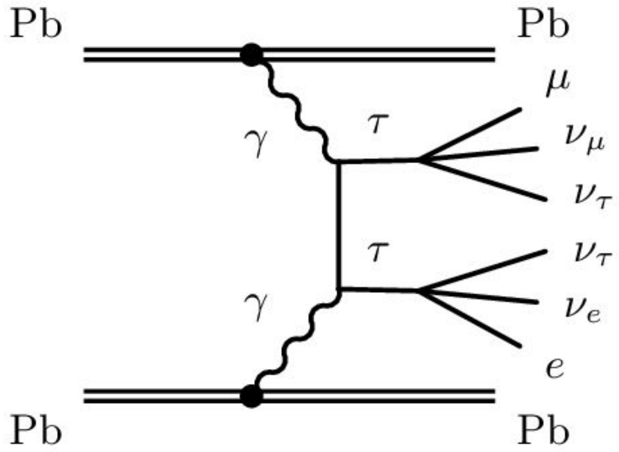
Tau g-2

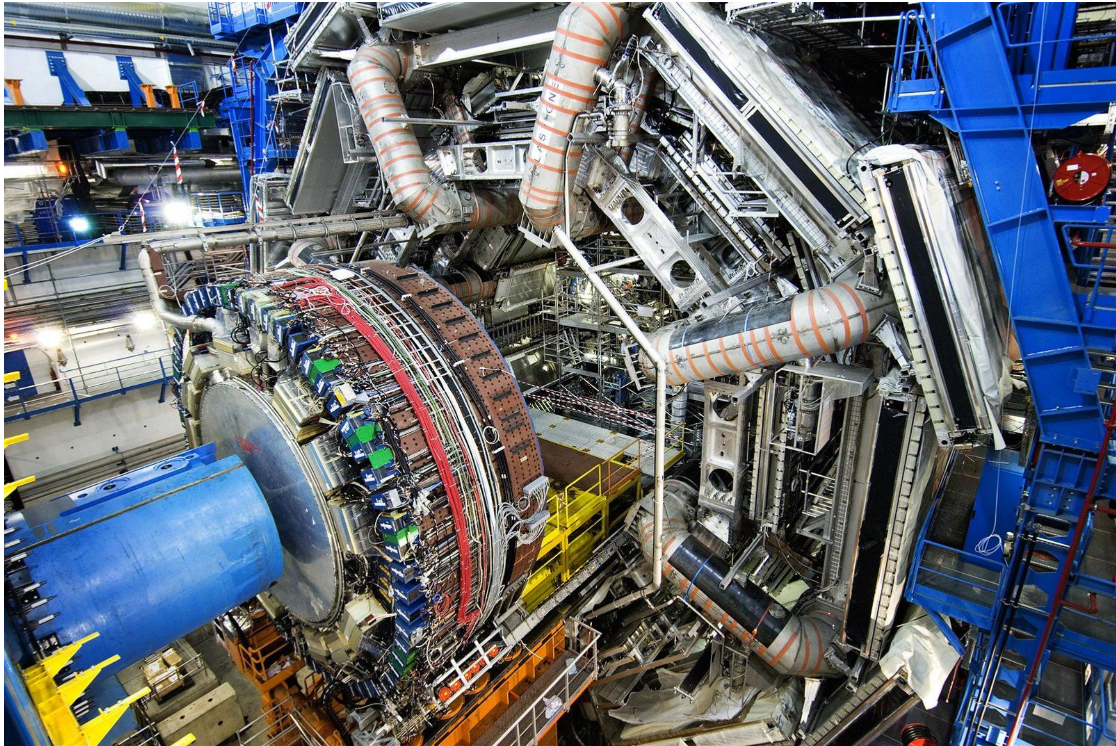


Large Hadron Collider (LHC)

