



SIM NS
FOUNDATION

The Role of Bell Turbulence Driven by Proton Cosmic Rays on Electron Acceleration at Oblique Shocks

Karol Fułat and Ellen Zweibel

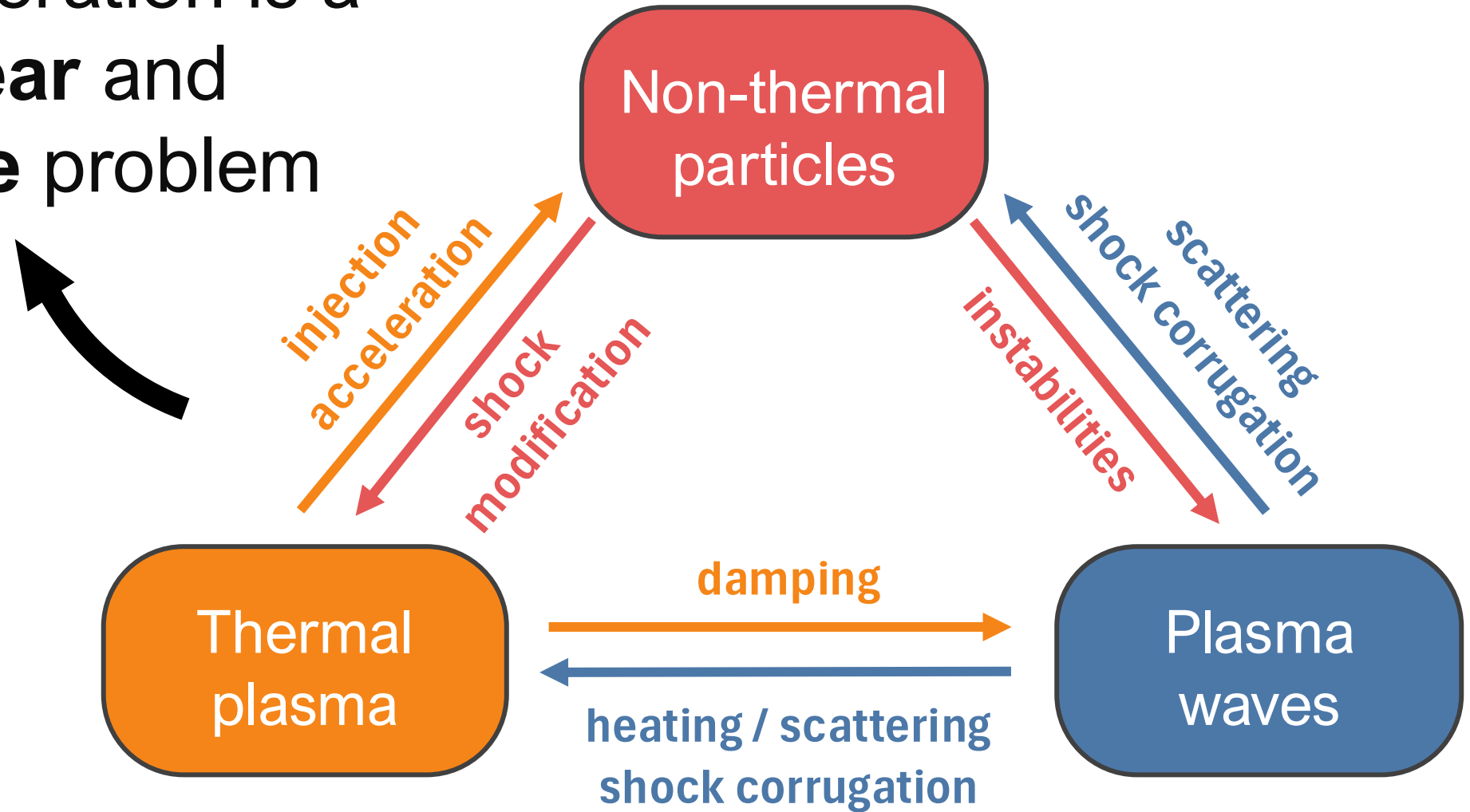
University of Wisconsin-Madison

Simons Collaboration on Extreme

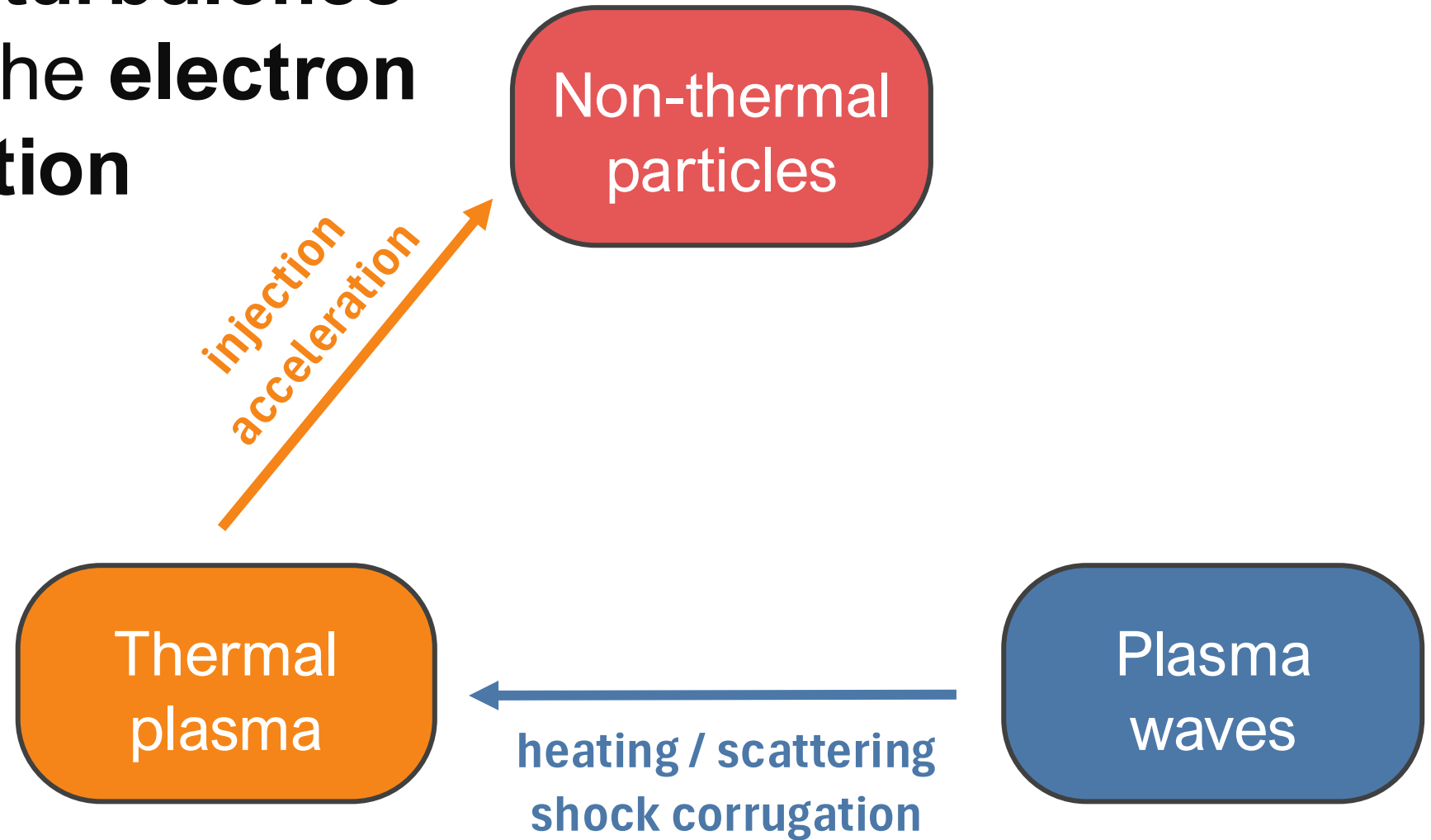
Electrodynamics of Compact Sources (SCEECS)

29 September 2026

Shock acceleration is a
non-linear and
multi-scale problem



Large-scale turbulence influence on the **electron** injection



e.g., Vladimirov et al. 2008, Bykov et al 2014

