Systematic approaches to new physics in cosmology

Will Barker

Ongoing work with **Will Handley**, **Mike Hobson**,
Anthony Lasenby, **Carlo Marzo** and **Alessandro Santoni**

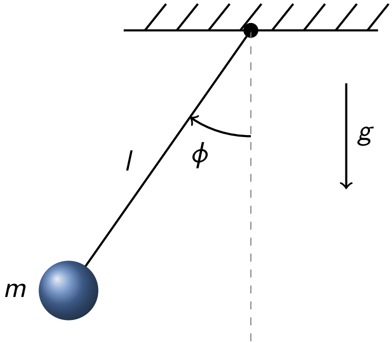
Theories are data models

- Physical theories have an **action** $\mathcal{S}[\phi]$
- Functional of **fields** ϕ
- Function of the **couplings** θ
- Example in the simple pendulum:

$$\mathcal{S}[\phi] = \int \mathrm{d}t \left[\underbrace{\tfrac{1}{2}\, m\, l^2\, \dot{\phi}^2}_{\text{kinetic}} - \underbrace{m\, g\, l\, (1 - \cos\phi)}_{\text{potential}} \right]$$

$$= \int \mathrm{d}t \left[\underbrace{\theta_1}_{\frac{1}{2} m l^2} \underbrace{\dot{\phi}^2}_{\mathcal{O}^1} + \underbrace{\theta_2}_{-\frac{1}{2} m g l} \underbrace{\phi^2}_{\mathcal{O}^2} + \underbrace{\theta_3}_{\frac{1}{24} m g l} \underbrace{\phi^4}_{\mathcal{O}^3} + \cdots \right]$$

$$= \int \mathrm{d}t \sum_i \theta_i\, \mathcal{O}^i$$



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$$= \int \mathrm{d}t \sum_i \theta_i\, \mathcal{O}^i$$

- Now consider paradigm shift:

$$\mathcal{S}[\phi] \longrightarrow \mathcal{S}(\theta)$$

- Seek **Bayesian evidence**:

$$\mathcal{Z}(D) = \int \mathrm{d}\theta\, \mathcal{L}(D|\theta)\, \pi(\theta)$$

- **Likelihood** $\mathcal{L}(D|\theta)$ of data D
- **Prior** $\pi(\theta)$ on the couplings
- How to compute the prior?

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$$= \int \mathrm{d}t \sum_i \theta_i\, \mathcal{O}^i$$

- First two terms capture essence
- Inverted **tachyon** $\theta_2 > 0$
- Negative KE **ghost** $\theta_1 < 0$
- Known as **unitarity conditions**

$$\theta_1 > 0, \quad \theta_2 < 0$$

- Leads to field of **polology**

Symbolic polology: the basic idea

- Polology can be done with **software**
- You get some crucial **science products**:
  - Symmetries
  - Unitarity
  - Masses
- Current version developed with Carlo Marzo, Giorgos Karananas, Claire Rigouzzo and Haochen Tu:

PSALTer Particle Spectrum for Any Tensor Lagrangian

	$\phi^{\#1}_{0^+}$	$h^{\#1}_{0^+}$	$h^{\#2}_{0^+}$	$\mathcal{A}^{\#1}_{0^+}$
$\phi^{\#1}_{0^+}\dagger$	0	$\frac{1}{2}\sqrt{3}\,k^2\,\theta_6$	$-\frac{Y_1 k^2}{2}$	0
$h^{\#1}_{0^+}\dagger$	$\frac{1}{2}\sqrt{3}\,k^2\,\theta_6$	$\frac{1}{2}(Y_2+Y_3 k^2)$	$\frac{1}{2}(\sqrt{3}\,Y_4 k^2+2\sqrt{3}\,\theta_5)$	$\frac{1}{2}\,i\,\sqrt{3}\,k\,\theta_6$
$h^{\#2}_{0^+}\dagger$	$-\frac{Y_1 k^2}{2}$	$\frac{1}{2}(\sqrt{3}\,Y_4 k^2+2\sqrt{3}\,\theta_5)$	$\frac{1}{2}(Y_5+Y_6 k^2)$	$-\frac{1}{2}\,i\,Y_1\,k$
$\mathcal{A}^{\#1}_{0^+}\dagger$	0	$-\frac{1}{2}\,i\,\sqrt{3}\,k\,\theta_6$	$\frac{i\,Y_1\,k}{2}$	0

	$h^{\#1}_{1^-\,\alpha}$	$\mathcal{A}^{\#1}_{1^-\,\alpha}$
$h^{\#1}_{1^-}\dagger^\alpha$	$\frac{\theta_1}{2}$	$-\frac{i\,\theta_1\,k}{\sqrt{2}}$
$\mathcal{A}^{\#1}_{1^-}\dagger^\alpha$	$\frac{i\,\theta_1\,k}{\sqrt{2}}$	$\theta_1\,k^2$

	$h^{\#1}_{2^+\,\alpha\beta}$
$h^{\#1}_{2^+}\dagger^{\alpha\beta}$	$\frac{1}{2}(\theta_1-k^2\,\theta_3)$

Abbreviations used in matrices

$Y_1 == 2\,\theta_1-\theta_6$ && $Y_2 == \theta_1+6\,\theta_5$ && $Y_3 == 6\,\theta_2-\theta_3$ &&
$Y_4 == 2\,\theta_2+\theta_4$ && $Y_5 == \theta_1+2\,\theta_5$ && $Y_6 == 2\,\theta_2+\theta_3+2\,\theta_4$ &&
$\mathrm{Det}(0^+) == -\frac{1}{4}k^2(1+k^2)(2\,\theta_1{}^3+2\,\theta_1{}^2(k^2(6\,\theta_2-\theta_3)+6\,\theta_5-\theta_6)+k^2\,\theta_3\,\theta_6{}^2+2\,\theta_1\,\theta_6(k^2(\theta_3+3\,\theta_4)+\theta_6))$ &&
$\mathrm{Det}(2^+) == \frac{1}{2}(\theta_1-k^2\,\theta_3)$