ATLAS: $-0.057 < a_T < 0.024$ @95% CL

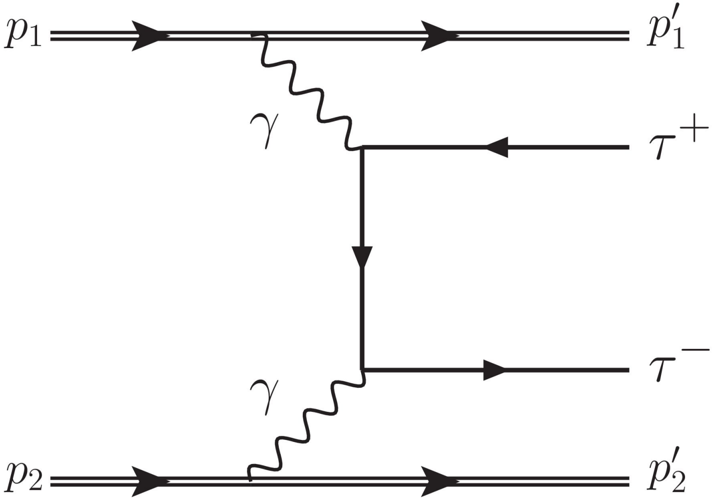
- Doesn't even say whether a_T is negative or positive…!!!

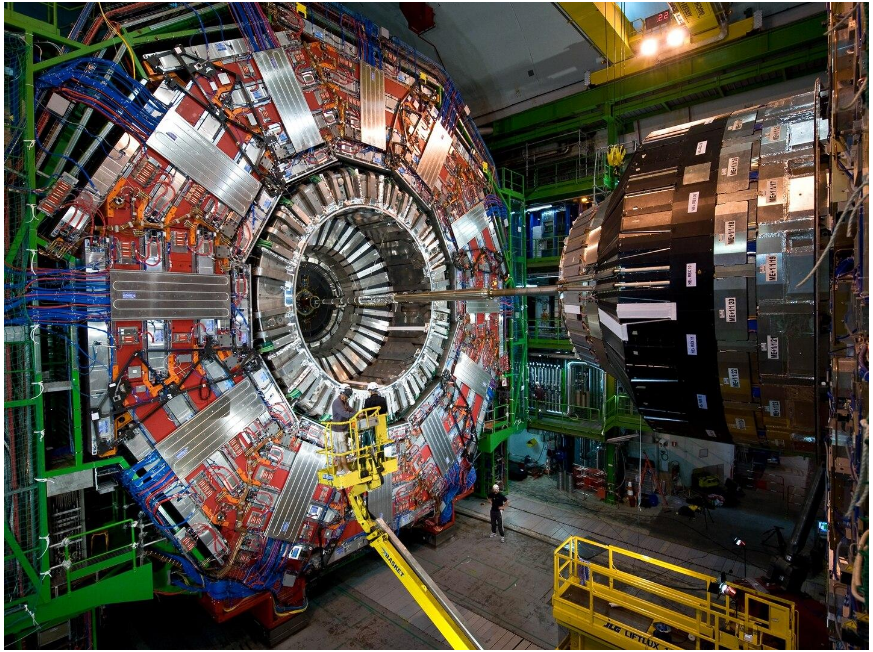




CMS: $-0.0042 < a_\tau < 0.0062$ @95% CL

- Order of magnitude more precise than ATLAS
- Still doesn't say the sign of a_τ....!!!





Conclusion

$$
\begin{aligned}
a_e &= 0.001\,159\,652\,180\,59 &\pm&\ 0.000\,000\,000\,000\,13 \\
a_\mu &= 0.001\,165\,920\,59 &\pm&\ 0.000\,000\,000\,22 \\
a_\tau &= 0.0009 &\pm&\ 0.0032
\end{aligned}
$$

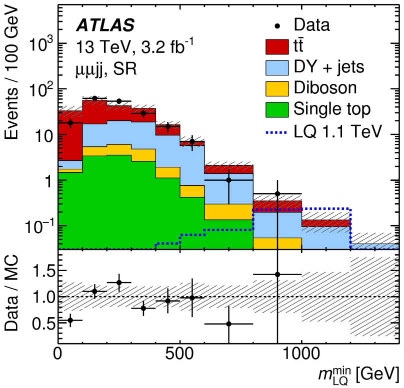
$$
\begin{aligned}
a_e &= 0.001\,159\,652\,180\,59 &\pm\ & 0.000\,000\,000\,000\,13 \\
a_\mu &= 0.001\,165\,920\,59 &\pm\ & 0.000\,000\,000\,22 \\
a_\tau &= 0.0009 &\pm\ & 0.0032
\end{aligned}
$$

