A tour of holographic duality

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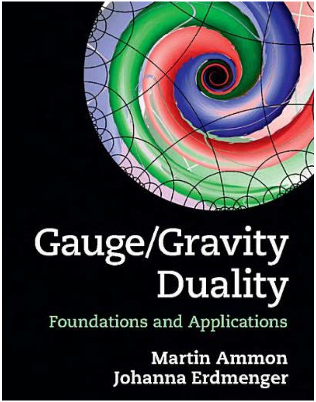
MINISTRY OF EDUCATION, YOUTH AND SPORTS

Disclaimer

- $\mathcal{O}(80,000)$ papers
- personal bias
- not qualified to talk about certain topics but will do it anyway



What is holographic duality?

Equivalence of two theories:

1. a **quantum theory of gravity**
2. a **quantum field theory without gravity** living on a lower-dimensional subspace or **holographic screen**

i.e. equivalence of Hilbert spaces, Hamiltonians, correlation functions, . . .
Also

- gauge-gravity correspondence
- AdS/CFT correspondence

OUTLINE

- History and basics
- Standard applications
- Recent developments

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History and basics

History I

- Black hole thermodynamics and **Bekenstein's bound** (1981)
  Bekenstein-Hawking black hole entropy

$$S_{BH} = \frac{\text{horizon area}}{4G}$$

system E, S, V

black hole

Second law:

$$S \leq S_{BH} = \frac{\text{area}}{4G}$$

- 't Hooft and Susskind (1993-94): **gravity is holographic**: its true degrees of freedom live on a holographic screen

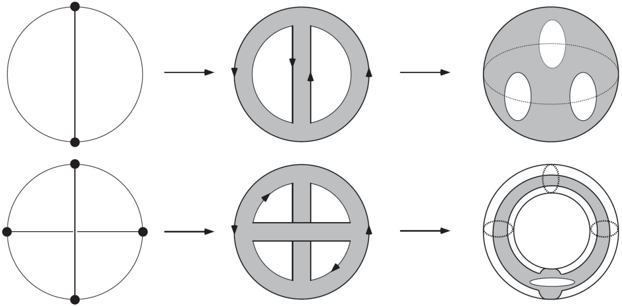
History II

’t Hoofts **large-N** (number of colours) expansion (1974)

$SU(N)$ Yang-Mills theory in the limit

$$N \to \infty, \qquad \lambda \equiv g_{YM}^2 N \ll 1 \text{ fixed}$$

looks like a string theory (with $G_N \sim \frac{1}{N^2}$)



$\sim N^2$

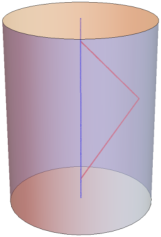
$\sim N^0$

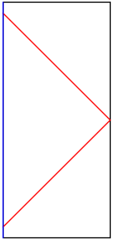
ANTI-DE SITTER SPACETIME

AdS_{d+1} is **maximally symmetric** solution to Einsteins equations with **negative cosmological constant**

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = 0, \qquad \Lambda = -\frac{d(d-1)}{2R_{AdS}^2}$$

$$\begin{aligned} ds^2_{d+1} &= R_{AdS}^2 \left(-\cosh^2 \rho dt^2 + d\rho^2 + \sinh^2 \rho d\Omega^2_{d-1} \right) \\ &= \frac{R_{AdS}^2}{\cos^2 \theta} \left(-dt^2 + d\theta^2 + \sinh^2 \theta d\Omega^2_{d-1} \right), \qquad \tan\theta = \sinh\rho \end{aligned}$$





CONFORMAL SYMMETRY

$$ds^2_{d+1} = R^2_{AdS} \frac{dz^2 + \eta_{\mu\nu} dx^\mu dx^\nu}{z^2}$$

Isometries:

- d-dimensional Poincaré symmetry (i.e. Lorentz + translations) on x^μ
- dilatations D

$$x^\mu \to \lambda x^\mu, \qquad z \to \lambda z$$

- 'inversion'

$$x^\mu \to \frac{x^\mu}{\eta_{\rho\sigma} x^\rho x^\sigma + z^2}, \qquad z \to \frac{z}{\eta_{\rho\sigma} x^\rho x^\sigma + z^2}$$

Isomorphic to **conformal group in d dimensions**
Act as standard conformal transformations on boundary $z = 0$

