Applications of Unitary & Analytic model

Describing the fundamental properties of particles

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Overview

Electromagnetic form factors (EM FF) of spin 0 and spin $\frac{1}{2}$ particles are fundamental quantities related to the charge and magnetization distribution inside the particle. In the limit of Q^2 goes to zero, the slope of the charge form factor determines the charge radius of the corresponding particle.

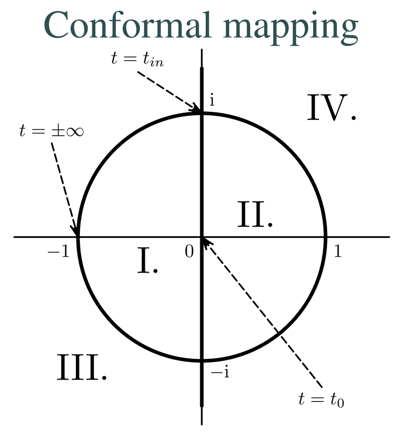
Understanding the electromagnetic structure of particles in the terms of their correct parameterization as EM form factors and uncertainties help us to study the data from polarized or unpolarized scattering and annihilation experiments. In the recent time the Unitary & Analytic (U&A) model was successfully applied for various problems in the field of phenomenology of particle physics.

Construction of U&A model

- $\mathrm{F}(t)$ – analytic function in the whole complex t-plane besides the cuts, e. g., $4m_\pi^2 \to \infty$
- $|\mathrm{F}(t)|_{t\to\infty} \sim t^{n_q-1}$ – asymptotic behavior of FF (n_q – number of quarks)
- Normalization in $t=0$, reality and unitarity conditions.
- All experimental data information in space-like and time-like regions.
- The experimental fact of creation of unstable vector meson resonances in the $\mathrm{e}^+\mathrm{e}^-$ annihilation processes into hadrons.