Lagrangian

$$\frac{1}{2}\theta_1\,h_{\alpha\beta}\,h^{\alpha\beta}+\theta_5\,h^\alpha{}_\alpha\,h^\beta{}_\beta+\theta_6\,h^\beta{}_\beta\,\partial_\alpha\mathcal{A}^\alpha-\theta_6\,h^\beta{}_\beta\,\partial_\alpha\partial^\alpha\phi-\theta_1\,\partial_\alpha\mathcal{A}^\alpha\,\partial_\beta\mathcal{A}^\beta+2\,\theta_1\,h^{\alpha\beta}\,\partial_\beta\partial_\alpha\phi-$$
$$2\,\theta_1\,h_{\alpha\beta}\,\partial^\beta\mathcal{A}^\alpha+\theta_1\,\partial_\beta\mathcal{A}_\alpha\,\partial^\beta\mathcal{A}^\alpha+\theta_2\,\partial_\beta h^\chi{}_\chi\,\partial^\beta h^\alpha{}_\alpha+\theta_3\,\partial_\alpha h^{\alpha\beta}\,\partial_\chi h_\beta{}^\chi+\theta_4\,\partial^\beta h^\alpha{}_\alpha\,\partial_\chi h_\beta{}^\chi-\frac{1}{2}\theta_3\,\partial_\chi h_{\alpha\beta}\,\partial^\chi h^{\alpha\beta}$$

Added source term(s):	$\phi\,\rho+\mathcal{A}^\alpha\,\mathcal{J}_\alpha+h^{\alpha\beta}\,\mathcal{T}_{\alpha\beta}$		
Source constraint(s)	# constraint(s)	Covariant form	
$i\,\rho^{\#1}_{0^+}+k\,\mathcal{J}^{\#1}_{0^+} == 0$	1	$\rho == \partial_\alpha\mathcal{J}^\alpha$	
$\mathcal{J}^{\#1\alpha}_{1^-}-2\,i\,k\,\mathcal{T}^{\#1\alpha}_{1^-} == 0$	3	$\partial_\beta\partial^\alpha\mathcal{J}^\beta+2\,\partial_\chi\partial^\chi\partial_\beta\mathcal{T}^{\alpha\beta} == \partial_\beta\partial^\beta\mathcal{J}^\alpha+2\,\partial_\chi\partial_\beta\partial^\alpha\mathcal{T}^{\beta\chi}$	
Total # constraint(s):	4		
Resolved pole(s)	# polarization(s)	Square mass	Residue
$J^P=0^+$	1	$-\frac{2\,\theta_1(\theta_1{}^2+6\,\theta_1\,\theta_5-\theta_1\,\theta_6+\theta_6{}^2)}{2\,\theta_1{}^2(6\,\theta_2-\theta_3)+2\,\theta_1(\theta_3+3\,\theta_4)\,\theta_6+\theta_3\,\theta_6{}^2}$	(Hidden for brevity)
$J^P=2^+$	5	$\frac{\theta_1}{\theta_3}$	$-\frac{2}{\theta_3}$
Resolved unitarity condition(s):		$\theta_1<0$ && $\theta_3<0$ && $\theta_5<\frac{-\theta_1{}^2+\theta_1\,\theta_6-\theta_6{}^2}{6\,\theta_1}$ && $\theta_2>\frac{2\,\theta_1{}^2\,\theta_3-2\,\theta_1\,\theta_3\,\theta_6-6\,\theta_1\,\theta_4\,\theta_6-\theta_3\,\theta_6{}^2}{12\,\theta_1{}^2}$	

Numerical pololoy: textbook examples

- Simple example of **Fierz–Pauli**:

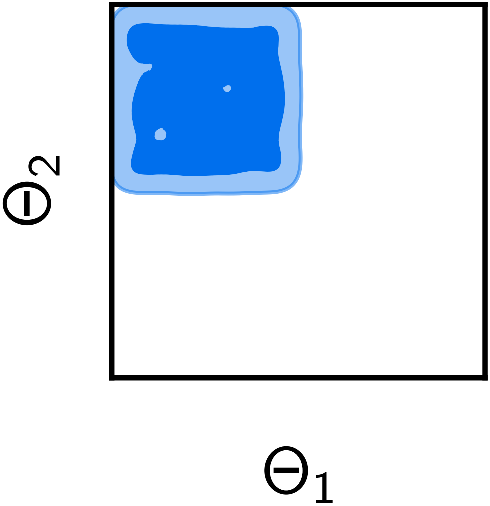
$$\mathcal{S}(\theta) = \int \mathrm{d}^4 x \left\{ \theta_1 \left[\frac{1}{2} \partial_\beta \mathcal{H} \partial^\beta \mathcal{H} - \partial^\alpha \mathcal{H}_{\alpha\beta} \partial^\beta \mathcal{H} \right.\right.$$
$$\left. - \frac{1}{2} \partial_\gamma \mathcal{H}^{\alpha\beta} \partial^\gamma \mathcal{H}_{\alpha\beta} + \partial_\beta \mathcal{H}^{\alpha\beta} \partial^\gamma \mathcal{H}_{\alpha\gamma} \right]$$
$$\left. - \theta_2 \left[\mathcal{H}_{\alpha\beta} \mathcal{H}^{\alpha\beta} - \mathcal{H}^2 \right] \right\}$$