Scales of PIC simulations

Time steps shorter than any characteristic frequencies in the plasma

$$\omega_{pe}^{-1} \simeq 10^{-4} \text{ s (ISM)}$$

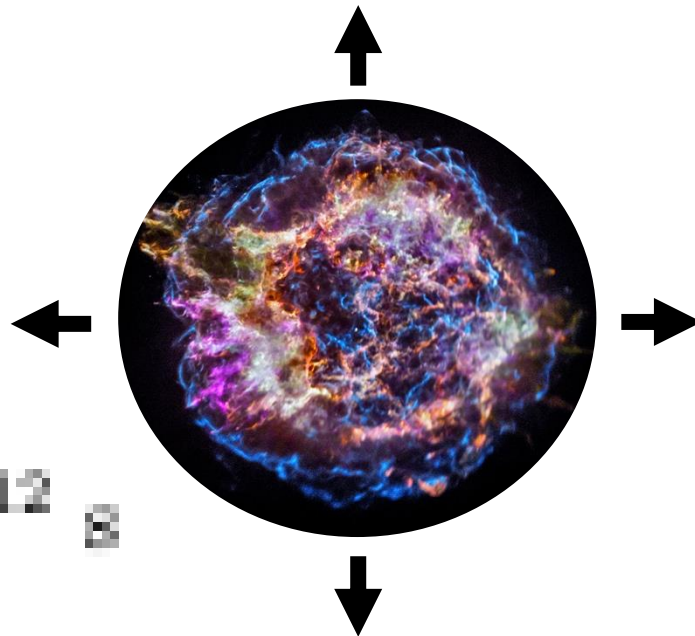
$$\delta t \sim 0.1 \omega_{pe}^{-1} \simeq 10^{-5} \text{ s}$$

$$t_{\text{sim}} \sim 10 \text{ s}$$

Grid discretization effects require to resolve electron scales

$$\lambda_{se} = \frac{c}{\omega_{pe}} \simeq 10^3 \text{ m (ISM)}$$

$$L_{\text{sim}} \sim 10^3 \lambda_{se} \approx 10^6 \text{ m}$$



Global PIC not possible,
so how to **improve** them?