History III: AdS_3 and black hole microstates

- Brown-Henneaux (1986): asymptotic symmetry of 3D AdS gravity is infinite-dimensional symmetry algebra of 2D CFT:

$$[L_m, L_n] = (m-n)L_{m+n} + \frac{c}{12}m(m^2-1)\delta_{m,-n}, \qquad [\bar{L}_m, \bar{L}_n] = (m-n)\bar{L}_{m+n} + \frac{c}{12}m(m^2-1)\delta_{m,-n}$$

with $m, n \in \mathbb{Z}$ and

$$c = \frac{3R_{AdS}}{2G_N}$$

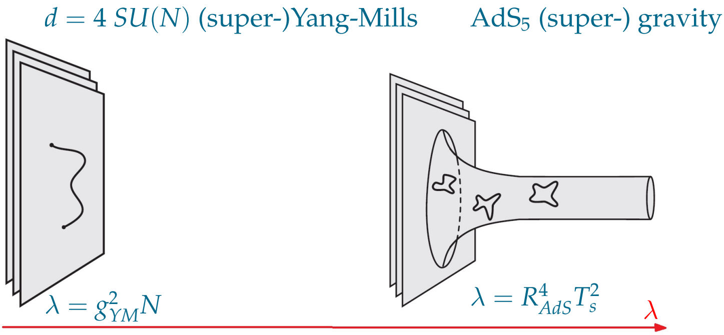
- Strominger-Vafa (1995): 5D black hole in string theory with 3 charges Δ, Q_1, Q_5 and entropy

$$S_{BH} = 2\pi\sqrt{\Delta Q_1 Q_5}$$

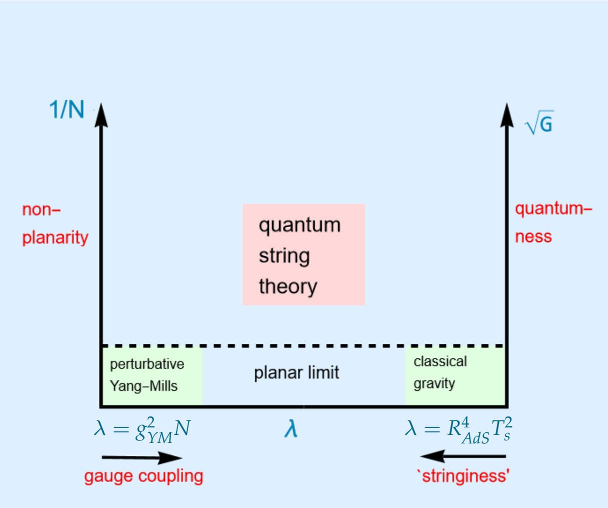
  - contains AdS_3 factor
  - specific 2D CFT $(Sym^{Q_1Q_5}(T^4))$ with $c = 6Q_1Q_5$
  - S_{BH} from Cardy's formula for CFT state degeneracy

$$\log d(\Delta) = 2\pi\sqrt{\frac{\Delta c}{6}}$$

N solitonic objects (D3-branes) in string theory, for $N \to \infty$
in different regions of parameter space



AdS/CFT parameter space



Equality of generating functions

$$\langle e^{-\int d^d x J(x)\mathcal{O}(x)}\rangle_{CFT} = Z_{grav}\Big|_{\Phi_{bdy}=J}$$

The AdS/CFT dictionary

Equality of **generating functions**

$$\int [DX] e^{-S_{CFT} - \int d^d x J(x) \mathcal{O}(x)} = \int [D\Phi]|_{\Phi_{bdy}=J}\, e^{-S_{AdS}}$$

The AdS/CFT dictionary

Equality of **generating functions**

$$\int [DX] e^{-S_{CFT} - \int d^d x J(x)\mathcal{O}(x)} = \int [D\Phi]|_{\Phi_{bdy}=J}\, e^{-S_{AdS}}$$

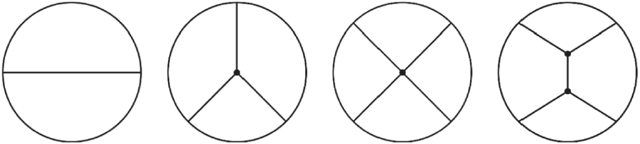
need to know **holographic dictionary**

bulk AdS field Φ	CFT operator $\mathcal{O}$	D-eigenvalue
metric $g_{\mu\nu}$	stress tensor T_{ij}	$\Delta = d$
gauge field A_μ	symmetry current J^i	$\Delta = d - 1$
scalar field mass m	scalar operator	$\Delta = \frac{d}{2} + \sqrt{\frac{d^2}{4} + m^2 R_{AdS}^2}$
spinor mass m	spinor operator	$\Delta = mR_{AdS} + \frac{d}{2}$

