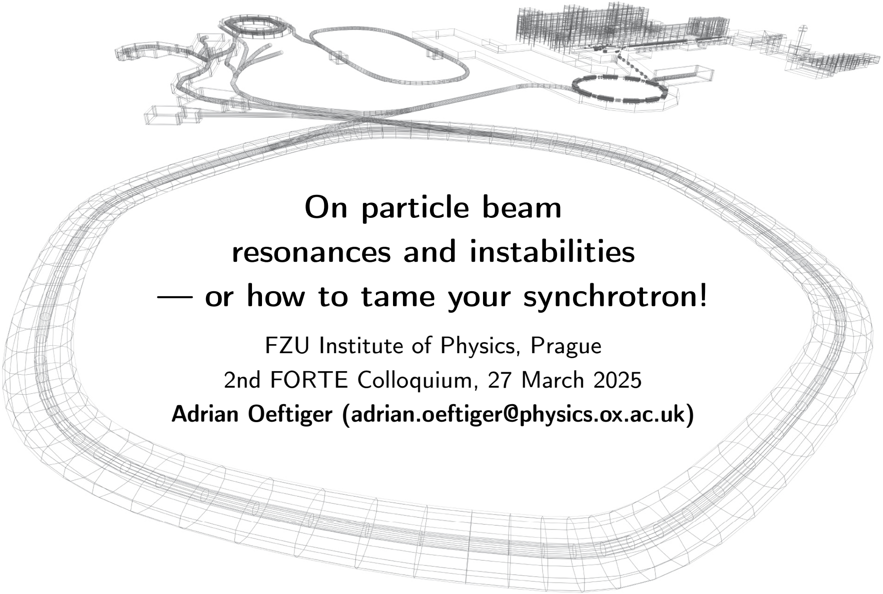

On particle beam resonances and instabilities — or how to tame your synchrotron!

FZU Institute of Physics, Prague

2nd FORTE Colloquium, 27 March 2025

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Abstract

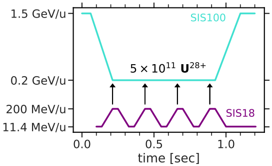
Space charge is a major performance limitation for hadron synchrotrons, e.g., at CERN, GSI, ISIS, SNS, CSNS, BNL, JPARC and FNAL. The interaction of the beam particles with the beam self-fields, which are typically nonlinear, leads to a betatron tune spread. This spread in the transverse particle oscillation frequencies increases with the bunch intensity, which eventually makes the bunch suffer from nearby betatron resonances. A maximum intensity, i.e., the space charge limit, is reached when these resonances excite the particle distribution to large enough amplitudes inducing beam loss.

This talk reviews the key resonance mechanisms identified over the recent years, demonstrating them with modern modelling tools used for the prediction of the space charge limit. We then discuss compensation methods to increase the space charge limit. As a highlight we cover a recently published approach with pulsed electron lenses, which is currently pushed forward at the dedicated test facility IOTA (FNAL) and GSI.

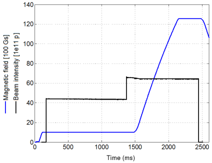
Context

Accumulating synchrotrons operating close to space charge limit:

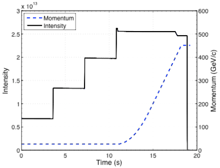
- **long duration**: model up to seconds of storage time (accumulation)
- **bunched beam**: large space charge tune footprints
- **complex dynamics** due to synchrotron motion
- **goal**: understand and alleviate detrimental impact of space charge induced crossing of betatron resonances ($\Rightarrow$ beam halo generation, $\Rightarrow$ beam loss, $\Rightarrow$ emittance growth)



(a) FAIR SIS18-SIS100 cycling



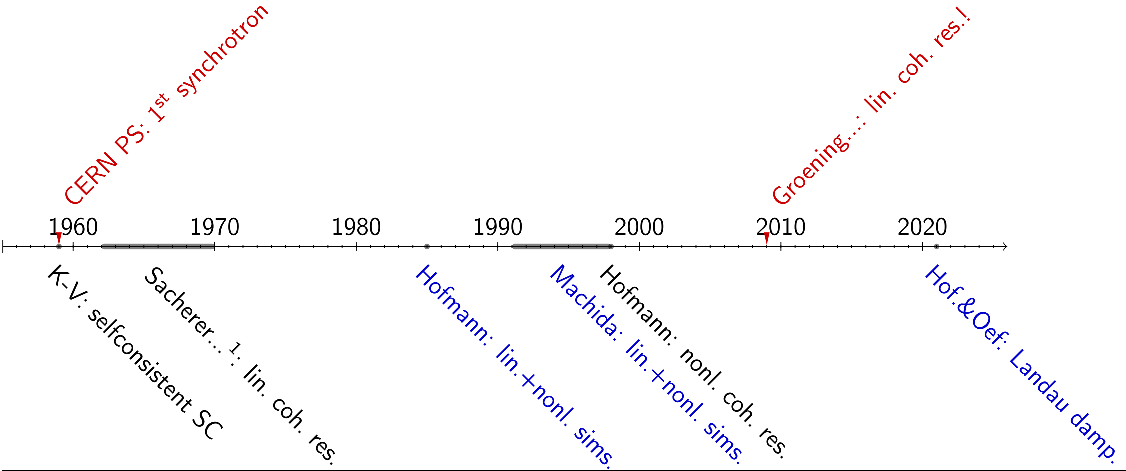
(b) CERN PS cycle (pre LS2)



(c) CERN SPS cycle (pre LS2)

Figure: Synchrotron examples in strong space charge regime

The History of the “Conflict”



[1]) 1962 Morin, 1963 Lapostolle, 1963 Smith, 1968 Sacherer, 1970 Gluckstern

The Situation ≈ 5 Years Ago...

Challenges for the design of accelerators (& upgrades) operating close to space charge limit:

1. **resonance type**: apparent contradiction between theory & operational experience
   - ⟶ theory: only coherent resonances are relevant
   - ⟶ operation: absence of coherent resonance signatures, "everything incoherent"
   - ⟶ also operation: pure incoherent picture in theory, $\Delta Q^{\mathsf{SC}} < 0.25$, predicts too low limit!
   - ⇝ how to determine the "forbidden" tunes?
   - ⟹ detailed numerical simulations required to predict space charge limit!

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2. "do it the right way": **self-consistent** numerical models
   - ⟶ computationally expensive (+ numerical intra-beam scattering, noise effects)
   - ⟹ not suitable for parameter scans (design!)

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   - ⇝ how

   ...

   ... so what now? ...

   - ⟹ detailed numerical simulations required to predict space charge limit!
2. "do it the right way": **self-consistent** numerical models
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Golden Thread

In this talk on (accumulating) synchrotrons I will argue as follows:

- coherent resonances play no role (up to now)
  $\Longrightarrow$ **Part I: Landau Damping**
- simulate incoherent resonances with fast, non-selfconsistent models
  $\Longrightarrow$ **Part II: Frozen Space Charge Models**
- identify the space charge limit with simulations
  $\Longrightarrow$ **Part III: The Space Charge Limit...**
- test space charge mitigation methods
  $\Longrightarrow$ **Part IV: ... & how to push it further!**

Setting the Scene...