Incorporating macroscopic effects into PIC

Macroscopic phenomena set the local conditions
→ How to include effects on scales larger than electron kinetic scales in PIC simulations?

Example: turbulence driven at MHD scales that cascades to kinetic scales

1. STIR PLASMA IN A PERIODIC BOX

particle velocity:

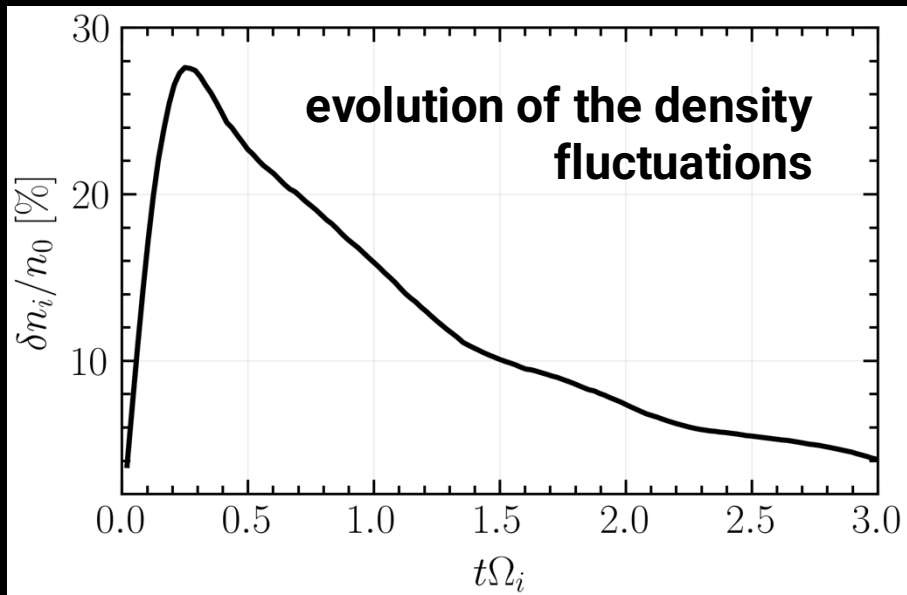
$$\mathbf{v} = \mathbf{v}_b + \delta \mathbf{v}_b + \mathbf{v}_{th}$$

↓ bulk ↓ thermal

local bulk disturbance
(compressive and shear waves)



2. DECAYING COMPRESSIVE TURBULENCE



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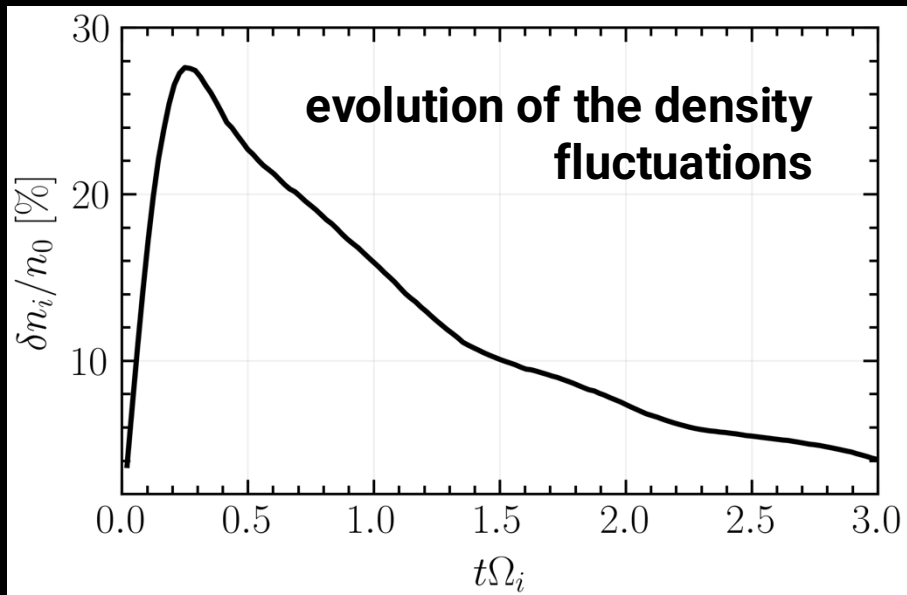
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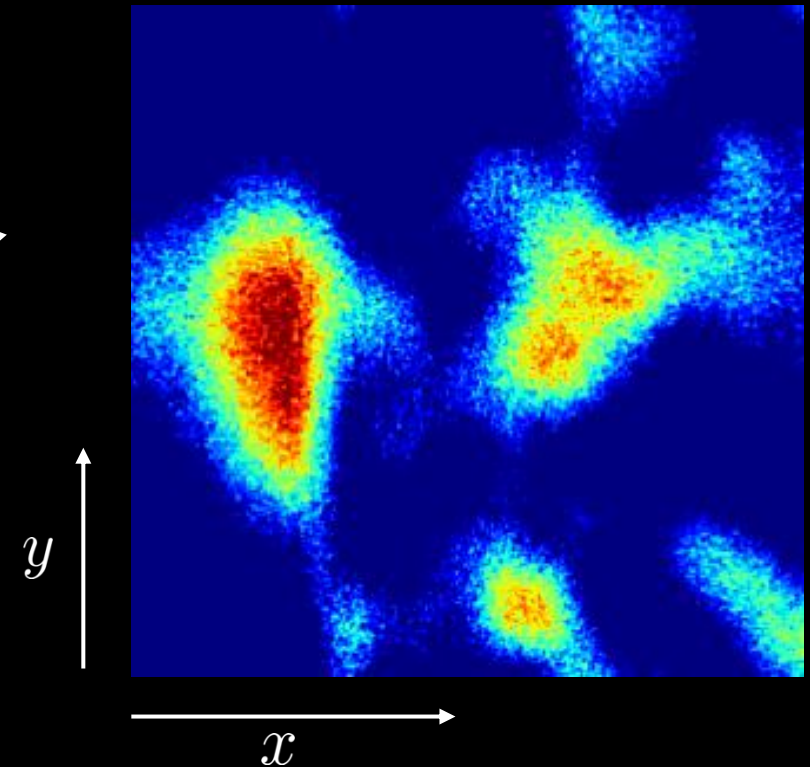
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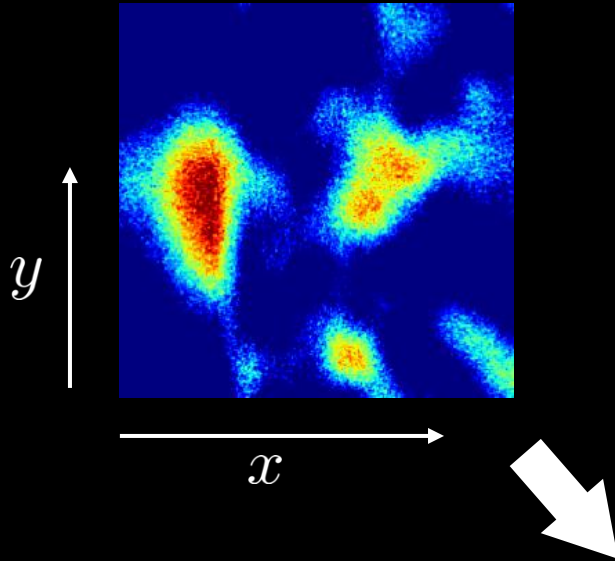
2. DECAYING COMPRESSIVE TURBULENCE