Pion U&A model

Conformal mapping

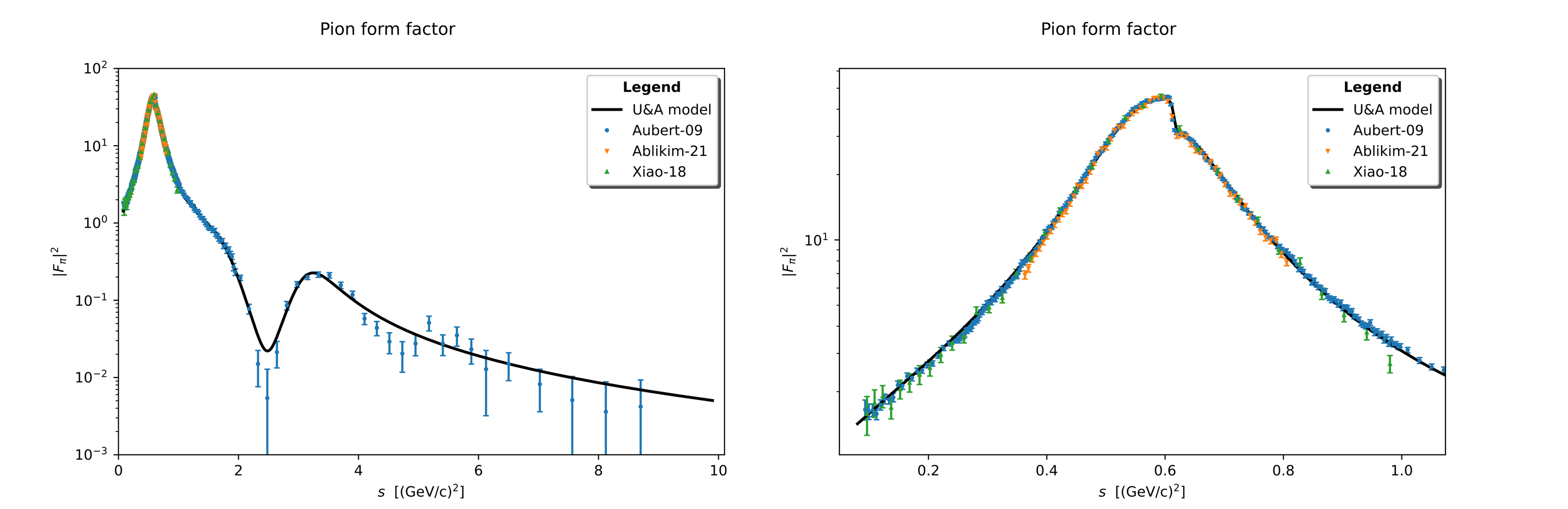


$$\mathrm{F}[W(t)] = \left(\frac{1-W^2}{1-W_N^2}\right)^2 \frac{(W-W_Z)(W_N-W_P)}{(W_N-W_Z)(W-W_P)}$$
$$\times \Big\{ \frac{(W_N-W_\rho)(W_N-W_\rho^*)}{(W-W_\rho)(W\_W_\rho^*)} \frac{(W_N-1/W_\rho)(W_N-1/W_\rho^*)}{(W-1/W_\rho)(W-1/W_\rho^*)} \left(\frac{f_{\rho\pi\pi}}{f_\rho}\right)$$
$$+ \sum_{v=\rho',''} \Big[\frac{(W_N-W_v)(W_N-W_v^*)}{(W-W_v)(W\_W_v^*)} \frac{(W_N+W_v)(W_N+W_v^*)}{(W+W_v)(W+W_v^*)} \Big] \left(\frac{f_{v\pi\pi}}{f_v}\right) \Big\},$$

$$0 < W_{Z,P} < 1, \quad W_N \ldots t=0, \quad \left(\frac{f_{\rho'\pi\pi}}{f_{\rho'}}\right), \left(\frac{f_{\rho''\pi\pi}}{f_{\rho''}}\right) \sim \left(\frac{f_{\rho\pi\pi}}{f_\rho}\right)$$

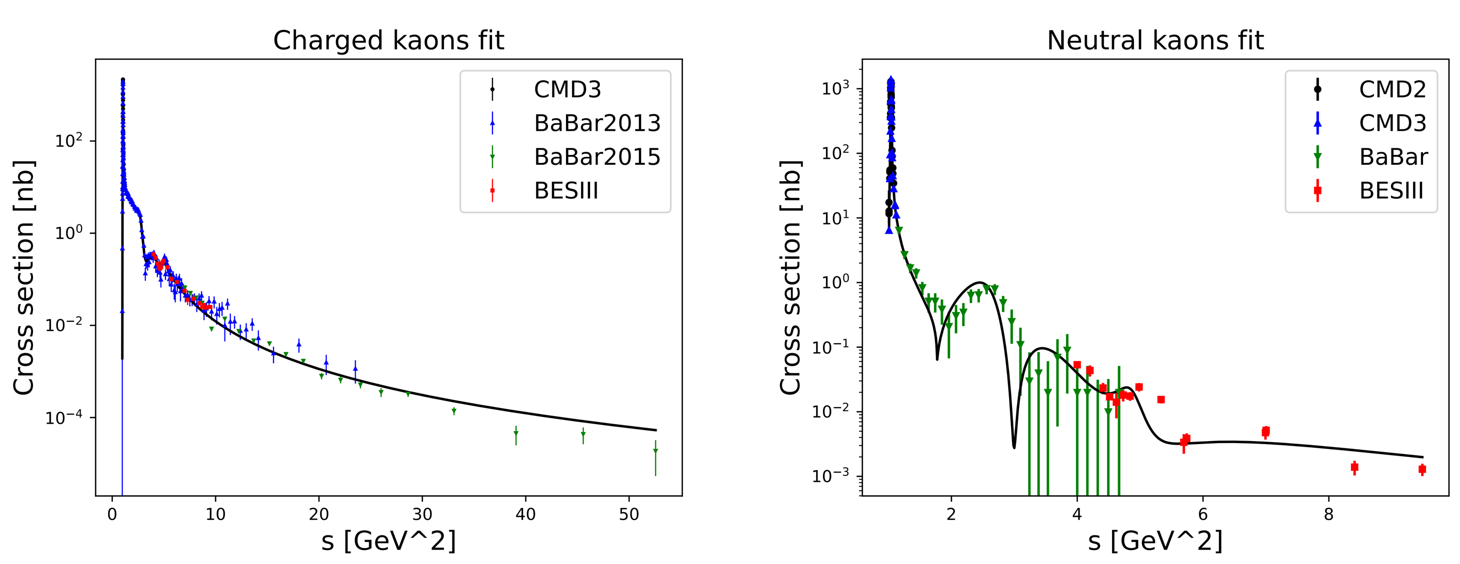
$$W(t) = \mathrm{i}\frac{\sqrt{q_{in}+q}-\sqrt{q_{in}-q}}{\sqrt{q_{in}+q}+\sqrt{q_{in}-q}},$$
$$q = \sqrt{\frac{t-t_0}{4}},\ q_{in} = \sqrt{\frac{t_{in}-t_0}{4}}$$