Tune

- Particles are guided around accelerator ring with linear external focusing $K_{x,y}(s)$
  $\Longrightarrow$ Hill equation of motion:

$$x'' + K_x(s)x = 0$$

- *Tune* $Q_{x,y}$ describes number of particle oscillations per turn

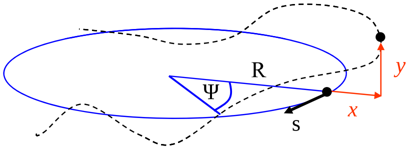


Figure: sample particle trajectory

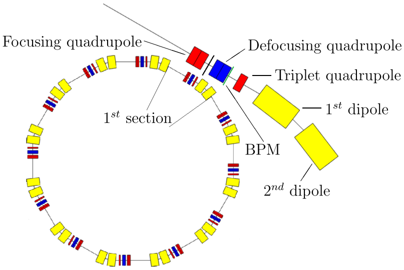
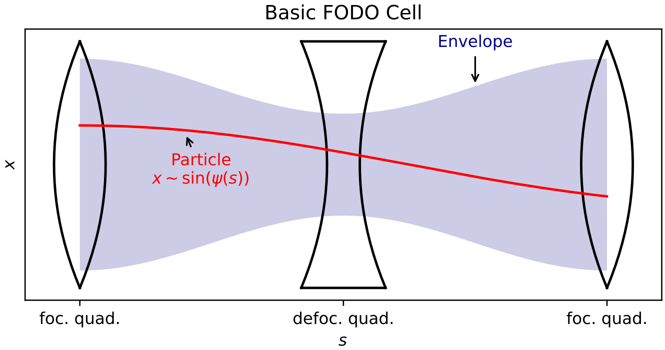


Figure: SIS18 synchrotron lattice

Basic Focusing Cell



- FODO = periodic structure of quadrupoles: focusing - defocusing - focusing
- phase advance per cell $k < 180°$ (unstable overfocusing) $\implies$ $Q_{x,y} = n_{\text{cells}} \cdot k_{x,y}$

Space Charge Potential

- Bunch of particles of distribution ρ features a space charge potential ϕ, determined by Poisson Equation

$$\nabla^2 \phi = -\frac{\rho}{\epsilon_0}$$

- Particles feel internal beam potential ϕ:

$$x'' + K_x(s)x = -\frac{q}{E_0 \beta_0^2 \gamma_0^3} \frac{d\phi}{dx}(s)$$

⟶ stronger at lower energies

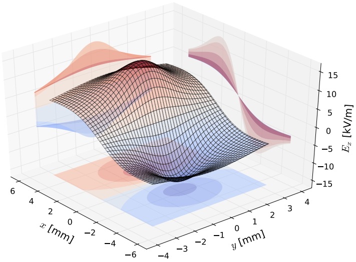


Figure: space charge field $E_x = -\frac{d\phi}{dx}$ of Gaussian bunch

Particle Distribution Functions ρ

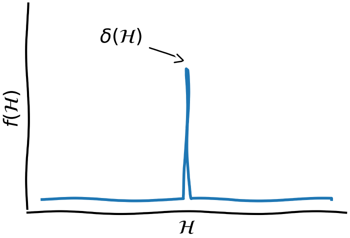


Figure: K-V distribution

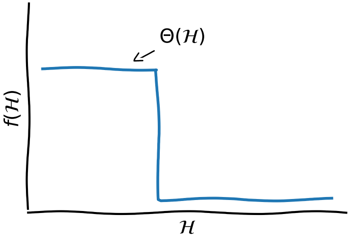


Figure: Waterbag distribution

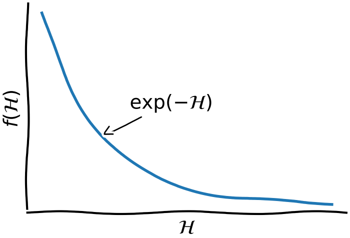


Figure: Thermal or Gaussian distribution

Transverse Profiles

Typically Gaussian distributed $\Longrightarrow$ nonlinear SC field:

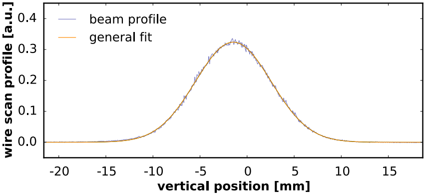


Figure: CERN PS

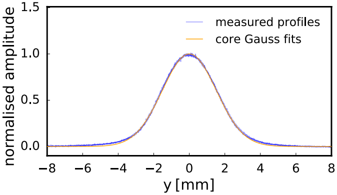


Figure: CERN SPS

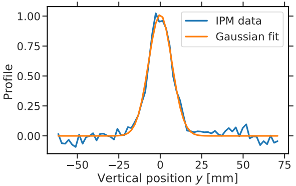


Figure: GSI SIS18

Space Charge Tune Spread

Maximum space charge tune shift in Gaussian bunch:

$$\Delta Q_y^{SC} = -\frac{r_c \lambda_{max}}{\beta_0^2 \gamma_0^3} \oint \frac{ds}{2\pi} \frac{\beta_y(s)}{\sigma_y(s)(\sigma_x(s)+\sigma_y(s))}$$

r_c : classical particle radius

λ_{max} : maximum line density

β_0 : speed in $[c]$

γ_0 : Lorentz factor

β_y : local beta-function

$\sigma_{x,y}$: local rms beam size

$\Longrightarrow$ bunch distribution may suffer from nearby betatron resonances

$\Longrightarrow$ **aim** of space charge studies: predict non-resonant working points to avoid detrimental amplitude growth!

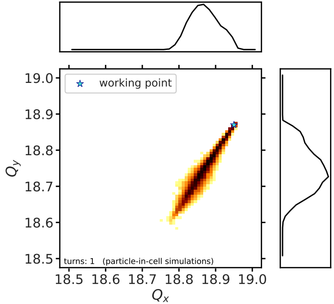
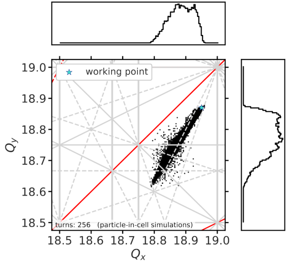


Figure: Tune footprint in FAIR SIS100, $\Delta Q_y^{SC} = -0.3$

Incoherent vs. Coherent Resonances

incoherent perspective



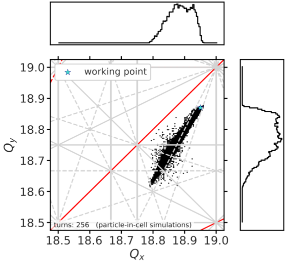
incoherent resonance condition:

$$mk_{xy} = h \cdot 360^\circ$$

$$mQ_{xy} = h$$

Incoherent vs. Coherent Resonances

incoherent perspective



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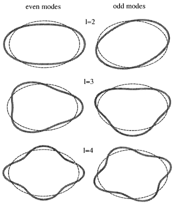
coherent perspective



coherent resonance condition:

$$m\underbrace{\left(k_0 - C_m \Delta k_{KV}\right)}_{\text{mode tune}} = h \cdot 360^\circ$$

$$m\overbrace{\left(Q_0 - C_m \Delta Q_{KV}\right)} = h$$

Incoherent vs. Coherent Resonances

incoherent perspective

coherent perspective

... the dilemma ...

- with coherent resonance condition, more intensity before resonance
  ⟶ higher space charge limit?