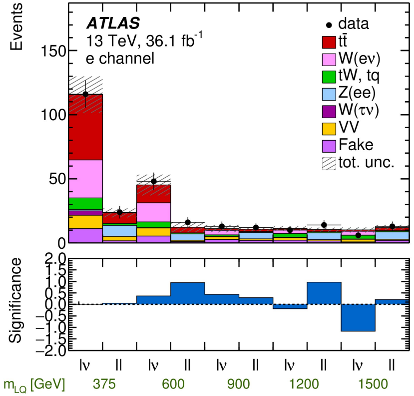
Contributions of new heavy particles

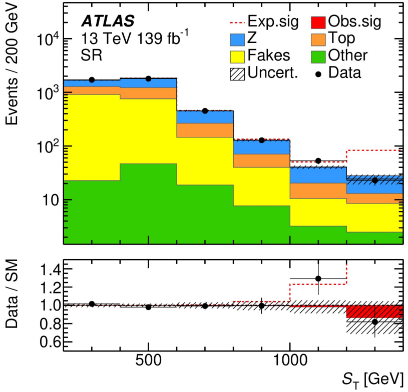
$$
\begin{aligned}
\left(\frac{m_e}{m_{\mathrm{NP}}}\right)^2 &= \mathrm{x} \\
\left(\frac{m_\mu}{m_{\mathrm{NP}}}\right)^2 &= 43000\ \mathrm{x} \\
\left(\frac{m_\tau}{m_{\mathrm{NP}}}\right)^2 &= 12000000\ \mathrm{x}
\end{aligned}
$$

New Physics: e.g. Leptoquarks…

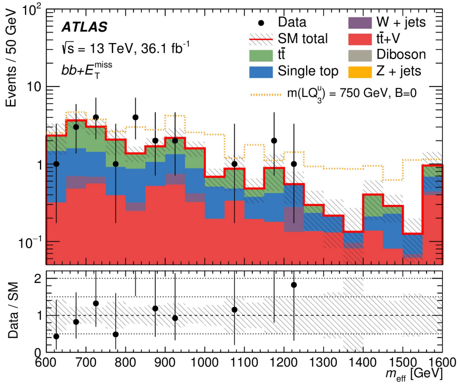
Analyzer:

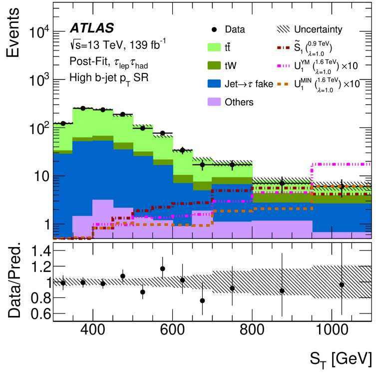






Editorial Board member:





BACKUP

Magnetický moment částice (animace, čas 34:00)

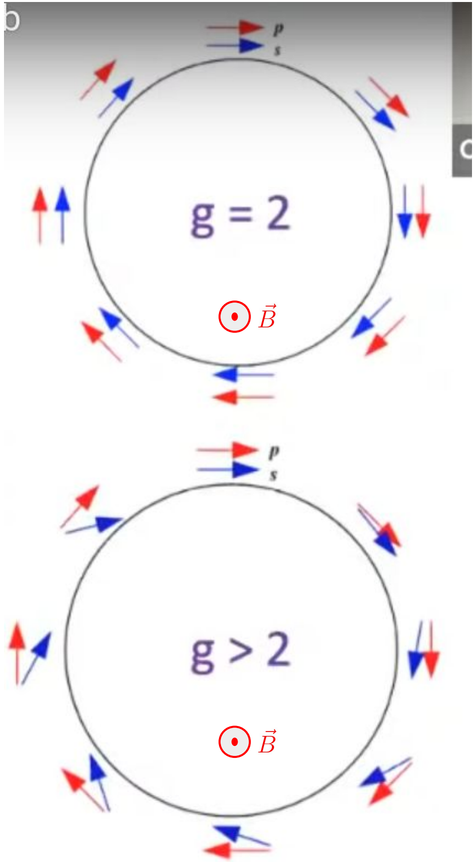
Muon g - 2 tricks (few out of many…)