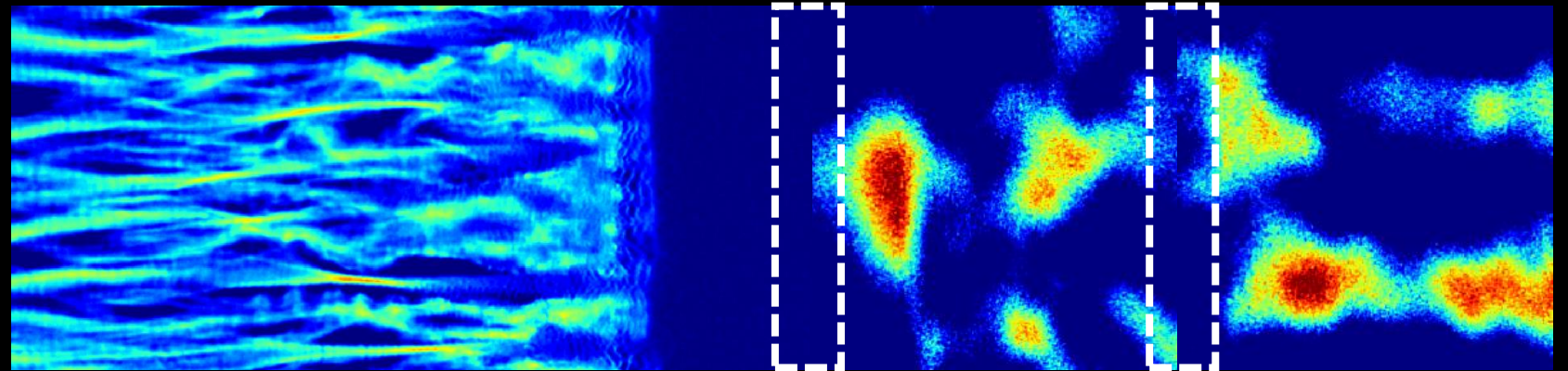
3. TURBULENT PLASMA SLAB



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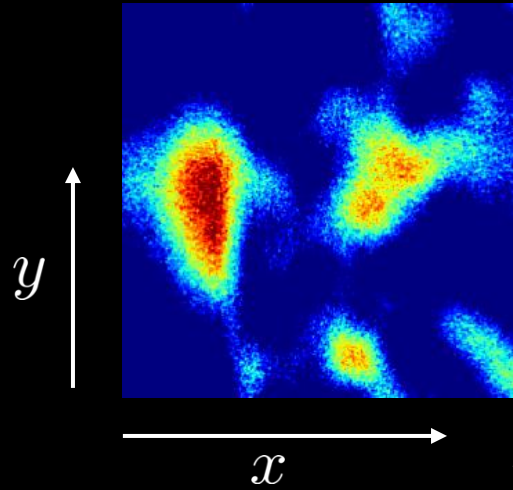
4. SLAB INJECTION INTO A SHOCK SIMULATION