⟶ operational accelerators seem limited by space charge before this condition

⟶ no experimental evidence for nonlinear coherent resonances yet

⟶ BUT calculated incoherent tune footprints can overlap with e.g. integer resonance without apparent issue! $\Longrightarrow$ incoherent condition too restrictive!

$$mk_{xy} = h \cdot 360^\circ$$

$$mQ_{xy} = h$$

mode tune

$$m\big(Q_0 - C_m \Delta Q_{KV}\big) = h$$

Incoherent vs. Coherent Resonance

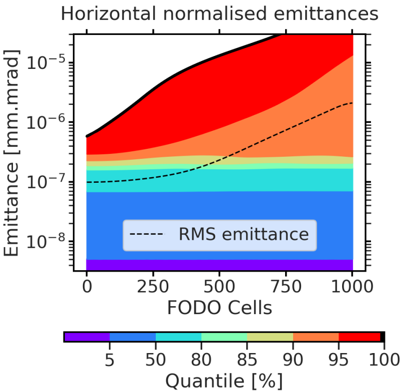


Figure: Incoherent Resonance

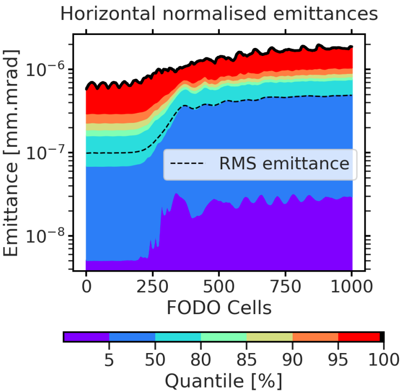
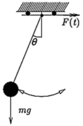


Figure: Coherent Resonance

Parametric Resonance

non-parametric resonance

Forced harmonic oscillator:



governed by e.o.m. of type

$$x'' + K\,x = sin(t)$$

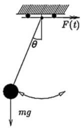
$\longrightarrow$ amplitude on resonance: **linear** growth

$\Longrightarrow$ beam dynamics example:
integer (dipole error) resonance

Parametric Resonance

non-parametric resonance

Forced harmonic oscillator:



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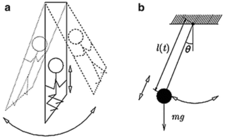
parametric resonance

Parametric harmonic oscillator:



governed by e.o.m. of type

$$x'' + (K + sin(t))\,x = 0$$

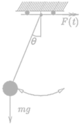
$\longrightarrow$ amplitude on res.: **exponential** growth

$\Longrightarrow$ beam dynamics example: 180° stop band in FODO cell (Mathieu instability)

Parametric Resonance

non-parametric resonance

Forced harmonic oscillator:



governed by e.o.m. of type

$$x'' + K\,x = sin(t)$$

$\longrightarrow$ amplitude on resonance: **linear** growth

$\Longrightarrow$ beam dynamics example: integer (dipole error) resonance

parametric resonance

Parametric harmonic oscillator:

Attention!

resonance frequency halved!

$\rightsquigarrow$ driving harmonic $h \longmapsto h/2$, consequence for synchrotrons:

$\Longrightarrow$ parametric resonances in tune diagram appear twice as dense

$\Longrightarrow$ parametric coherent resonance of order m is close to incoherent resonance of order $2m$

I. Landau Damping

FODO Space Charge Study

PHYSICAL REVIEW ACCELERATORS AND BEAMS **24,** 024201 (2021)

Self-consistent long-term dynamics of space charge driven resonances in 2D and 3D

Ingo Hofmann and Adrian Oeftiger
GSI Helmholtzzentrum für Schwerionenforschung GmbH, Planckstrasse 1, 64291 Darmstadt, Germany

Oliver Boine-Frankenheim
Technische Universität Darmstadt, Schlossgartenstrasse 8, 64289 Darmstadt, Germany

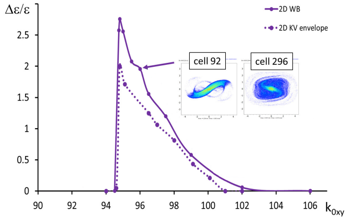
(Received 8 November 2020; accepted 29 January 2021; published 15 February 2021)

Computer experiment with a (perfect) FODO cell:

- keep tune per cell below $Q < 0.5$, i.e. phase advance $k < 180°$
- only possible source of resonant dynamics: space charge, $\Delta k_{KV} = 12°$
- compare distribution functions f:
  KV $f = \delta(\mathscr{H})$, waterbag $f = \Theta(\mathscr{H})$ and Gaussian (thermal) $f = \exp(-\mathscr{H})$

Param. Coh. Resonance Order

Fig.: waterbag distribution in FODO



resonance condition

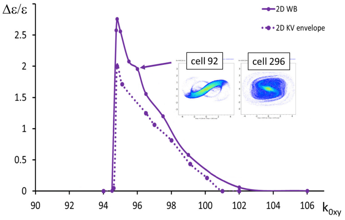
$$m\left(k_0 - C_m\,\Delta k_{KV}\right) = \frac{1}{2}360^\circ$$

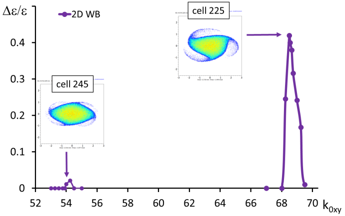
Waterbag distribution:

- $m = 2$: envelope instability $\Longrightarrow$ 90° stop band

Param. Coh. Resonance Order

Fig.: waterbag distribution in FODO





resonance condition

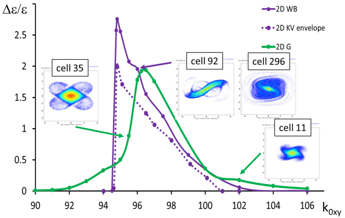
$$m\left(k_0 - C_m \Delta k_{KV}\right) = \frac{1}{2} 360^\circ$$

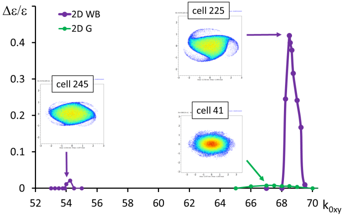
Waterbag distribution:

- $m = 2$: envelope instability $\Longrightarrow$ 90° stop band
- $m = 3$: sextupole moment instability $\Longrightarrow$ 60° stop band
- $m = 4$: octupole moment instability $\Longrightarrow$ 45° stop band

Param. Coh. Resonance Order

Fig.: Gaussian distribution in FODO





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Waterbag distribution:

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- $m = 3$: sextupole moment instability $\Longrightarrow$ 60° stop band
- $m = 4$: octupole moment instability $\Longrightarrow$ 45° stop band

$\Longrightarrow$ Gaussian distribution: no coherent response for nonlinear orders!