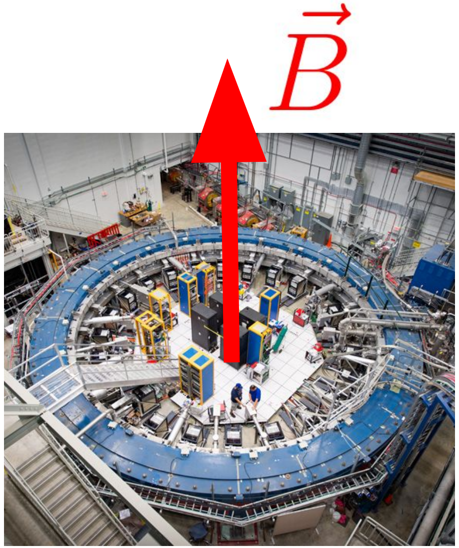
$$\vec{\omega}_a \equiv \vec{\omega}_s - \vec{\omega}_c = -a_\mu \frac{q\vec{B}}{m_\mu}$$

$$\vec{\omega}_a \equiv \vec{\omega}_s - \vec{\omega}_c = -\frac{q}{m_\mu}\left[a_\mu \vec{B} - a_\mu \left(\frac{\gamma}{\gamma+1}\right)(\vec{\beta}\cdot\vec{B})\vec{\beta} - \left(a_\mu - \frac{1}{\gamma^2-1}\right)\frac{\vec{\beta}\times\vec{E}}{c}\right]$$

Perpendicular $\vec{\beta}$ and $\vec{B}$

$$\gamma = \sqrt{1+\frac{1}{a_\mu}} \approx 29.3$$





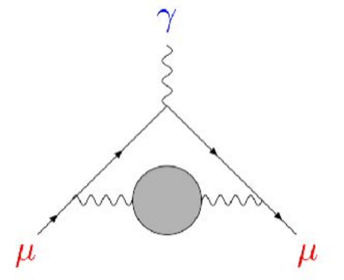
From talk by M. Fertl at PANIC 2021

Units: xxx 10^{-11}

QED ($\mathcal{O}(\alpha^5)$, > 12000 digrams): 116584718.931 ± 0.104

Electroweak: 153.6 ± 1.0

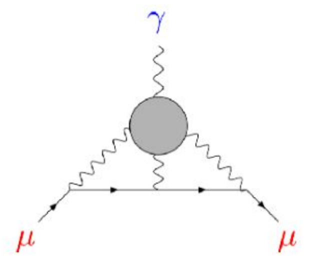
LO hadronic vacuum polarization: 6931 ± 40



NLO HVP: -98.3 ± 0.7

NNLO HVP: 12.4 ± 0.1

LO hadronic light-by-light scattering: 92 ± 19



NLO hLbL scattering: 2 ± 1

Uncertainty dominated by hadronic physics contributions!