- Unitarity conditions:

$$\theta_1 < 0, \quad \theta_2 > 0$$

- Visualise on the compactified hypercube:

$$\Theta \equiv \tan^{-1} \theta$$



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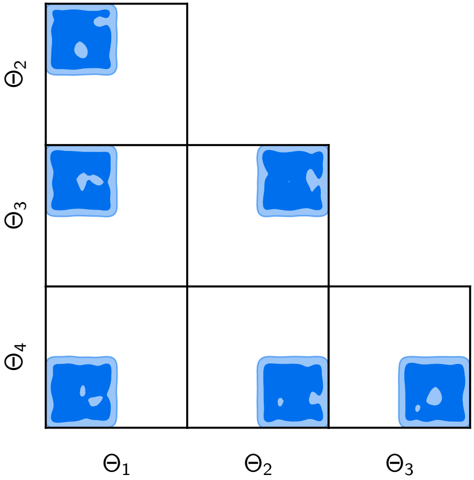
- Simple example of **Proca**:

$$\mathcal{S}(\theta) = \int \mathrm{d}^4x \left\{ \theta_3 \left[-\frac{1}{2}\partial_\alpha \mathcal{A}_\beta \partial^\alpha \mathcal{A}^\beta + \frac{1}{2}\partial_\alpha \mathcal{A}^\alpha \partial_\beta \mathcal{A}^\beta \right] - \frac{1}{2}\theta_4 \mathcal{A}_\alpha \mathcal{A}^\alpha \right\}$$

- Unitarity conditions:

$$\theta_1 < 0, \quad \theta_2 > 0, \quad \theta_3 > 0, \quad \theta_4 < 0$$



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Numerical polology: new physics

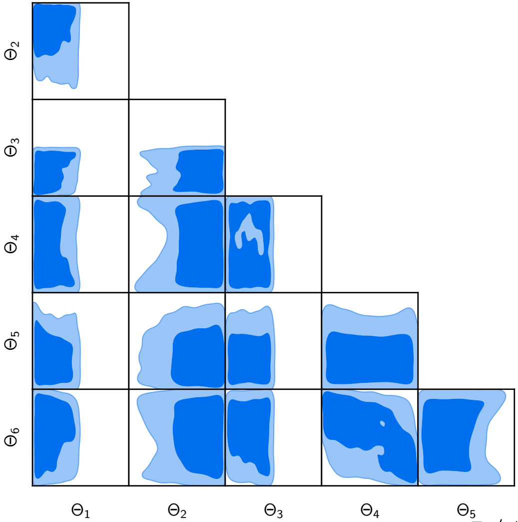
- Shiny **new** theory of physics:

$$\begin{aligned}\mathcal{S}(\theta) = \int \mathrm{d}^4x \Big[&\frac{\theta_1}{2}\mathcal{H}_{\alpha\beta}\mathcal{H}^{\alpha\beta} + \theta_5\mathcal{H}^2 + \theta_6\mathcal{H}\partial_\alpha\mathcal{A}^\alpha \\ &- \theta_6\mathcal{H}\partial_\alpha\partial^\alpha\phi - \theta_1\partial_\alpha\mathcal{A}^\alpha\partial_\beta\mathcal{A}^\beta \\ &+ \theta_1\partial_\beta\mathcal{A}_\alpha\partial^\beta\mathcal{A}^\alpha + 2\theta_1\mathcal{H}^{\alpha\beta}\partial_\beta\partial_\alpha\phi \\ &- 2\theta_1\mathcal{H}_{\alpha\beta}\partial^\beta\mathcal{A}^\alpha + \theta_2\partial_\beta\mathcal{H}\partial^\beta\mathcal{H} \\ &+ \theta_3\partial_\alpha\mathcal{H}^{\alpha\beta}\partial_\gamma\mathcal{H}_\beta{}^\gamma + \theta_4\partial^\beta\mathcal{H}\partial_\gamma\mathcal{H}_\beta{}^\gamma \\ &- \frac{\theta_3}{2}\partial_\gamma\mathcal{H}_{\alpha\beta}\partial^\gamma\mathcal{H}^{\alpha\beta}\Big]\end{aligned}$$

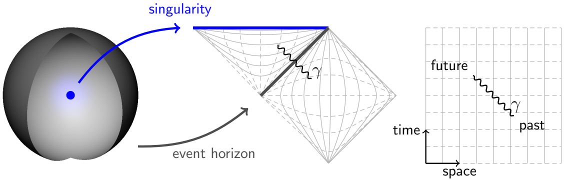
- Unitarity conditions:

$$\theta_1 < 0, \quad \theta_3 < 0, \quad \theta_5 < \frac{-\theta_1{}^2 + \theta_1\theta_6 - \theta_6{}^2}{6\theta_1},$$

$$\theta_2 > \frac{2\theta_1{}^2\theta_3 - 2\theta_1\theta_3\theta_6 - 6\theta_1\theta_4\theta_6 - \theta_3\theta_6{}^2}{12\theta_1{}^2}$$

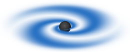


Numerical polology: black hole superradiance constraints

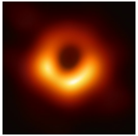


general BH idea | curved spacetime | flat spacetime

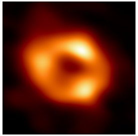
- BH mass M and angular momentum J
- Size of event horizon is $r_g \sim GM$