Landau Damping

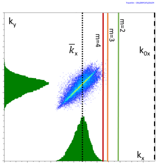


Fig.: waterbag

Landau damping requires mode frequency inside incoherent spectrum $f_0(k_{xy})$ on descending flank:

$$\partial f_0 / \partial k_{xy} < 0$$

- waterbag distribution: $m \leq 4$ outside spectrum!
  $\Longrightarrow$ nonlinear modes unstable

Landau Damping

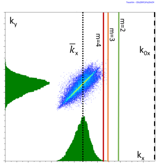


Fig.: waterbag

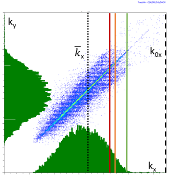


Fig.: Gaussian

Landau damping requires mode frequency inside incoherent spectrum $f_0(k_{xy})$ on descending flank:

$$\partial f_0 / \partial k_{xy} < 0$$

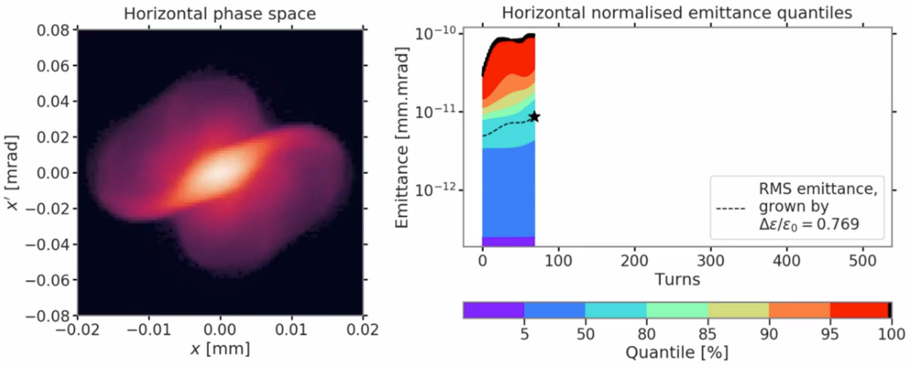
- waterbag distribution: $m \leq 4$ outside spectrum!
  $\Longrightarrow$ nonlinear modes unstable
- Gaussian distribution: $m > 2$ inside spectrum
  $\Longrightarrow$ nonlinear modes stabilised via Landau damping

This is the reason for...

... absence of coherent parametric resonances $m > 2$ in operational machines! ($m = 2$ or 90° stop band at GSI UNILAC: PRL **102** 234801)

Animation: Envelope Instability

Animation ↗ of envelope instability and interplay with 4th order incoherent resonance:



From 2D to 3D

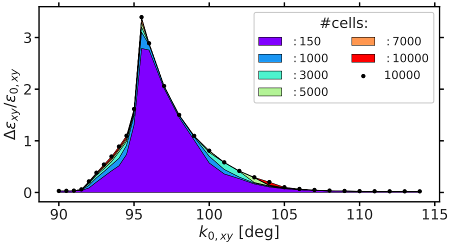


Fig.: 2D coasting beam

Coasting beam:

- short-term coherent dynamics (150 FODO cells sufficient)
- weak long-term incoherent resonances

From 2D to 3D

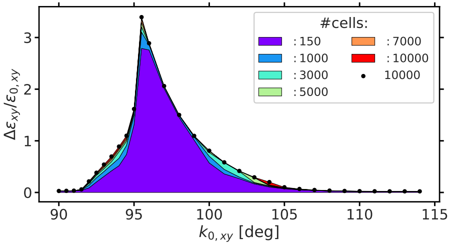


Fig.: 2D coasting beam

RF $\Longrightarrow$ on

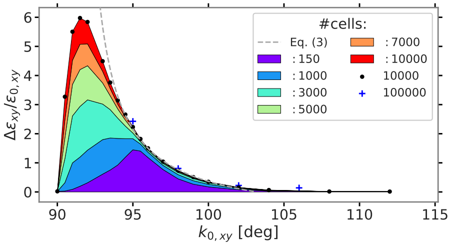


Fig.: 3D bunched beam ($k_{xy}/k_z = 300$)

Coasting beam:

- short-term coherent dynamics (150 FODO cells sufficient)
- weak long-term incoherent resonances

Bunched beam:

- short-term coherent dynamics overshadowed in the long term
- synchrotron oscillation reduces growth of coherent peak

Halo Dynamics: Scattering & Trapping

Impact of synchrotron motion on halo dynamics was presented by G. Franchetti and I. Hofmann in NIMA 561 195-202 (2006):

- key mechanism: particles **periodically cross the resonance** due to varying space charge strength along bunch
- large amplitudes due to: scattering off resonance, trapping in resonance islands

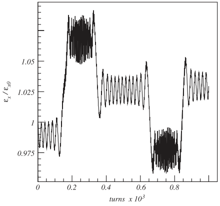


Fig. 7. Scattering of ε_x during 1 synchrotron oscillation.

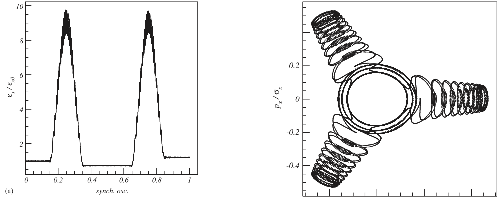


Fig. 6. Trapping of a particle during one synchrotron oscillation: single particle invariant (a); phase space (b).

(b)

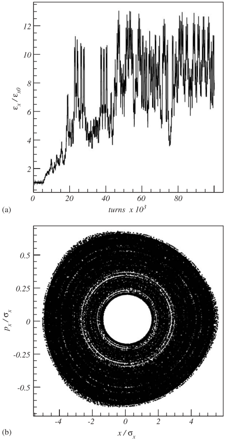
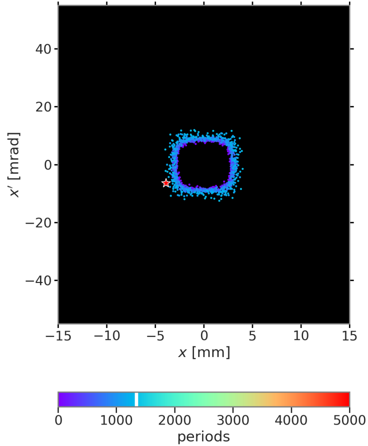


Fig. 8. Scattering and trapping of a particle during 100 synchrotron oscillations. The longitudinal tune is $Q_{z0} = 10^{-3}$.

Animation: Halo Particle

Animation of halo particle trapped in 4th order incoherent resonance islands:

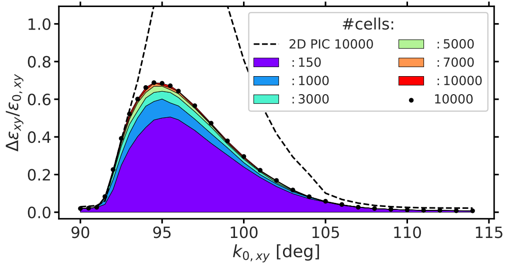


II. Frozen Space Charge Models

Frozen Space Charge

Model space charge with fixed frozen field map of Gaussian distribution:

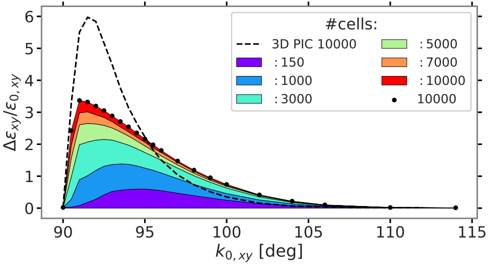
- 2D results mainly missing coherent dynamics (envelope instability)
  - ⟶ heavy underestimation of rms emittance growth (apart from halo)



Frozen Space Charge

Model space charge with fixed frozen field map of Gaussian distribution:

- 2D results mainly missing coherent dynamics (envelope instability)
  - ⟶ heavy underestimation of rms emittance growth (apart from halo)
- 3D results mainly missing change of distribution (rms, profile)
  - ⟶ underestimation in halo, overestimation in core
  - ⟹ frozen model useful for conservative prediction of resonance-free tunes



Summary of FODO Study

Long-term evolution of 3D Gaussian bunches subject to space charge:

- (parametric) coherent resonances are Landau damped for nonlinear orders $m > 2$
  - $\Longrightarrow$ only $m = 2$ envelope instability remains (90° stop band)
  - $\Longrightarrow$ intrinsic space charge limit: $\Delta Q_{KV} = 0.25 \Longleftrightarrow \Delta Q_{SC,Gauss} = 0.5$
- coherent resonances are short-term effects (fast saturation)
- coherent stop band *embedded within* incoherent stop band
  - $\Longrightarrow$ resonance-free tune areas *bounded by incoherent* resonance stop bands
  - $\Longrightarrow$ resonance-free tunes in incoherent prediction should be free of coherent resonance