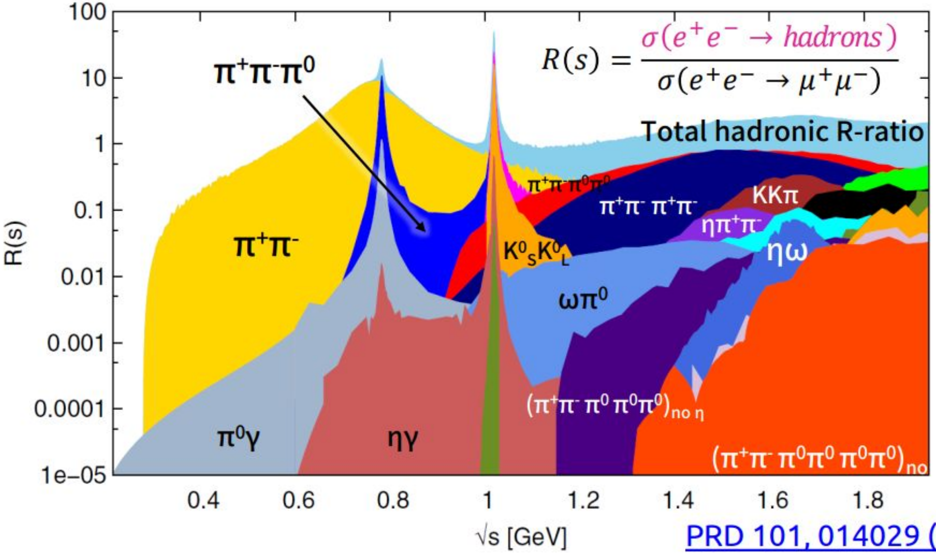
Taken from Yuki Sue’s ICHEP 2024 talk

Cross section measurements of exclusive channels

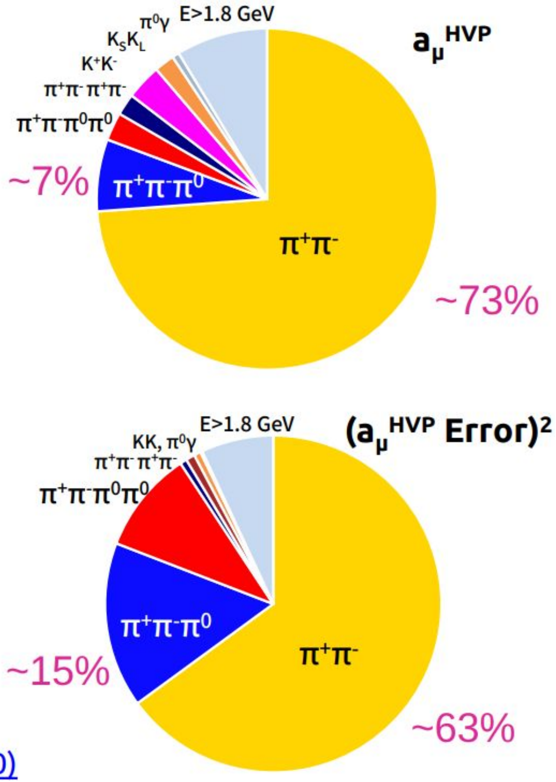
Leading order HVP contribution

$$a_\mu^{\mathrm{HVP,LO}} = \left(\frac{\alpha}{3\pi}\right)^2 \int_{m_\pi^2}^{\infty} \frac{\hat{K}(s)}{s^2} \frac{\sigma(e^+e^- \to hadrons)}{\sigma(e^+e^- \to \mu^+\mu^-)} ds$$

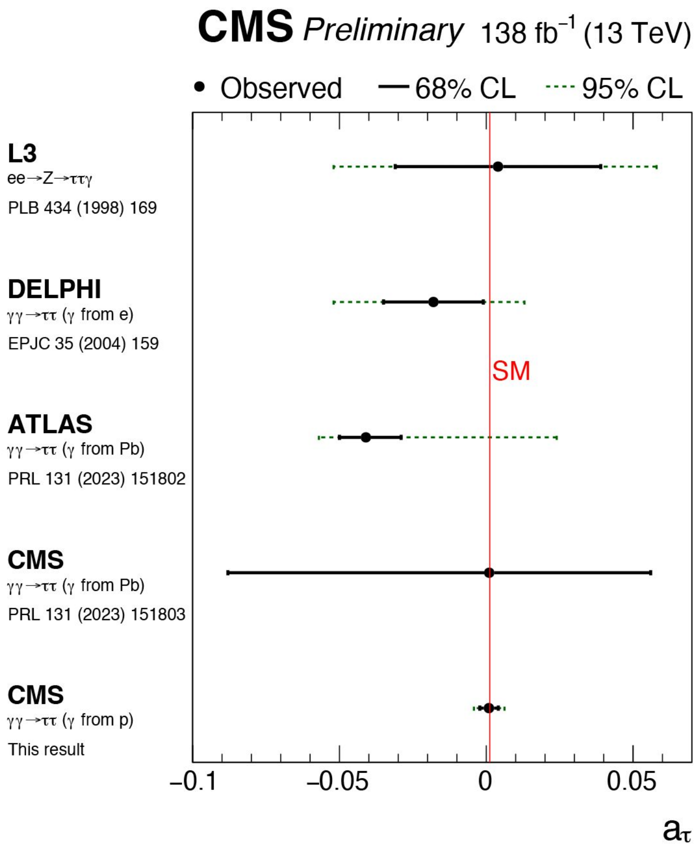
- Verify cross sections at Belle II
- As a first step, we begin with $e^+e^- \to \pi^+\pi^-\pi^0$ channel