Saddle point approximation

$$\langle e^{-\int d^d x J(x)\mathcal{O}(x)}\rangle_{CFT} \sim e^{-S_{grav}}\Big|_{\Phi_{bdy}=J}$$

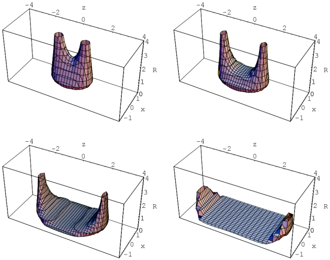
Witten diagrams for CFT correlators



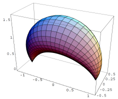
(a) 2-point (b) 3-point (c) 4-point, 1 vertex (d) 4-point, 2 vertices

STRONG-WEAK DUALITY IS HARD! (A PERSONAL NOTE)

D3-brane probe solutions in AdS_5



Drukker, Fiol: nonperturbative contribution to Euclidean circular Wilson loop



EXAMPLES

Top-down: known holographically dual theories

- String-theory based:
  - strings on $AdS_5 \times S^5 \leftrightarrow \mathcal{N} = 4\ SU(N)$ super Yang-Mills
  - M-theory on $AdS_4 \times S^7 \leftrightarrow$ ABJM theory
  - tensionless strings on $AdS_3 \times S^3 \times T^4 \leftrightarrow$ 2D CFT $Sym^N(T^4)$
- Higher spin holography
  - AdS_4 Vasiliev theory $\leftrightarrow O(N)$ vector model
  - AdS_3 Vasiliev theory $\leftrightarrow W_N$ minimal models
  - dS_4 Vasiliev theory $\leftrightarrow Sp(N)$ vector model

Examples

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⚠ Many applications are bottom-up!

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non-supersymmetric

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proven

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not straightforward limit of string theory

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not restricted to AdS

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Standard applications

Hawking-Page and deconfinement

Euclidean continuation: time translation → Boltzmann weight factor of canonical ensemble

$$t \to i\tau \qquad e^{iHt} \to e^{-\tau H}$$

Yang-Mills **free energy** on a sphere S^{d-1} = **Euclidean gravity action**

$$\beta F[S^{d-1}] = S^E_{grav}\left[\text{boundary} = S^{d-1} \times S^1_\beta\right]$$

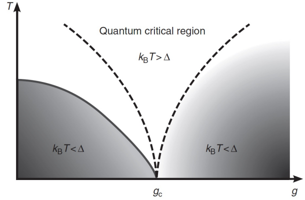
Two contributing solutions (Hawking-Page 1979):

- Global AdS: $F = \mathcal{O}(N^0)$: dominates for $T < T_{HP}$
- AdS-Schwarzschild: $F = \mathcal{O}(N^2)$: dominates for $T > T_{HP}$

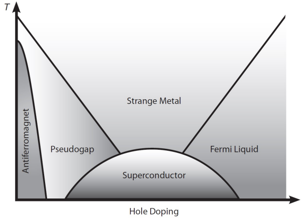
First order phase transition at $T_{HP} = \frac{d-1}{2\pi R_{AdS}}$. From gauge theory p.o.v. it is a type of **confinement-deconfinement** transition

AdS / condensed matter

Condensed matter applications: holographic 'understanding' of quantum phase transitions



Example: high-T_c superconductivity



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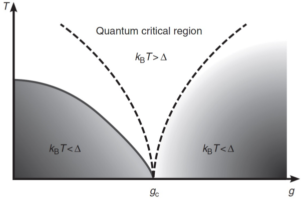

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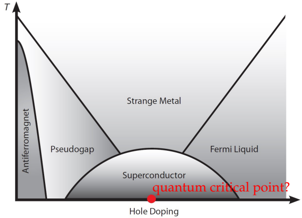


Holographic superconductors

- Related to **violation of 'no-hair theorem'** in AdS
- Setting: $U(1)$ **charged** (Reissner-Nordstrom) **black hole** in AdS
- Asymptotic value of gauge field A_t = **chemical potential** μ