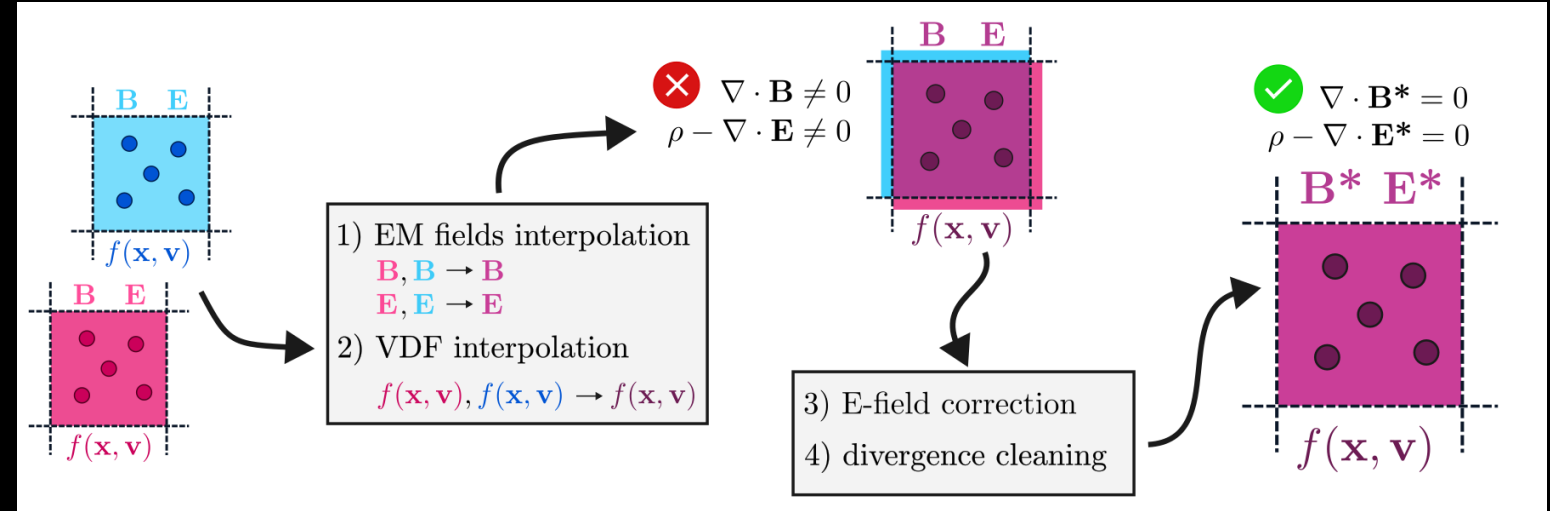
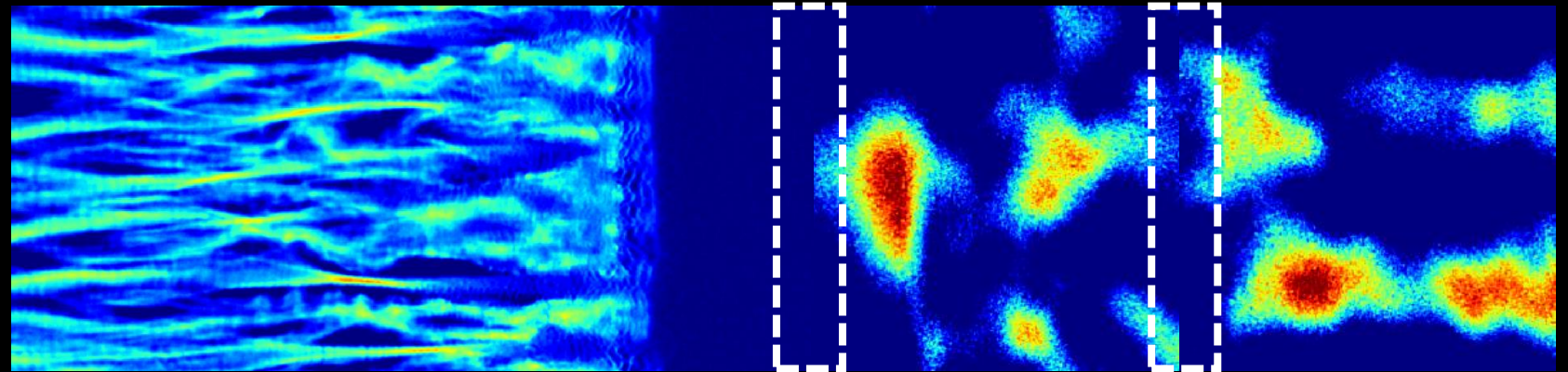
5. STITCH PLASMA AT THE INTERFACE