How to identify resonance-free tunes

1. scan tunes with fast non-selfconsistent frozen model in long-term simulations (!)
2. validate resonance-free tune areas with selfconsistent model

III. The Space Charge Limit...

Turning to Realistic Accelerator

PHYSICAL REVIEW ACCELERATORS AND BEAMS **25,** 054402 (2022)

Simulation study of the space charge limit in heavy-ion synchrotrons

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Vladimir Kornilov,[1] Dmitrii Rabusov,[2] and Stefan Sorge[1]

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Nuclear Inst. and Methods in Physics Research, A 1040 (2022) 167290

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journal homepage: www.elsevier.com/locate/nima

Characterization and minimization of the half-integer stop band with space charge in a hadron synchrotron

Dmitrii Rabusov [a,*], Adrian Oeftiger [b], Oliver Boine-Frankenheim [a,b]

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About FAIR

Facility for **A**ntiproton and **I**on **R**esearch:

- under construction at GSI, Germany
- key: heavy ion synchrotron SIS100

⟶ operation close to *space charge limit*

supersymmetry	$S = 6$
circumference	1083.6 m
particles	from $A = 1$ (protons) to $A = 238$ (U^{28+})
injection energy	200 MeV/u
extraction energy	≤2.7 GeV/u (U^{28+})
intensity	$\leq 5 \times 10^{11}\, U^{28+}$/cycle
max. SC tune shift	$\Delta Q_y^{SC} = -0.3$

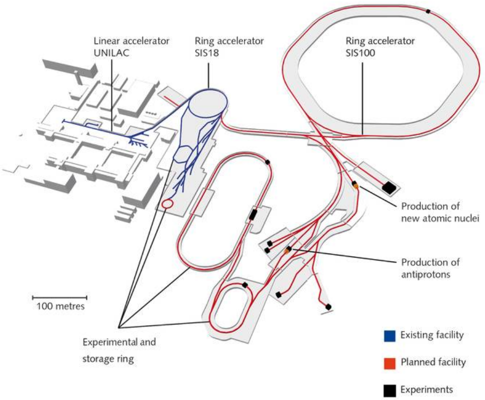
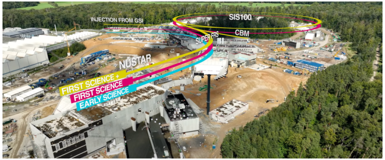


Figure: FAIR facility

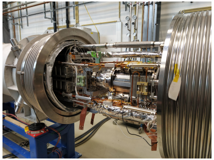
FAIR Status

- String test of full SIS100 arc cell established
- SIS100 accelerator sections being installed since Q2 2024:
  - dipoles in the tunnel
  - quadrupoles being supplied to GSI
- IPAC'23 paper on SIS100 status ↗

(a) video construction site ↗



(b) video experiments ↗



(c) string test at GSI

Motivation for FAIR Space Charge Study

FAIR:

- SIS100: deliver high-intensity hadron beams

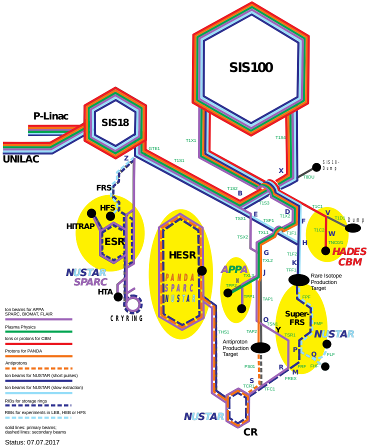


Figure: FAIR complex

Motivation for FAIR Space Charge Study

FAIR:

- SIS100: deliver high-intensity hadron beams
- crucial for performance: maintain beam quality during 1-sec injection plateau
  $\Longrightarrow$ 160 000 turns or 13 440 000 basic focusing cells
- reference case: uranium U^{28+} beam
  - largest beam size vs. transverse aperture
  - space charge induced losses
    $\rightsquigarrow$ important: dynamic vacuum stability
    $\Longrightarrow$ low-loss operation $< 5\%$!

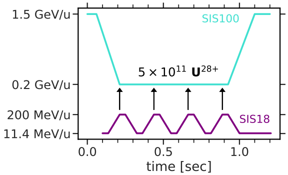


Figure: SIS18 to SIS100 transfer

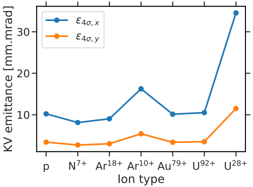


Figure: scaled beam sizes at 18 T m

Motivation for FAIR Space Charge Study

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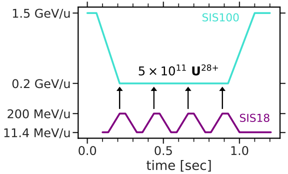
- reference case: uranium U^{28+} beam
  - largest beam size vs. transverse aperture
  - space charge induced losses
    $\rightsquigarrow$ important: dynamic vacuum stability
    $\Longrightarrow$ low-loss operation $< 5\%$!

$\Longrightarrow$ What is the maximum tolerable intensity at the space charge limit? (And can we increase it?)



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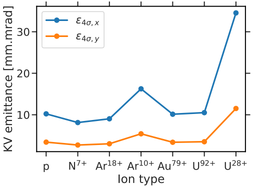


Figure: scaled beam sizes at 18 T m

"Our" Uranium Beam

Relevant beam parameters:

Hor. norm. rms emittance ϵ_x	5.9 mm mrad
Vert. norm. rms emittance ϵ_y	2.5 mm mrad
Rms bunch length σ_z	13.2 m
Bunch intensity N_0 of U^{28+} ions	6.25×10^{10}
Max. space charge ΔQ_y^{SC}	-0.30
Rms chromatic $Q'_{x,y} \cdot \sigma_{\Delta p/p_0}$	0.01
Synchrotron tune Q_s	4.5×10^{-3}
Kinetic energy	$E_{kin} = 200\,\mathrm{MeV/u}$
Relativistic β_0 factor	0.568
Revolution frequency f_{rev}	157 kHz

Numerical Simulation Model

Simulation model:

- track macro-particles (m.p.) through accelerator lattice & space charge kicks

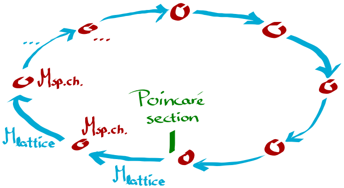


Figure: sketch of simulation model

Numerical Simulation Model

Simulation model:

- track macro-particles (m.p.) through accelerator lattice & space charge kicks
- nonlinear 3D space charge (SC) models:
  - *self-consistent **PIC**:* particle-in-cell for open-boundary Poisson equation
  - *fixed frozen (**FFSC**):* constant field map independent of m.p. dynamics

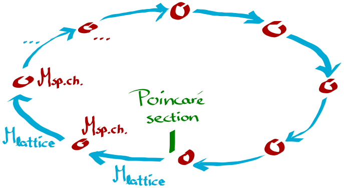


Figure: sketch of simulation model

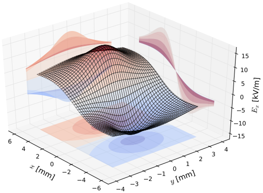


Figure: horizontal space charge field

Only Space Charge

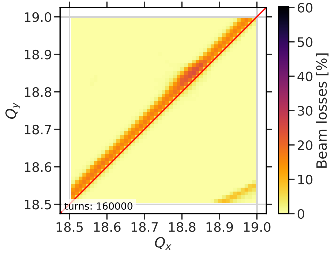


Figure: tune diagram of beam loss

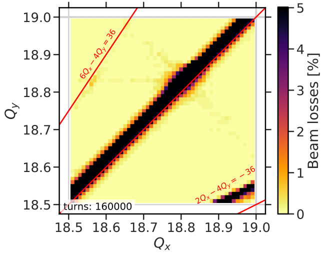
Symmetric error-free SIS100 lattice:

- perfect dipole and quadrupole magnets
- exact symmetry of $S = 6$
- space charge → only source for resonances
- simulated for 160'000 turns = 1 second

$\Longrightarrow$ mainly Montague resonance visible

$\Longrightarrow$ absence of low-order structure resonances!