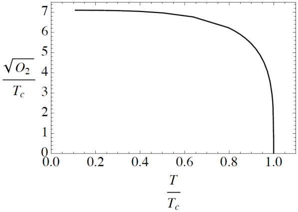
$$\int A_t J^t = A_t \int J^t = \mu\rho$$

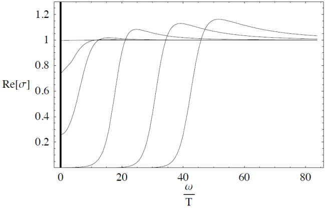
- Charged scalar field

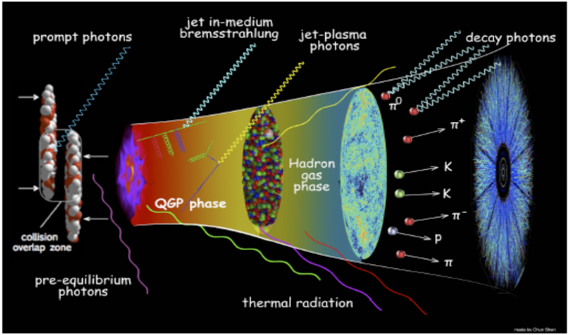
$$\begin{aligned} \mathcal{L} &= (\partial_\mu - A_\mu)\Phi(\partial^\mu - A^\mu)\bar{\Phi} + m^2|\Phi|^2 \\ &= g^{zz}\partial_z\Phi\partial_z\bar{\Phi} + m^2_{eff}|\Phi|^2 + \ldots \\ m^2_{eff} &= m^2 + \mu^2 g^{tt} \end{aligned}$$

  can become 'tachyonic'
- Condenses to black hole with scalar 'hair'

HOLOGRAPHIC SUPERCONDUCTORS

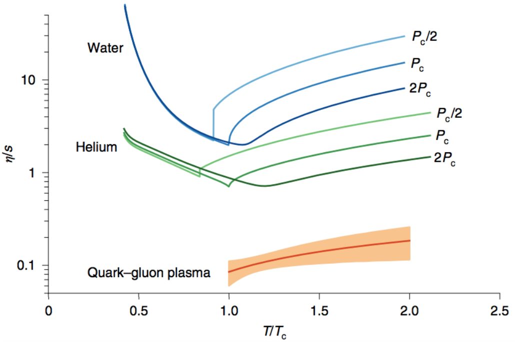






QGP = The hottest, densest, smallest, most perfect liquid

$$\frac{\text{shear viscosity}}{\text{entropydensity}} \equiv \frac{\eta}{s} \sim 0.05 - 0.2$$



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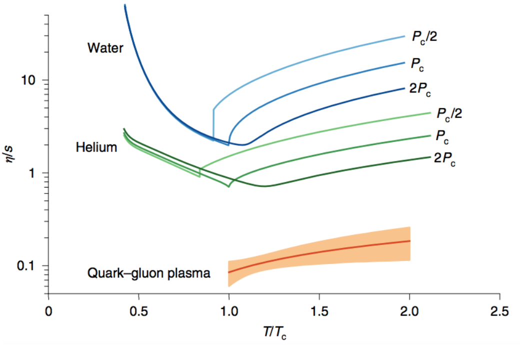

in holographic fluid

$$\frac{\eta}{s} = \frac{1}{4\pi} \sim 0.08$$

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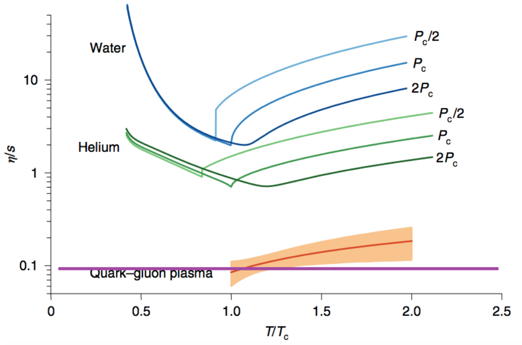
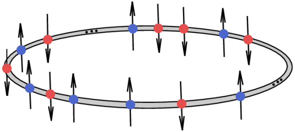


In holographic fluid

$$\frac{\eta}{s} = \frac{1}{4\pi} \sim 0.08$$

INTEGRABILITY

Spectrum = eigenvalues of dilatation operator D
In planar limit: mapped to Hamiltonian of **integrable spin-chain**