10 parameters: $t_{in}, m_\rho, \Gamma_\rho, m_{\rho'}, \Gamma_{\rho'}, m_{\rho''}, \Gamma_{\rho''}, f_{\rho\pi\pi}/f_\rho, W_Z, W_P$



Pion fit of quadratic value of FF, 11-parametric U&A model, $\chi^2 = 1.753$. The model was used in the search for dumped oscillating structure for charged pion electromagnetic FF which were not confirmed.

Charged and neutral Kaons



K^0-meson and $K^\pm$-meson (mean squared) charge radius

$\langle r^2 \rangle = 6\frac{\mathrm{d}F(s)}{\mathrm{d}s}\Big|_{s=0}$ values compared with other methods

Charge radius [fm^2]	PDG	U&A method	Dispersion method
$\langle r^2 \rangle_{\pi^\pm}$	$0.434(5)$	$0.428(10)$	—
$\langle r^2 \rangle_{K^0}$	$-0.077(10)$	$-0.123(10)$	$-0.060(4)$
$\langle r^2 \rangle_{K^\pm}$	$0.314(35)$	$0.403(5)$	$0.359(3)$

Corrections to Dashen's theoreme

EM mass difference for kaons: $(\Delta M_K^2)_{\mathrm{EM}} = (M_{K^\pm}^2 - M_{K^0}^2)_{\mathrm{EM}}$ in the formula for the EM corrections to Dashen's theoreme

$$\epsilon = \frac{(\Delta M_K^2)_{\mathrm{EM}}}{(\Delta M_\pi^2)_{\mathrm{EM}}} - 1.$$

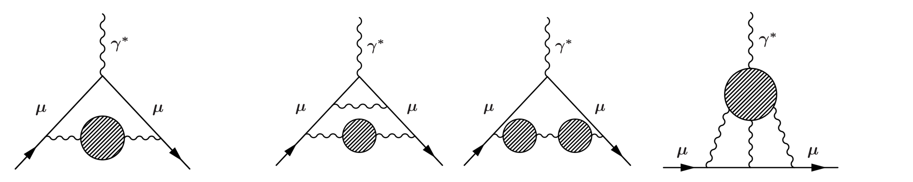
While $(\Delta M_\pi^2)_{\mathrm{EM}} = (M_{\pi^\pm}^2)_{\mathrm{EM}}$, EM contributions of K-mesons can be estimated via the Cottingham formula, which connects the EM self energies and the forward Compton tensor

$$(M_P^2)_{\mathrm{EM}} = \frac{\alpha}{8\pi}\int_0^\infty \mathrm{d}s \big[F_P(-s)\big]^2 \Big(4W + \frac{s}{M_P^2}(W-1)\Big), \quad W = \sqrt{1+4M_P^2/s}$$

[GeV^2]	U&A method	Dispersion method
$(M_{K^\pm}^2)_{\mathrm{EM}}$	$1.99(3)\times10^{-3}$	$2.12(18)\times10^{-3}$
$(M_{K^0}^2)_{\mathrm{EM}}$	$4.3(11)\times10^{-5}$	$7(17)\times10^{-6}$
$(\Delta M_K^2)_{\mathrm{EM}}$	$1.95(3)\times10^{-3}$	$2.12(18)\times10^{-3}$
$(\Delta M_\pi^2)_{\mathrm{EM}}$	$1.18(2)\times10^{-3}$	$1.3(3)\times10^{-3}$
ϵ	$0.65(6)$	$0.63(40)$

Muon anomaly $a_\mu = (g-2)/2$

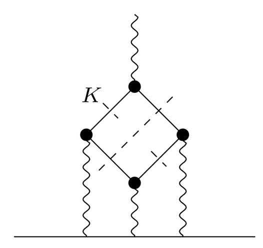
$$a_\mu^{\mathrm{SM,WP25}} = 116\,592\,033(62)\times10^{-11}$$
$$a_\mu^{\mathrm{SM,UA}} = 116\,591\,964(48)\times10^{-11}$$



$$a_\mu^{\mathrm{SM}} = a_\mu^{\mathrm{QED}} + a_\mu^{\mathrm{EW}} + a_\mu^{\mathrm{(LO)had}} + a_\mu^{\mathrm{(HO)had}} + a_\mu^{\mathrm{(LBL)had}}$$

The evaluation of muon anomaly calculated by novel approach and with U&A model of pseudoscalar meson nonet deviates from White Paper (WP25) value by $\sim 0.88\,\sigma$.