Only Space Charge



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⟹ mainly Montague resonance visible

⟹ absence of low-order structure resonances!

Magnet Field Error Model

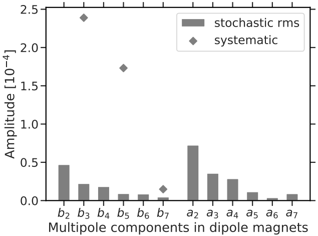


Figure: dipole magnets

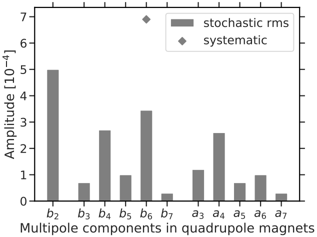


Figure: quadrupole magnets

Field error model extracted from cold bench measurements of main magnet units:

- stochastic amplitudes drive non-systematic resonances
- random number sequence → multipole errors for every dipole and quadrupole magnet

Full Model with Space Charge

Linear and nonlinear resonances driven by the field errors:

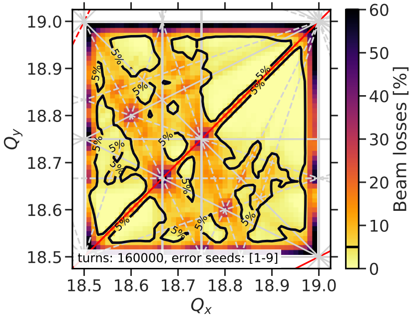


Figure: no space charge

Full Model with Space Charge

Linear and nonlinear resonances driven by the field errors:

⟶ SC broadens existing externally driven resonance stopbands

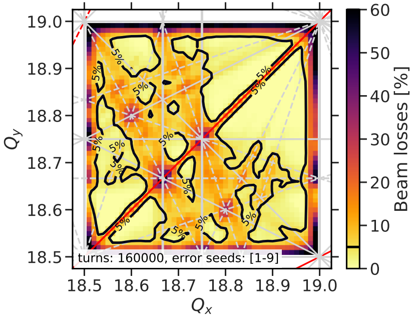


Figure: no space charge

intensity
⟹
on

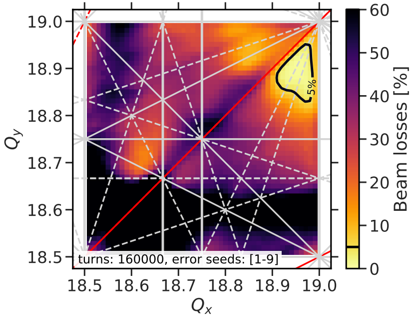


Figure: with fixed frozen space charge

Validation with Self-consistent PIC

Self-consistent PIC simulations:

⟶ now validate full error model FFSC predictions for beam loss

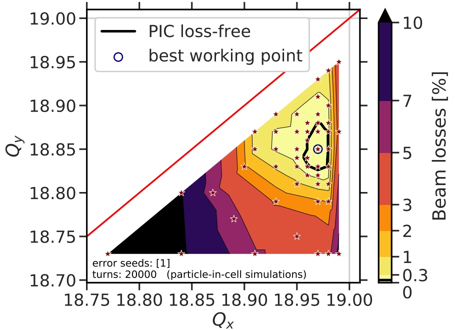


Figure: self-consistent PIC simulations

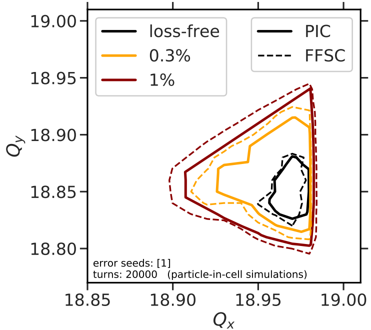


Figure: comparison between SC models

Space Charge Limit

dynamic definition of space charge limit

$\longrightarrow$ reached when loss-free working point area vanishes

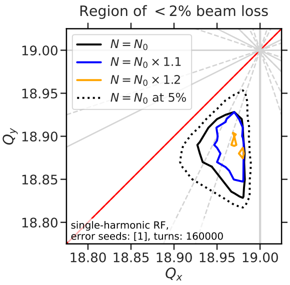


Figure: low-loss area for increasing N

Keeping all beam parameters identical, increasing N:

$\Longrightarrow$ U^{28+} space charge limit at **120%** of nominal bunch intensity N_0:

$$\max \left| \Delta Q_y^{SC} \right| = 0.36$$

IV. ... & how to push it further!

Options for Space Charge Mitigation

Conventional methods, in use:

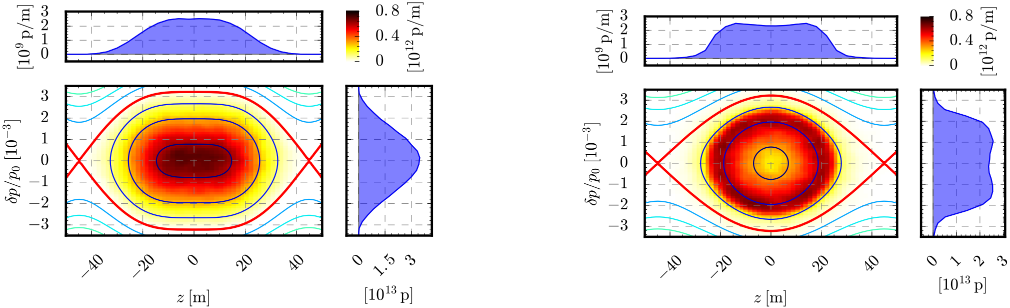
- bunch flattening (double harmonic RF, hollow bunches)
- resonance compensation (LEAR, LEIR, SPS, SIS100, JPARC MR)

Unconventional methods:

- charge neutralisation, e.g. ionised rest gas (CERN ISR 1971-84) or by electron columns
- electron lenses

Bunch Flattening

Flattened bunches with reduced $\lambda_{\max}$ mitigate space charge for $N = \text{const}$ as $\Delta Q^{\text{SC}} \propto \lambda_{\max}$:



(a) double-harmonic RF

(b) hollow bunch in single-harmonic RF

Figure: Simulated longitudinal phase space distributions

Double-harmonic RF

Add $h = 20$ harmonic in bunch lengthening mode:

$$V_{h=20} = V_{h=10}/2$$

$\Longrightarrow$ obtain reduced line density at 80% of nominal $\lambda_{\max}$.