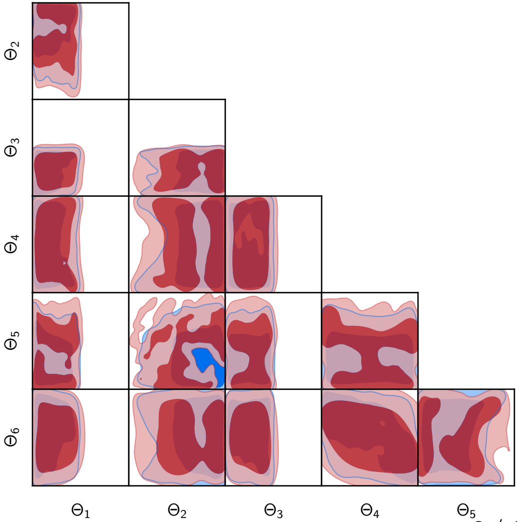
supermassive BHs in galaxy centres



a long time ago in a galaxy far far away



centre of our own galaxy



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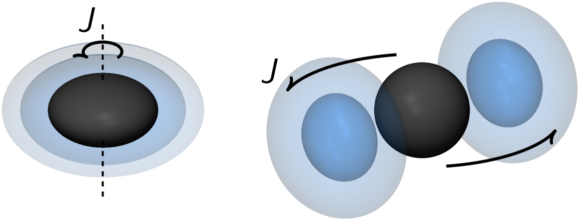
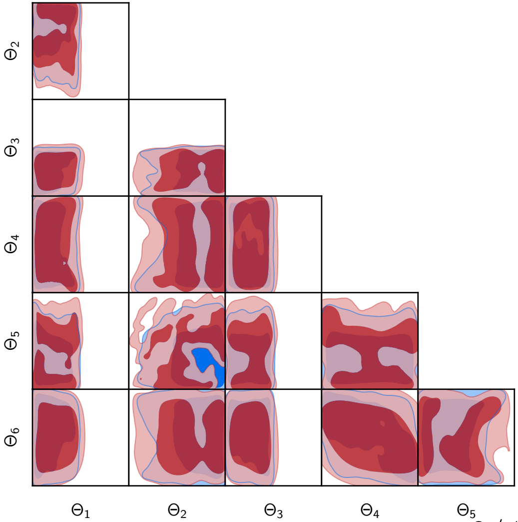


- Boson has Compton wavelength $\lambda \sim 1/m$
- BH event horizon $r_g \sim GM$
- Boson 'feels' BH when $\lambda \sim r_g$
- BH leaks J to boson at rate $\Gamma(M, m)$
- By astrophysics BH age is $\tau(M, J)$
- Observed J, M **rules out** m with $\tau\Gamma \gg 1$



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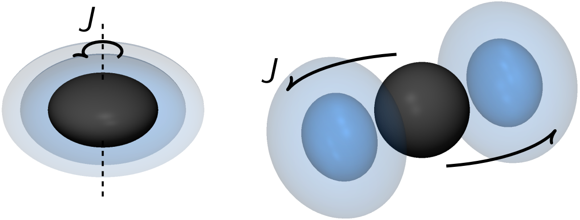

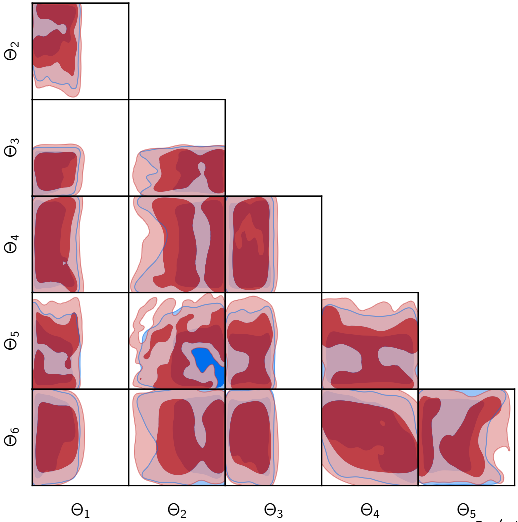

- Observed J, M **rules out** m with $\tau\Gamma \gg 1$
- By astrophysics, we expect J, M populations
- **Missing** J, M are a smoking gun for boson at m
- Two probes: **gravitational waves** and **astronomy**

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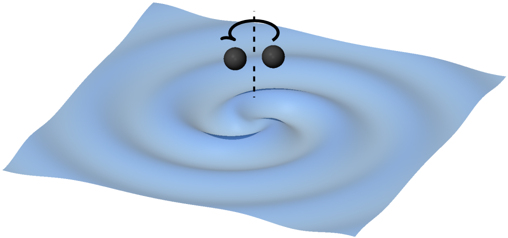
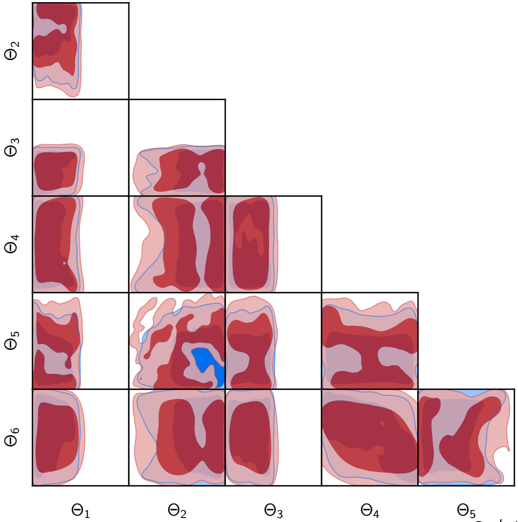
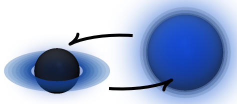
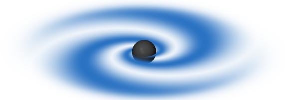
Numerical polology: black hole superradiance constraints



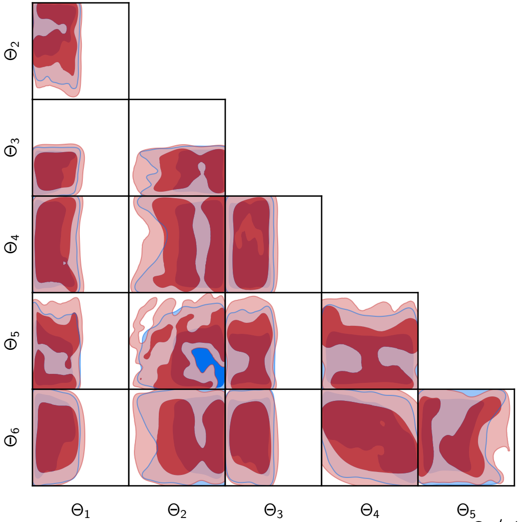
- Gravitational waves show **promise**
- Fairly small BHs merging
- J, M statistics inferred