Computable **exactly** for all λ. Planar limit of $\mathcal{N} = 4$ Yang-Mills is integrable

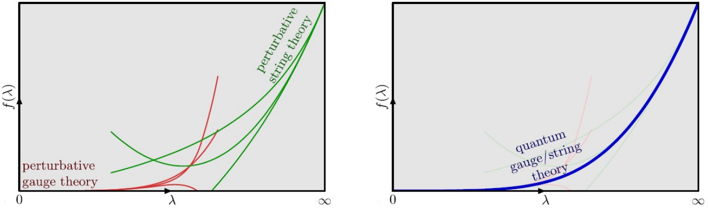
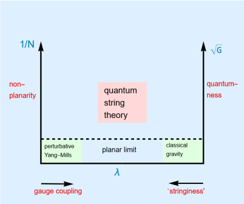
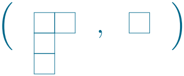
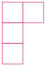
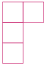
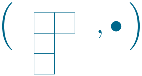


Figure 3: Weak coupling (3, 5, 7 loops) and strong coupling (0, 1, 2 loops) expansions (left) and numerically exact evaluation (right) of some interpolating function $f(\lambda)$.

Simple holography: higher spins in AdS$_3$



- Spin $2, 3 \dots N$ in AdS$_3$ $\leftrightarrow$ CFT$_2$ with extended W_N symmetry
- CFT representations labeled by two Young tableaux, eg. 
- In higher spin gravity, found 'conical' solutions labeled by single Young tableau, e.g. 
- Charges + symmetries, argued that  $\leftrightarrow$ 
- remaining CFT reps. from exciting matter in AdS$_3$

Local bulk geometry

- how does it emerge from CFT?
- what CFT properties does it encode?

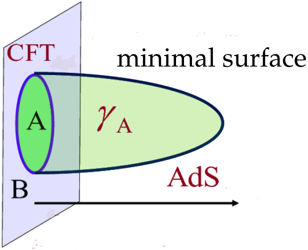
Answers related to quantum information theory.

- **entanglement entropy** S_A measures how strongly subsystem A is entangled with rest

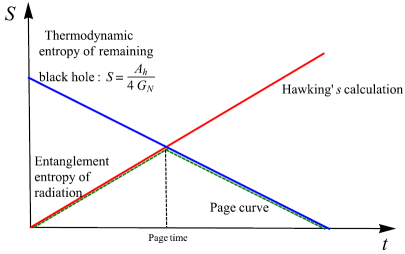
$$S_A = -\mathrm{tr}_A \rho_A \log \rho_A , \qquad \rho_A = \mathrm{tr}_{A^c}(|\Psi\rangle\langle\Psi|)$$

- Ryu-Takayanagi: **holographic formula** for entanglement

$$S_A = \frac{\mathrm{Area}(\gamma_A)}{4G_N}$$



- implies **Einstein equations** in bulk
- **Information paradox**: Page curve for $S_{\text{Hawking radiation}}$



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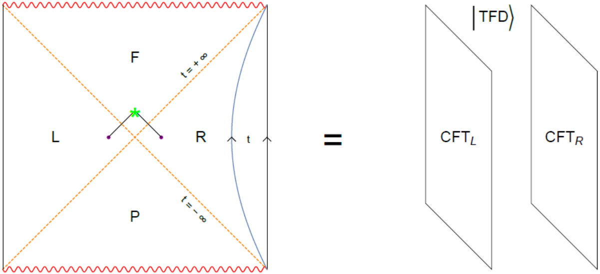
Recent developments

BEHIND THE HORIZON WITH VON NEUMANN



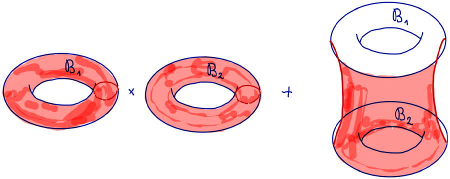
- Eternal black hole = highly entangled state in product of 2 CFTs
- In large N limit, algebras of obervables are 'type III' in von Neumann's classification
- allow time evolution beyond the horizon of R - involves operators from L!