Fulat et al. 2023

3. TURBULENT PLASMA SLAB



4. SLAB INJECTION INTO A SHOCK SIMULATION



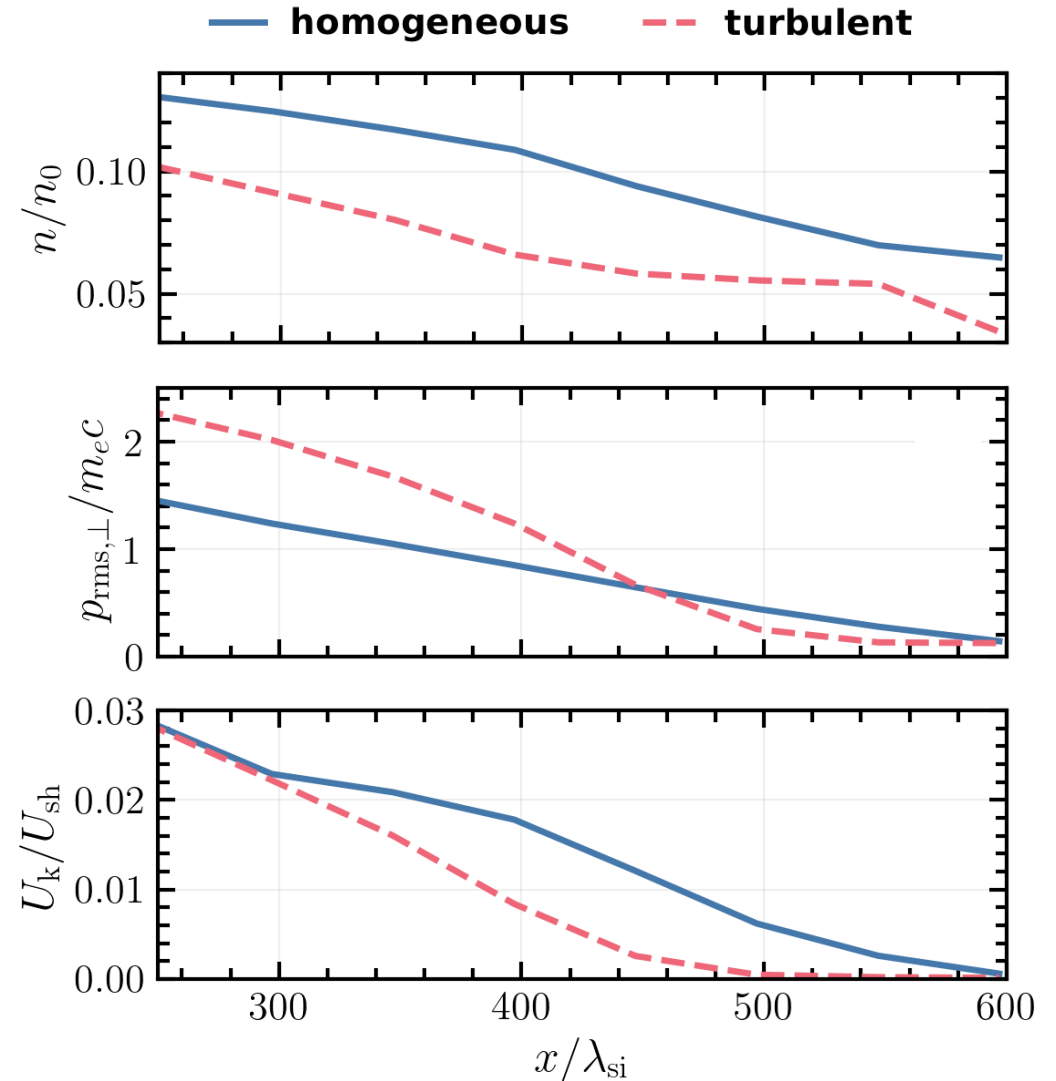
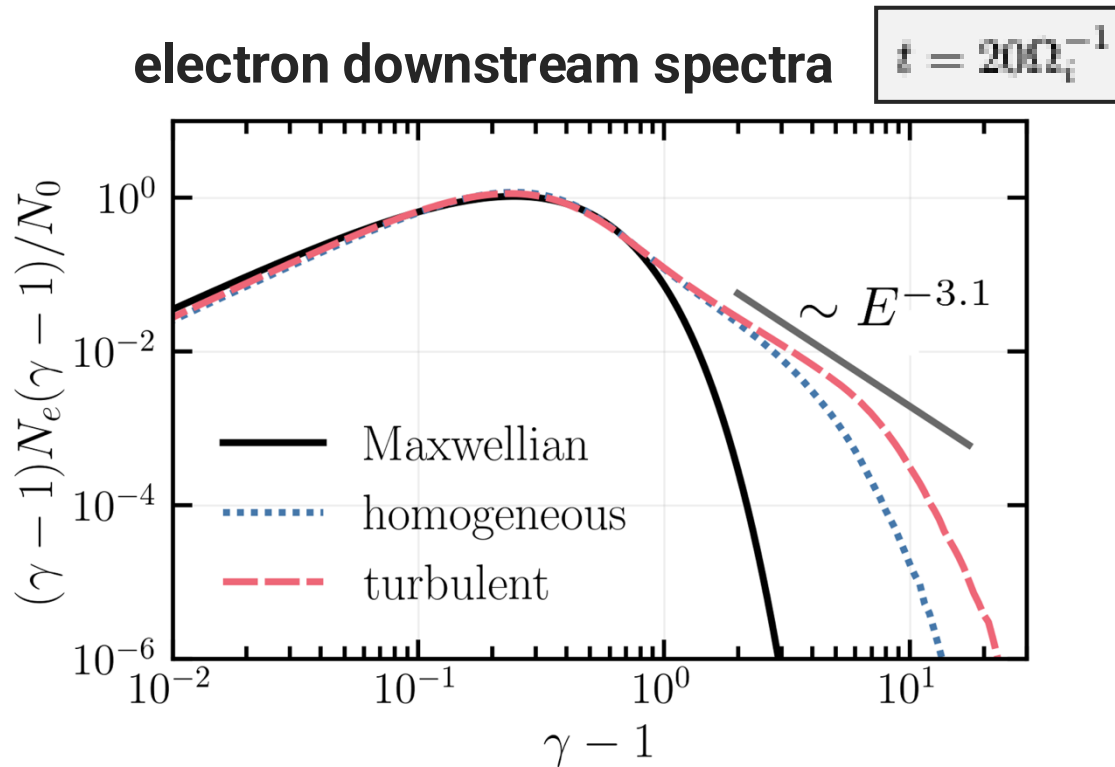
Oblique shocks