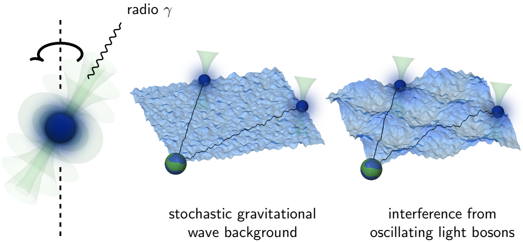


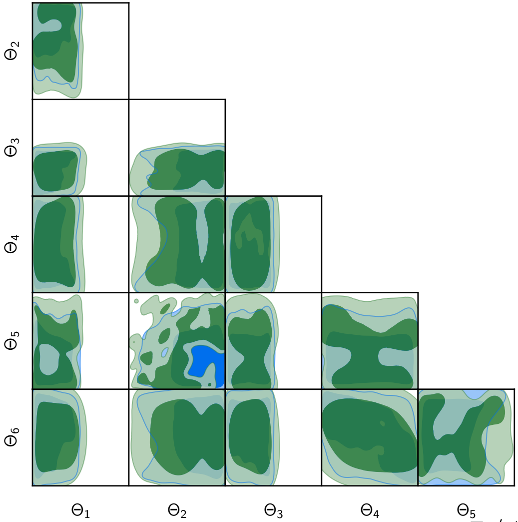


- Astronomy gives **crispest** cases
- Small BHs in X-ray binaries
- Supermassive BHs dominate galaxies
- Often high-J (which is good)
- BHSR codes from sebhoof/bhsr — see also 2406.10337



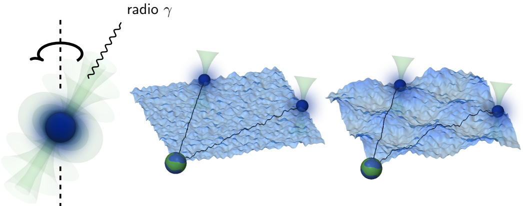
Numerical polology: pulsar timing array constraints





- Pulsars are rapidly rotating neutron stars
- Radio signals form precise clocks
- From astrophysics, expect GW background
- GWs delay/advance radio pulses
- New light bosons oscillate at $\omega \sim m$

Numerical polology: pulsar timing array constraints



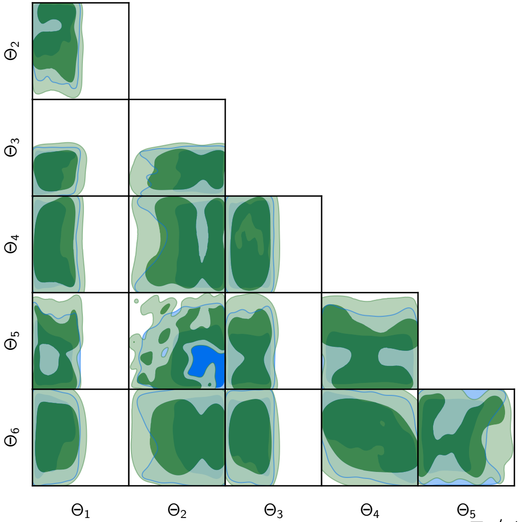
stochastic gravitational wave background

interference from oscillating light bosons

- Mass cut from 2312.12225

Arecibo (1963–2020)

FAST (2016–)



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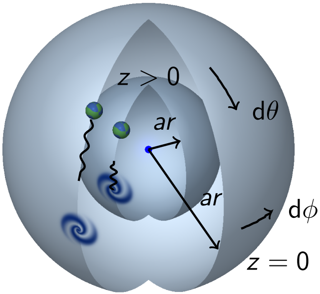
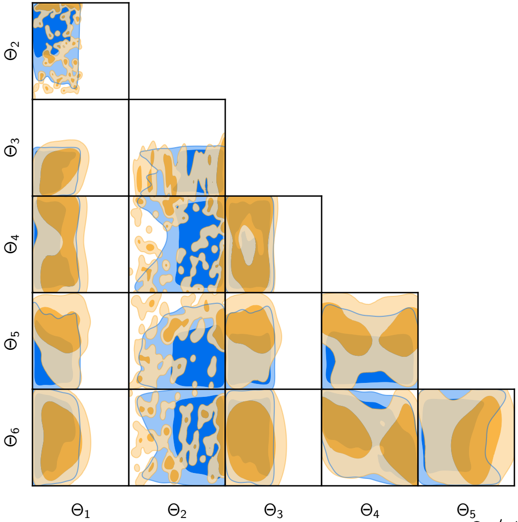
Numerical polology: dark energy constraints



- Distance $a^2 r^2 \left(d\theta^2 + \sin^2\theta \, d\phi^2\right)$
- Expansion via **scale factor** $a(z)$
- Photons **stretch** with redshift z
- Take $a(0) = 1$ **today**, $a(z > 0) < 1$ **past**



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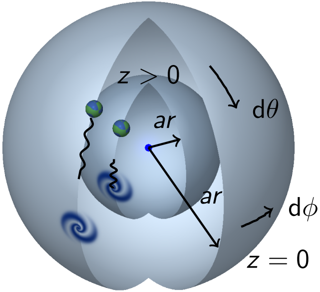
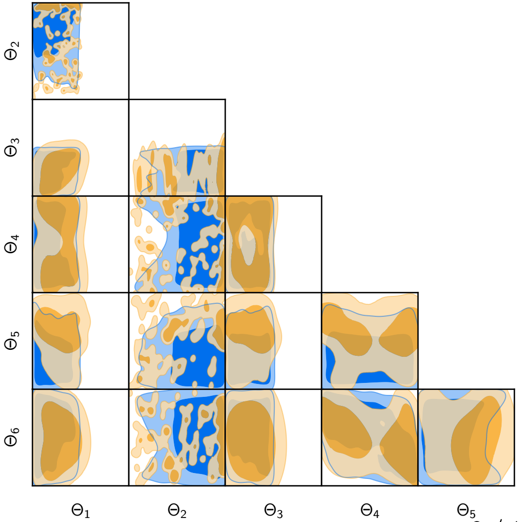