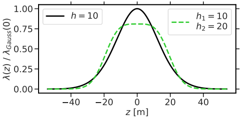


Figure: rms-equivalent line densities

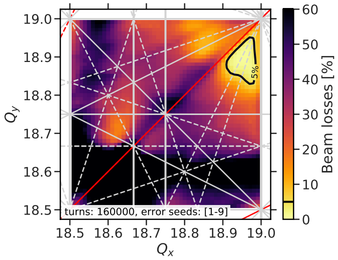


Figure: single-harmonic RF

flatten

$\Longrightarrow$

bunch

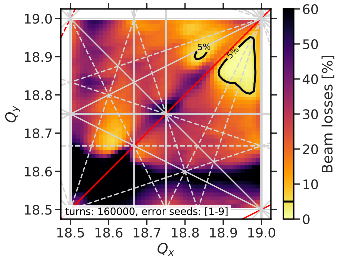


Figure: double-harmonic RF

SC Limit with Double-harmonic RF

Increasing N for double-harmonic RF:

- find space charge limit at **150%** of nominal intensity N_0

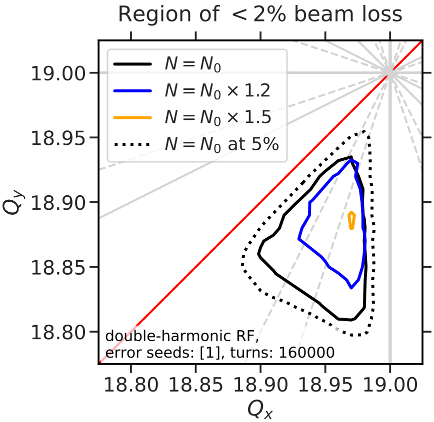
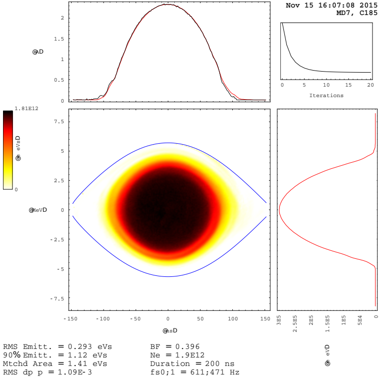
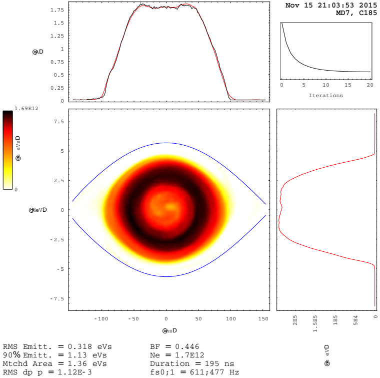


Figure: low-loss area for increasing N

Alternative bunch flattening: hollow longitudinal phase space distributions (CERN PS measurements, cf. my PhD thesis)



(a) Reconstructed parabolic longitudinal distribution.



(b) Reconstructed hollow longitudinal distribution.

Resonance Compensation

Dedicated resonance compensation is explored & used in many places, e.g.:

FNAL Booster

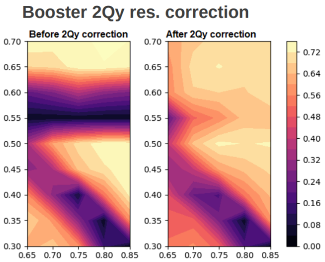


Figure: J. Eldred et. al., PRAB 24, 044001 (2021)

JPARC Main Ring

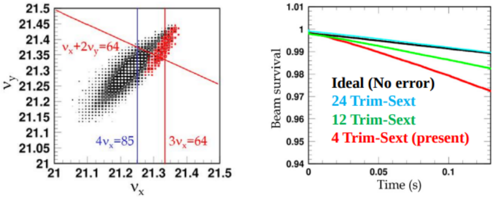
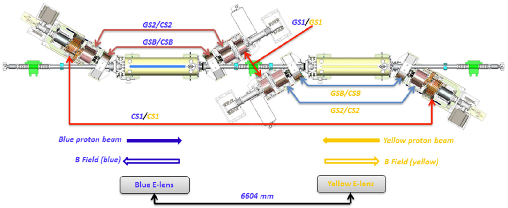


Figure: H. Hotchi et al., TUPM055 IPAC'23

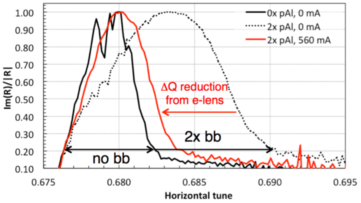
Electron Lenses

Electron lenses prove to be a versatile compensation technique:

- short insertions, electron beam overlaps hadron beam: provide focusing kick
- successful use in operation:
  - transverse Gaussian DC lens: head-on beam-beam effect compensation in colliders (FNAL Tevatron and BNL RHIC)
  - transverse hollow DC lens: beam halo removal (BNL RHIC, hopefully: CERN LHC)



(a) RHIC electron lenses



(b) beam-beam compensation

Figure: W. Fischer et al., PRL 115 (2015) 26, 264801

Electron Lenses

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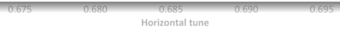
- short insertions, electron beam overlaps hadron beam: provide focusing kick
- successful use in operation:
  - transverse Gaussian DC lens: head-on beam-beam effect compensation in colliders

Further Proposals

Electron lenses investigated for use in

- nonlinear integrable optics element (FNAL IOTA)
- space charge compensation (FNAL IOTA and GSI SIS18 / FAIR SIS100, idea first proposed in 2001 by A. Burov et al.)
- Landau damping of dipole moment instabilities (V. Shiltsev et al., PRL 119 (2017) 13)

(a) RHIC electron lenses



(b) beam-beam compensation

Figure: W. Fischer et al., PRL 115 (2015) 26, 264801

Pulsed Electron Lenses for Space Charge Compensation

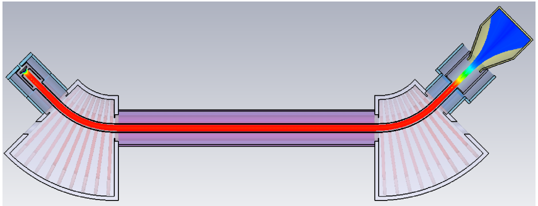


Figure: e-lens model for SIS18 [K. Schulte-Urlichs et al., IPAC'22] ↗

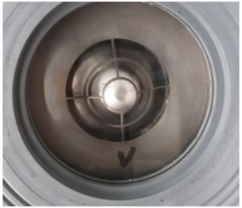


Figure: Modulation grid.

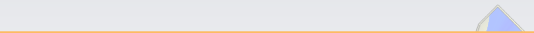
Short insertion (here $L = 3.36\,\mathrm{m}$) with co-propagating electron beam:

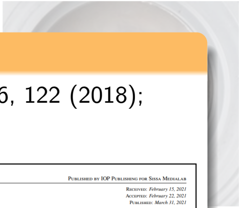
- transversely homogeneous distribution
- longitudinally modulated to match ion bunch profile

$\longrightarrow$ compensate longitudinal dependency of space charge

$\Longrightarrow$ **suppress periodic resonance crossing**

Pulsed Electron Lenses for Space Charge Compensation





Involved Publications

Idea of pulsed homogeneous lenses: O. Boine-Frankenheim et al, NIMA 896, 122 (2018);
Full feasibility studies:

PHYSICAL REVIEW LETTERS **132,** 175001 (2024)

Editors' Suggestion

Pulsed Electron Lenses for Space Charge Mitigation

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ICFA Beam Dynamics Newsletter#81 —
Electron Lenses for Modern and Future Accelerators

Pulsed electron lenses for space charge compensation in the FAIR synchrotrons

S. Artikova,[a] O. Boine-Frankenheim,[a,b,*] O. Meusel,[c] A. Oeftiger,[a] D. Ondreka,[a] K. Schulte-Urlichs[a] and P. Spiller[a]

Fi… …rid.