$$M_s, M_A \sim 30$$

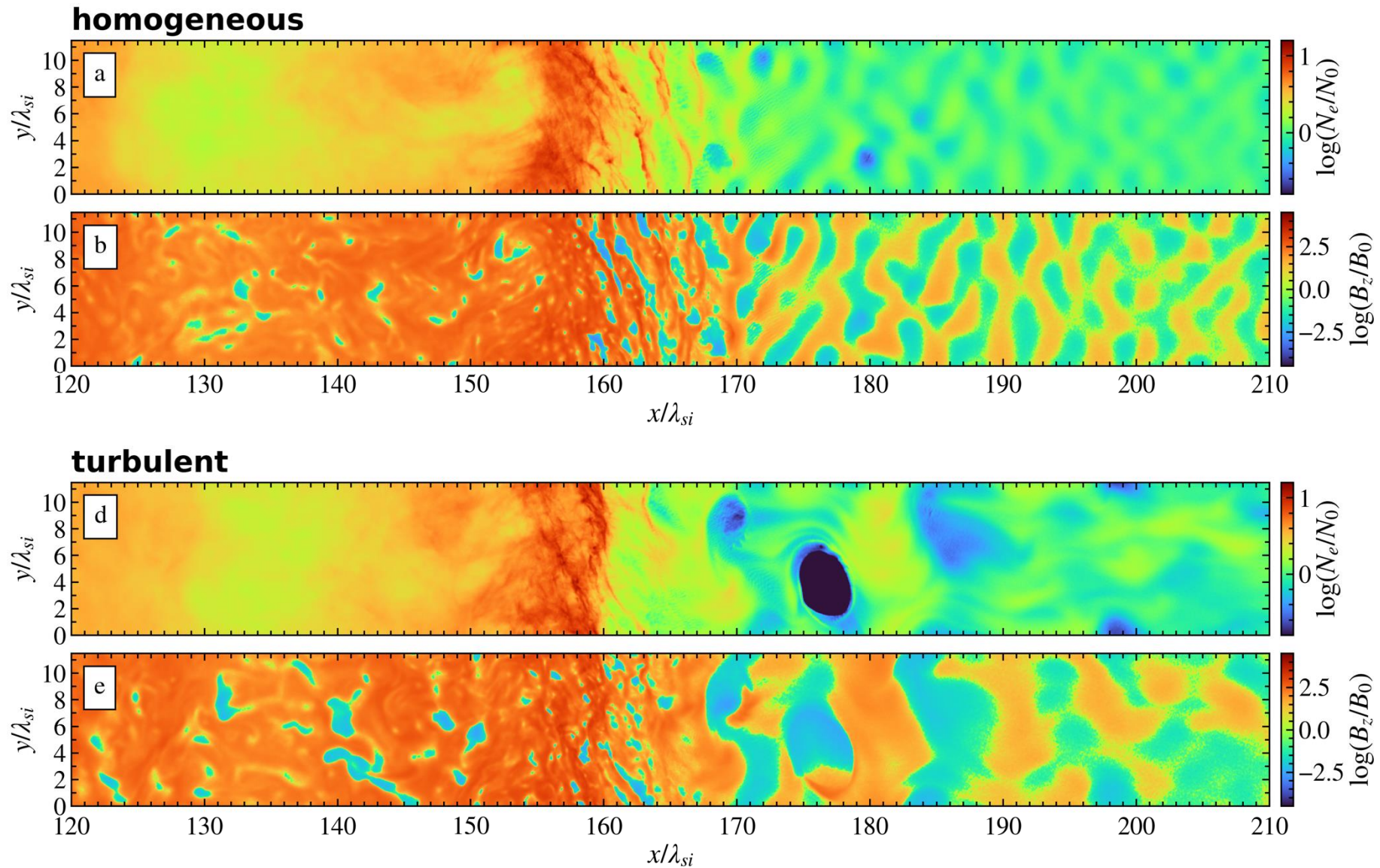
$$\theta_{Bn} = 60^\circ$$

Turbulent: less numerous but hotter reflected electrons

Reflected electrons drive instabilities
 → impact of the turbulence on the instabilities and then **particle scattering**
 → **more efficient acceleration**