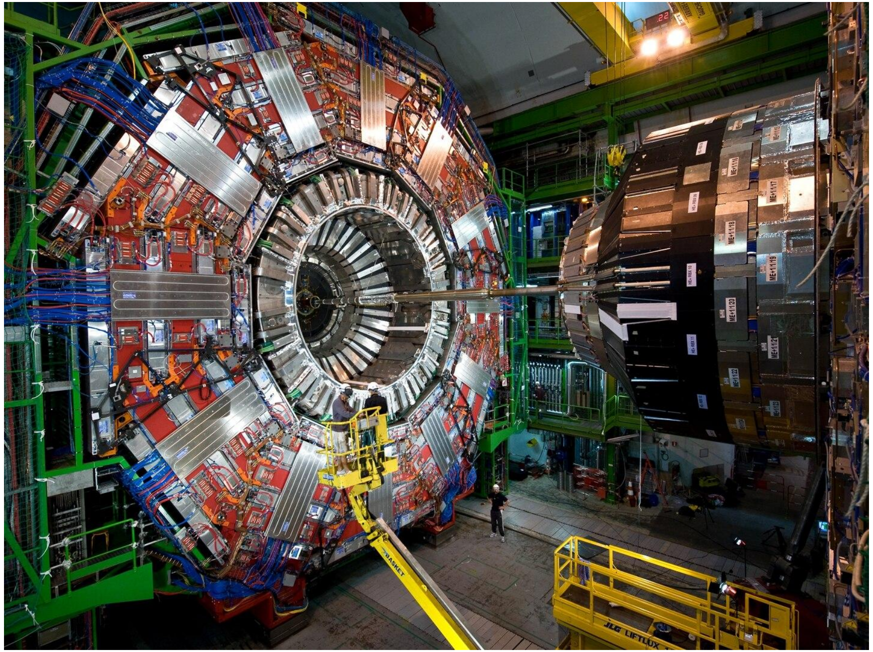


PRD 101, 014029 (2020)

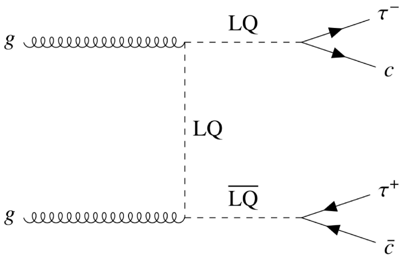


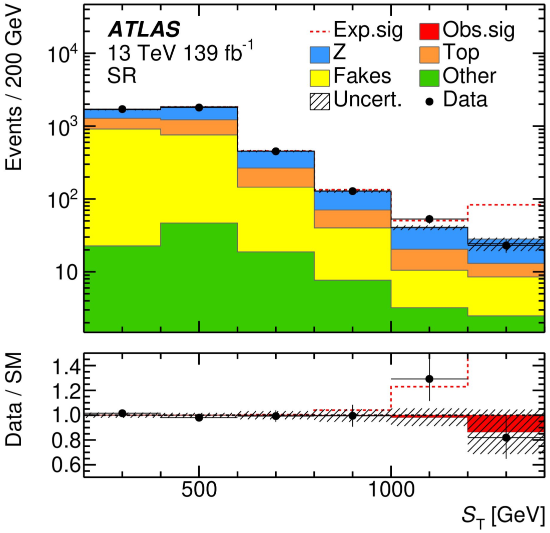
CMS: $-0.0022 < a_\tau < 0.0041$ @95% CL

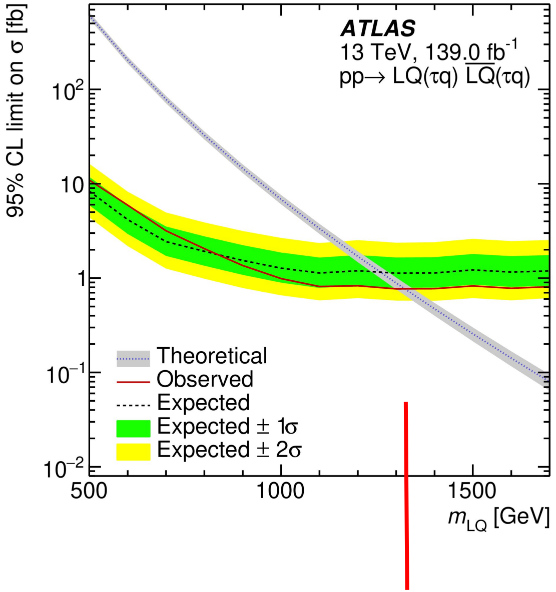




Přímá hledání nových částic/sil







…až příliš častý scénář…

Přesný test univerzality leptonů

$$R(D^*) = \frac{\mathcal{B}(B^0 \to D^{*-}\tau^+\nu_\tau)}{\mathcal{B}(B^0 \to D^{*-}\mu^+\nu_\mu)}$$

$$R(D) = \frac{\mathcal{B}(B^0 \to D\tau^+\nu_\tau)}{\mathcal{B}(B^0 \to D\mu^+\nu_\mu)}$$

Leptokvarky