Short

- transversely homogeneous distribution
- longitudinally modulated to match ion bunch profile

⟶ compensate longitudinal dependency of space charge

⟹ suppress periodic resonance crossing

Tune Footprint vs. E-Lens Compensation

Some n_{el} e-lenses with I_{e} current and rms beam size σ_{e} provide tune shift:

$$\Delta Q_y^{\mathrm{e}} = \frac{1}{4\pi}\sum_{k=1}^{n_{\mathrm{el}}}\beta_y(s_k)\frac{r_c}{Ze}\frac{I_{\mathrm{e}}}{\sigma_{\mathrm{e}}^2\gamma_0}\frac{1-\beta_{\mathrm{e}}\beta_0}{\beta_{\mathrm{e}}}\frac{L}{\beta_0 c}$$

Define linear compensation degree (for Gaussian bunches $\Delta Q^{\mathrm{KV}} = \Delta Q^{\mathrm{SC}}/2$):

$$\alpha \doteq \frac{\Delta Q^{\mathrm{e}}}{\left|\Delta Q^{\mathrm{KV}}\right|}$$

- dipole tune increases with

$$\Delta Q_{\mathrm{dip}} = \alpha \cdot \Delta Q^{\mathrm{e}}$$

- without chroma, $\alpha = 0.5$ yields smallest tune spread!

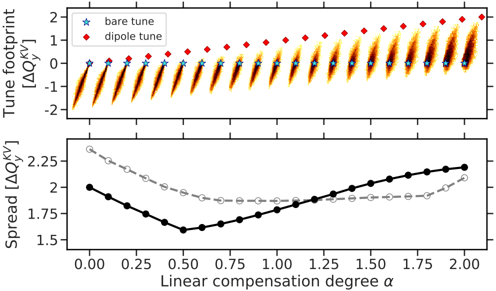
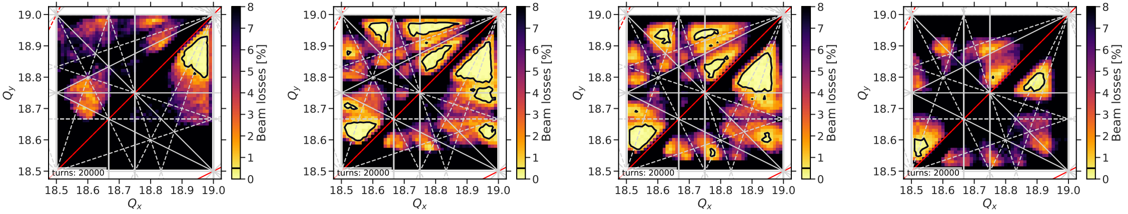


Figure: Gaussian bunch, tune footprint vs. e-lens strength (black: $\Delta p/p_0 = 0$, grey: with natural chromatic detuning)

Optimal E-Lens Configuration

In SIS100 with natural chromaticity:



(a) $\alpha = 0$ (b) $\alpha = 0.5$ (c) $\alpha = 0.7$ (d) $\alpha = 1$

Figure: FAIR design intensity $N = N_0$ with $n_{\mathrm{el}} = 3$ pulsed e-lenses.

- optimal choice of α depends on nearby resonances
  - $\Longrightarrow$ depends on particularities of synchrotron
- SIS100: at low $n_{\mathrm{el}} \leq 6$, $\alpha = 0.5$ optimal vs. high $n_{\mathrm{el}} > 6$, $\alpha = 0.7$ better

SC Limit with Pulsed E-Lenses

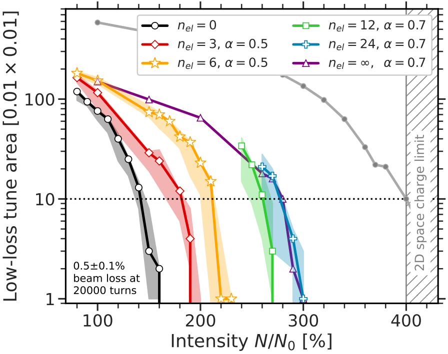


Figure: low-loss area for increasing N

Table: SC limit with electron lenses.

Number n_{el}	SC limit	Gain
0	$1.4 \cdot N_0$	100%
3	$1.8 \cdot N_0$	130%
6	$2.1 \cdot N_0$	150%
12	$2.6 \cdot N_0$	185%
$24, \infty$	$2.8 \cdot N_0$	200%

- SC limit scales well
- $n_{\mathrm{el}} = 24$ case saturates gain
- theoretical 2D limit ($Q_s = 0$, no e-lenses) = by construction no periodic resonance crossing

  $\Longrightarrow$ reached after $n_{\mathrm{el}} = 24, \infty$

Conclusion

Summary:

- apparent contradiction between theory and operational experience resolved:
  - ⟶ nonlinear (parametric) coherent modes are **Landau damped**
- established and validated strategy to identify space charge limit (via fast frozen modelling)
  - ⟶ evaluated **space charge limit** for FAIR SIS100: $\max\left|\Delta Q_y^{\mathrm{SC}}\right| = 0.36$
- explored mitigation methods with quantitative improvement estimates:
  - ⟶ nominal SIS100: +20% intensity
  - ⟶ double-harmonic RF: +50% intensity
  - ⟶ 3 pulsed electron lenses: +70..80% intensity
  - ⟹ compatibility: bunch flattening + electron lenses + resonance compensation!

⟹ electron lenses prove to be versatile and powerful mitigation tool for collective effects!

… Surprises …

One could also try to *fully compensate* the nonlinear space charge tune spread by employing transversely Gaussian, pulsed electron lenses:

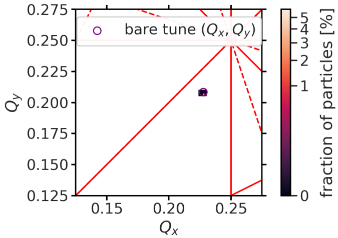


Figure: tune footprint, full nonlinear compensation

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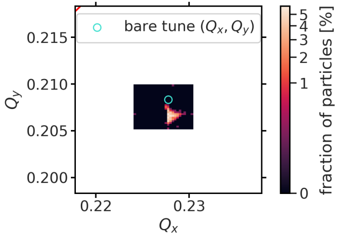

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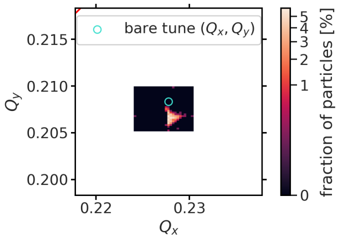

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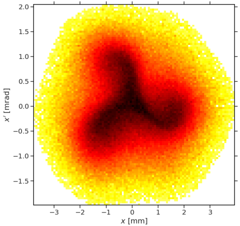
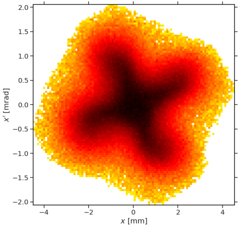


(a) sextupole $m = 3$



(b) octupole $m = 4$

Figure: parametric coherent resonances of $m > 2$ re-appear!

$\Longrightarrow$ … but then we have to deal with previously Landau-damped friends!

Thank you for your attention!