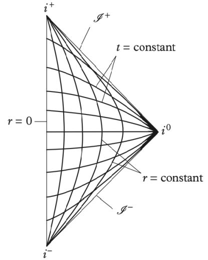
Ensemble holography and wormholes

- **Ensembles** reflect incomplete knowledge of details of the system
- In holography: when **gravity theory is UV-incomplete** theory. Gravity path integral computes an **average over ensemble of CFTs**
- Examples:
  - Jackiw-Teitleboim AdS_2 gravity $\leftrightarrow$ average over QM Hamiltonians
  - pure AdS_3 gravity?
- **non-factorization on disconnected boundaries**

$$Z[\mathcal{B}_1, \mathcal{B}_2] = \sum_{\{CFTs\}_I} \rho_I \, Z_I[\mathcal{B}_1] Z_I[\mathcal{B}_2] \neq Z[\mathcal{B}_1] Z[\mathcal{B}_2]$$

- comes from **wormhole geometries** connecting the boundaries



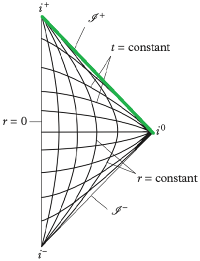


Asymptotic symmetry = BMS algebra: includes superstranslations $\mathcal{T}_{\alpha,\beta}$.

$$[L_m, L_n] = L_{m+n} \qquad [\bar{L}_m, \bar{L}_n] = \bar{L}_{m+n} \qquad m, n \in \{-1, 0, 1\}$$

$$[L_m \mathcal{T}_{\alpha,\beta}] = \left(\frac{m}{2} - \alpha\right) \mathcal{T}_{\alpha+m,\beta} \qquad [\bar{L}_m \mathcal{T}_{\alpha,\beta}] = \left(\frac{m}{2} - \beta\right) \mathcal{T}_{\alpha,\beta+m} \qquad m, n \in \mathbb{Z}/2$$

TWO ROADS TO FLAT HOLOGRAPHY



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Holographic screen at **future null infinity** $\mathcal{I}^+$ $\Leftrightarrow$ **conformal Carrollian field theory**

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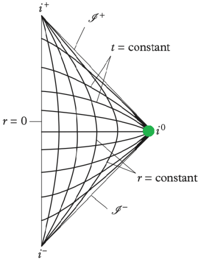


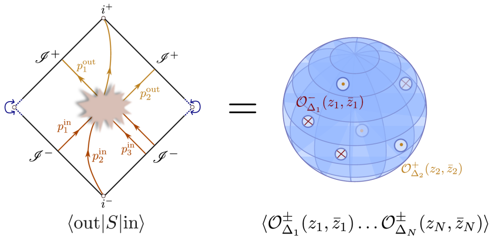
Asymptotic symmetry = extended BMS algebra

$$[L_m, L_n] = L_{m+n} \qquad [\bar{L}_m, \bar{L}_n] = \bar{L}_{m+n} \qquad m, n \in \mathbb{Z}$$

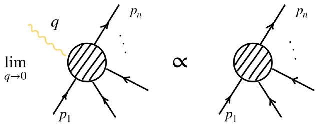
$$[L_m \mathcal{T}_{\alpha,\beta}] = \left(\frac{m}{2} - \alpha\right) \mathcal{T}_{\alpha+m,\beta} \qquad [\bar{L}_m \mathcal{T}_{\alpha,\beta}] = \left(\frac{m}{2} - \beta\right) \mathcal{T}_{\alpha,\beta+m} \qquad m, n \in \mathbb{Z}/2$$

Holographic screen at **spatial infinity** i^0 $\Leftrightarrow$ **Celestial CFT**

Celestial holography and soft theorems

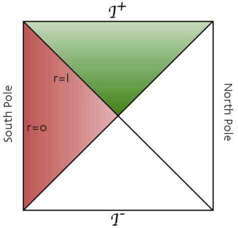


Weinberg's **soft theorems**



interpreted as **Ward identities** for extended BMS

DE SITTER AND WAVEFUNCTION OF THE UNIVERSE



- CFT lives at future infinity $\mathcal{I}^+$
- time is emergent!
- CFT computes wavefunction of the universe Ψ:

$$\langle e^{-\int d^d x J(x)\mathcal{O}(x)}\rangle_{CFT} = \Psi[t \to \infty, \Phi(x) = J(x)]$$

- $\rightarrow$ late-time dS correlation functions, e.g. near end of inflation

Thank you!