- Hubble **number** $H(z) = \dot{a}/a$
- Expansion history of the Universe
- Huge focus to understand $H(z)$



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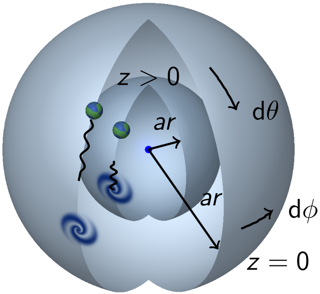
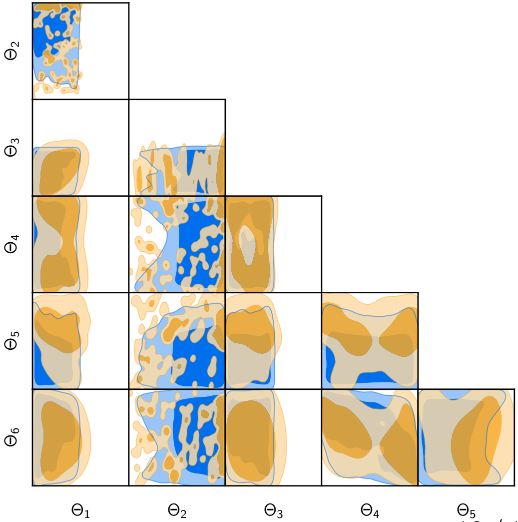


- Generally $H(z)^2$ equals **energy density**
- As matter dilutes away, $H(z)^2 \sim \Lambda$
- Bosons oscillate when $H(z) \lesssim m$
- These can **modify** expansion history



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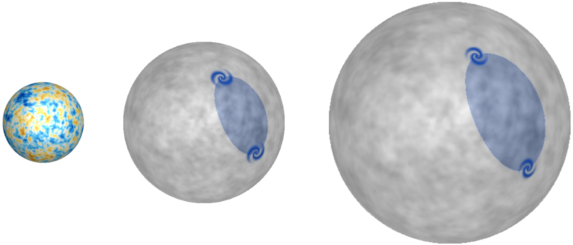
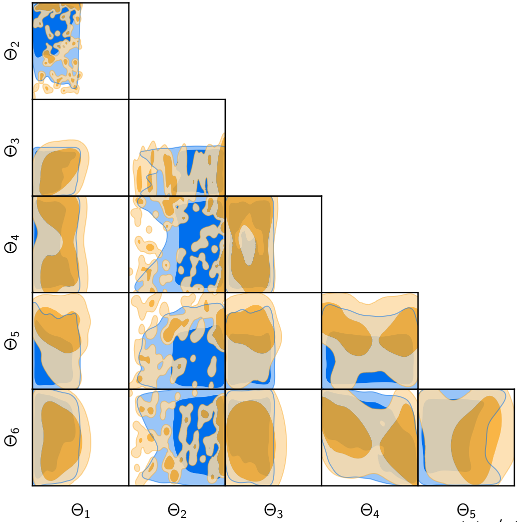
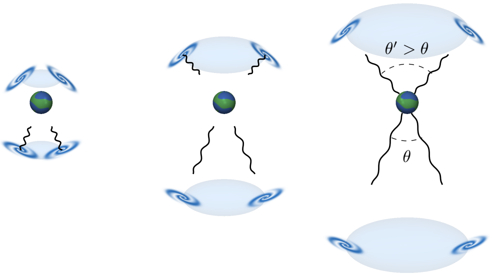


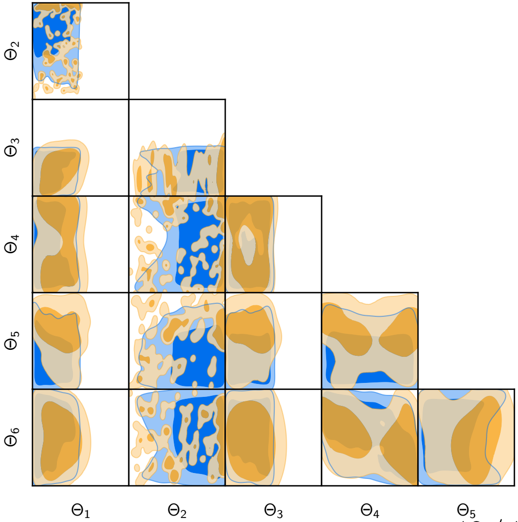


- Plasma and radiation at early times
- Correlated structure from acoustic waves
- Structures determine galaxy positions
- Galaxy positions stretch
- Galaxies do not! (gravitationally bound)

Numerical polology: dark energy constraints



- Photons also stretch with **redshift** z
- **Angular size shrinks** with z
- We can measure **expansion history**



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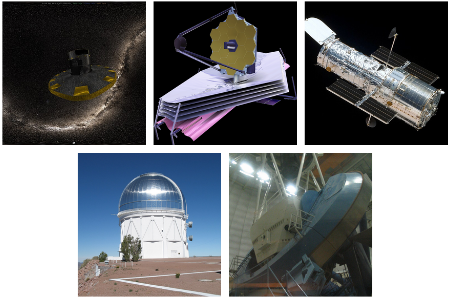
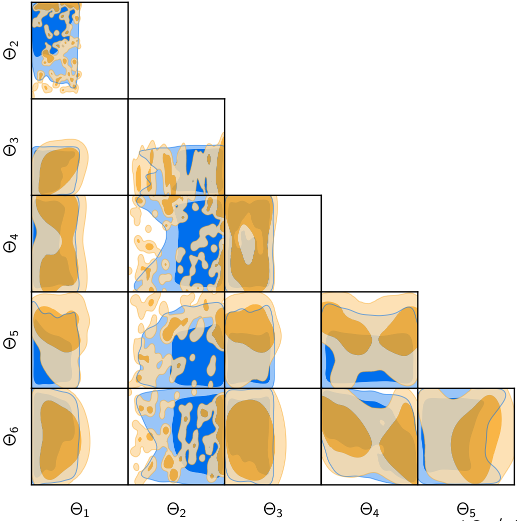
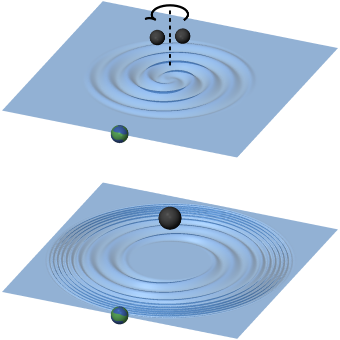
Numerical polology: dark energy constraints