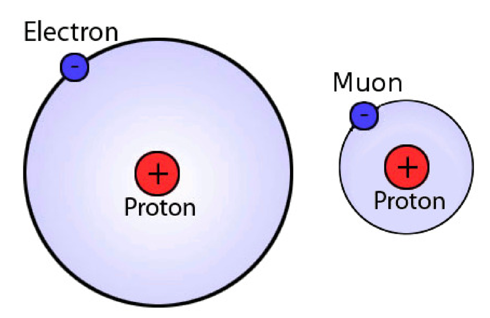
Box contribution to the hadronic light-by-light scattering



	U&A method	Dispersion method
$a_\mu^{K^\pm\text{–box}}$	$-0.451(22)\times10^{-11}$	$-0.484(11)\times10^{-11}$
$a_\mu^{K^0\text{–box}}$	$-4.6(14)\times10^{-15}$	$-0.50(45)\times10^{-15}$
$a_\mu^{\pi^\pm\text{–box}}$	$-16.0(2)\times10^{-11}$	$-15.9(2)\times10^{-11}$

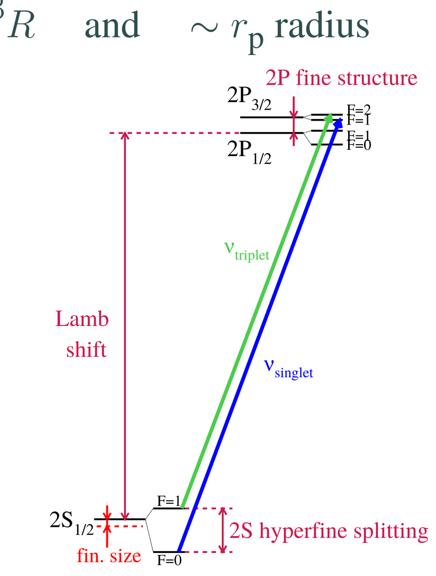
Proton

Size of proton – value of proton charge radius



Lamb shift: splitting $2S_{1/2} - 2P_{1/2}$

$\sim \alpha^3 R$ and $\sim r_p$ radius



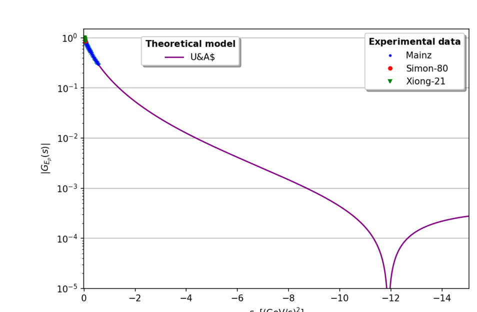
$m_\mu = 105.658\,371\,5$ MeV $\sim 200\,\mathrm{m_e}$

$\sim 200\times$ smaller Bohr orbits

$\sim 200^3\times$ larger overlap of wave functions

$r_p = 0.841\,84(67)$ fm	Lamb shift
$r_p = 0.8510(21)$ fm	U&A model
$r_p = 0.895(18)$ fm	CODATA

Existence of a zero in proton elastic form factor



Illustrative plot of electric Sachs form factor of proton $G_E^p(s)$ demonstrating the possible zero in the finite value of s.

Repeated investigations by U&A model with nine definitely experimentally confirmed vector-meson resonances present clear arguments in favor of the existence of the zero of $G_E^p(s)$ in the spacelike region.

Forthcoming Development

https://github.com/Mezek/datalock – World experimental data for selected physical processes.

https://github.com/Mezek/bvpasion – Theoretical models of form factors for elementary particles.

References

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[2] Erik Bartoš, Stanislav Dubnička, Anna Zuzana Dubničková, and Lukáš Holka. Prediction of Some Meson Physical Quantities by Analytic Continuation from Time-Like to Space-Like Region. *Int. J. Theor. Phys.*, 64(12):324, 2025.

[3] A. Z. Dubničková, S. Dubnička, Lukas Holka, and A. Liptaj. Advantages of a Novel Approach to the Numerical Evaluation of the Muon g – 2 Anomaly. *Phys. Part. Nucl.*, 53(4):811–814, 2022.

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