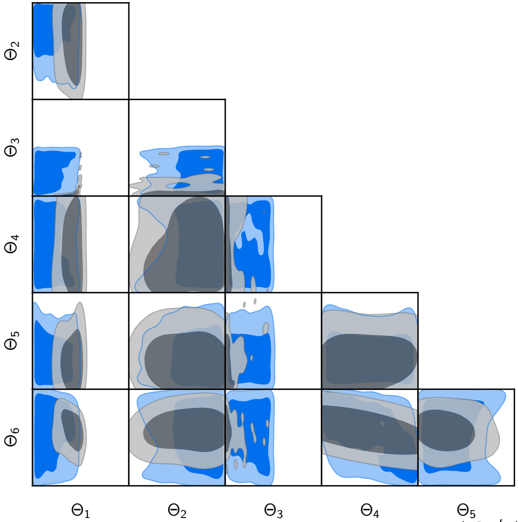


- Full picture is more complicated
- Calibrate supernovae in nearby galaxies
- Distant supernovae give $H(z \lesssim 2)$
- Distant galaxy correlations give $H(z \lesssim 3)$
- Techniques inspired by 2503.08658



Numerical polology: gravitational wave dispersion constraints





- Our theory also has a **massive graviton**!
- Gravitational waves **below** speed of light

Polology

- Think of the Lagrangian in **momentum space** representation of action:

$$\mathcal{L}(\phi;\, \theta) = \phi^{\dagger}(k) \cdot \mathsf{O}(\theta;\, k) \cdot \phi(k) + \mathcal{O}(\phi^3)$$

- Call $\mathsf{O}(\theta;\, k)$ the **wave operator**, then $\Pi(\theta;\, k) \equiv \mathsf{O}^{-1}(\theta;\, k)$ is the **propagator**

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- **Massive states** have **spin** and **parity**
- For a **scalar** field, a **vector** field, and a **symmetric tensor** field, the basis is:

$$\phi \sim 0^+, \quad \mathcal{A}_\mu \sim 1^- \oplus 0^+,$$
$$\mathcal{H}_{\mu\nu} \sim 2^+ \oplus 1^- \oplus 0^+ \oplus 0^+$$

- E.g. for a **rank-three** field:

$$\mathcal{C}_{\mu\nu\rho} \sim 3^- \oplus 2^+ \oplus 2^+ \oplus 2^+ \oplus 2^-$$
$$\oplus 2^- \oplus 1^+ \oplus 1^+ \oplus 1^+ \oplus 1^-$$
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- Wave operator may be **singular**:

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- Interpret as **gauge symmetries**:

$$\delta\phi \propto \mathrm{v} \implies \mathcal{L}(\phi + \delta\phi;\ \theta) = \mathcal{L}(\phi;\ \theta)$$

- Can invert **physical part** of $\mathrm{O}(\theta;\ k)$:

$$\underbrace{\begin{pmatrix} a & b & 0 \\ c & d & 0 \\ 0 & 0 & 0 \end{pmatrix}}_{\mathrm{O}} \to \underbrace{\frac{1}{ad - bc}\begin{pmatrix} d & -b & 0 \\ -c & a & 0 \\ 0 & 0 & 0 \end{pmatrix}}_{\mathrm{O}^{+}}$$

- (**Moore–Penrose pseudoinverse**)

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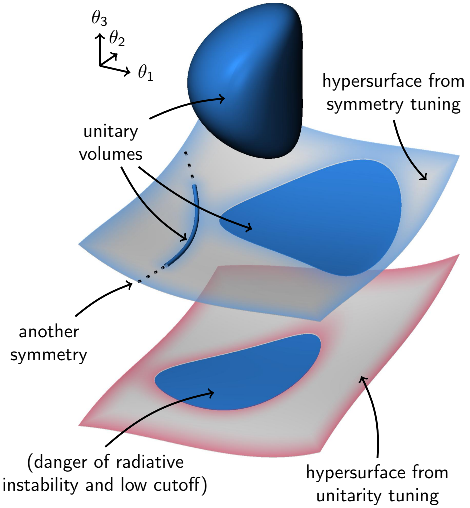

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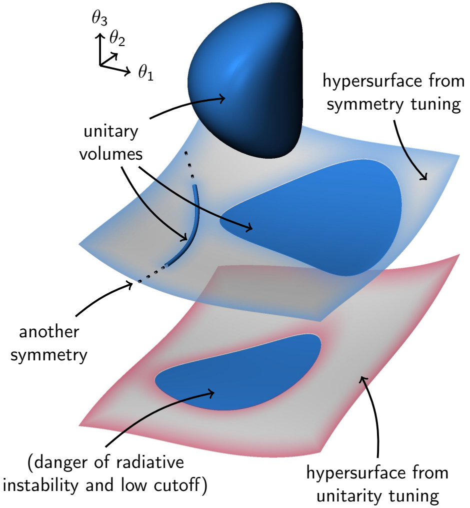
- Null vectors as **gauge symmetries**
- Symmetries protect against radiative corrections, and guide the EFT



Numerical polology: how it scales to unknown theories

- General vector theory:

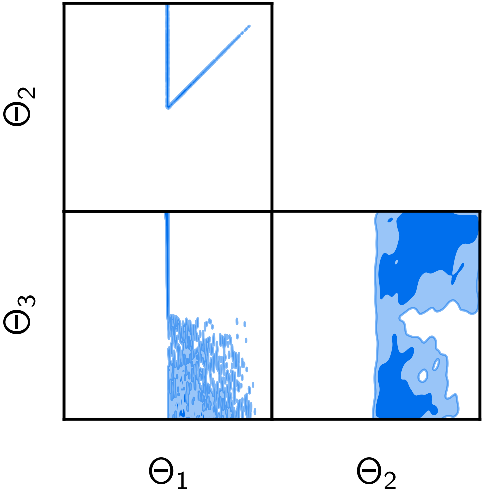
$$\mathcal{S}(\theta) = \int \mathrm{d}^4x \left[-\frac{1}{2}\theta_1 \partial_\mu \mathcal{A}_\nu \partial^\mu \mathcal{A}^\nu + \frac{1}{2}\theta_2 \partial_\mu \mathcal{A}^\mu \partial_\nu \mathcal{A}^\nu - \frac{1}{2}\theta_3 \mathcal{A}_\mu \mathcal{A}^\mu \right]$$

- Proca theory at $\theta_2 = \theta_1$:

$$\theta_1 > 0, \quad \theta_3 < 0$$

- Klein–Gordon theory at $\theta_1 = 0$:

$$\theta_2 > 0, \quad \theta_3 > 0$$



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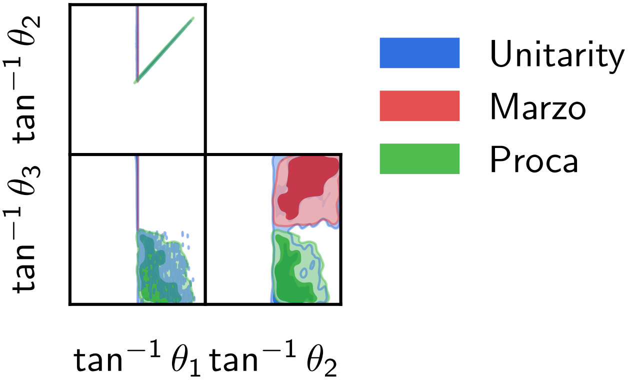

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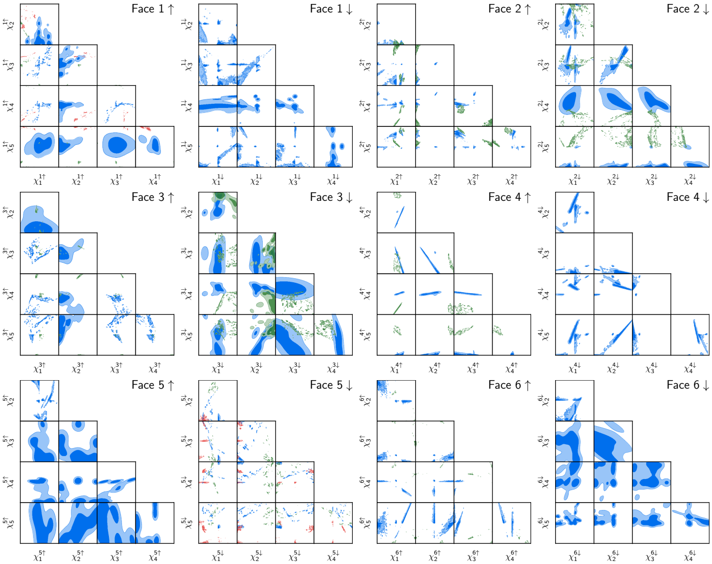

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Numerical polology: the full pipeline

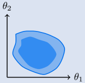
Basic HEP stack

- Unitarity
- Gauge symmetries
- Mass spectrum
- Spurion analysis

Basic pheno stack

- Non-interacting superradiance
- Pulsar timing arrays
- Background quintessence
- Graviton dispersion

Modern sampling



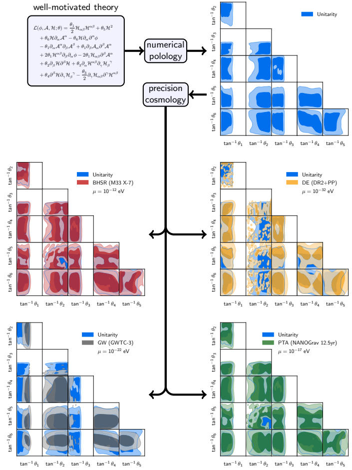
- θ-posterior — **always shrinking**
- Derived params — **always growing**

Advanced HEP stack

- Interaction ansatz
- Automated noether
- Dimensional regularisation with $\overline{\text{MS}}$
- Cutoff and renormalisation group

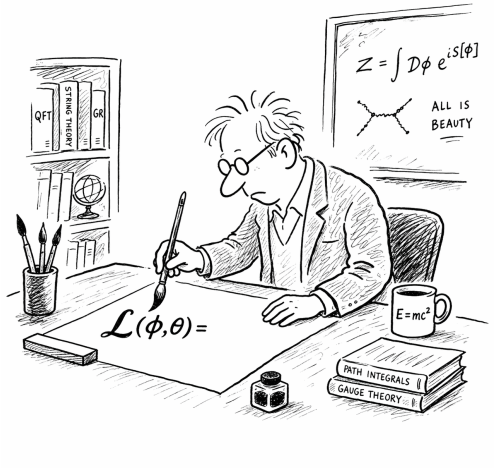
Advanced pheno stack

- Interacting superradiance
- Boltzmann integration
- Small-scale hydrodynamics
- Mode-mixing



Closing

- Aggressively mechanized model-building
- Computer algebra does not scale...
- ...but numerical alternatives do
- Theories must be subordinate to the observed phenomena
- Thank you for your attention!



Going, going, gone are the days...

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