Not much influence on the shock structure

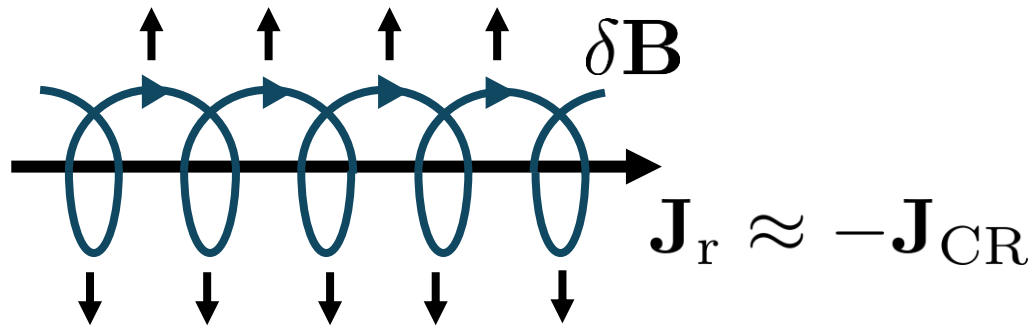


What if turbulence affects the shock structure?

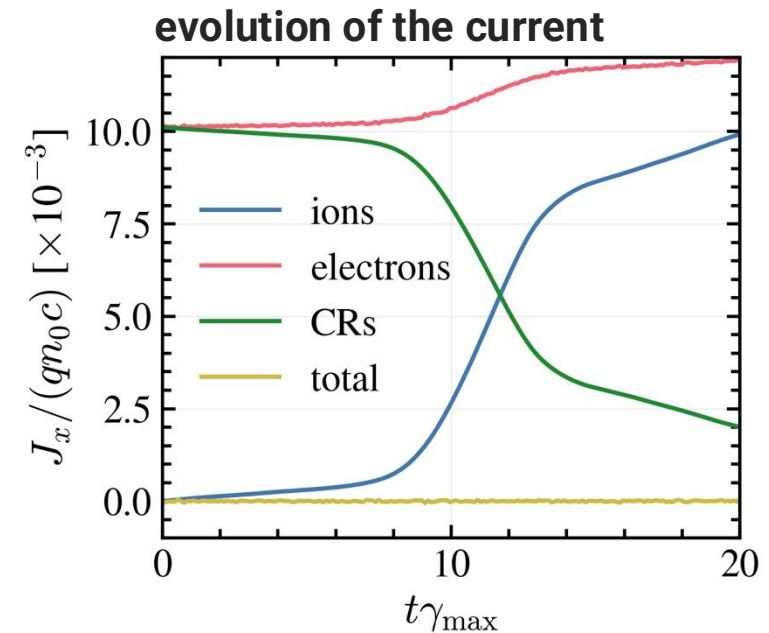
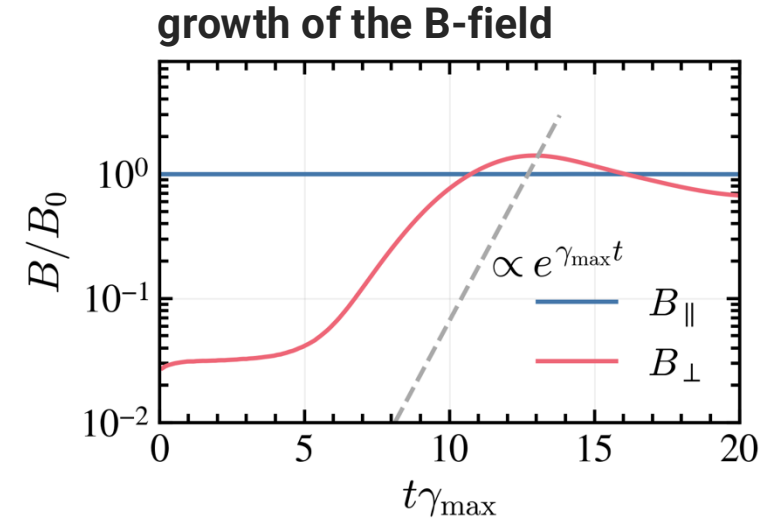
Bell instability

Bell 2004, 2005

CRs are streaming with a velocity larger than the Alfvén speed, the background plasma reacts to reduce the speed. The instability leads to growth of modes, that are **non-resonant** with the beam.



The background plasma experiences the $\mathbf{E} \times \mathbf{B}$ drift, which reduces the relative drift between CRs and the plasma.



e.g., Riquelme and Spitkovsky 2009,
Stroman et al. 2009, Ohira et al. 2009

Bell instability at shocks in PIC

Bell instability

$$\frac{\gamma_{\max}}{\Omega_i} = k_{\max} \lambda_{\text{si}} \simeq \frac{1}{2} M_A \left(\frac{n_{\text{CR}}}{n_0} \right)$$

$$\gamma_{\max} / \Omega_i = k_{\max} \lambda_{\text{si}} \propto J_{\text{CR}}$$

For SNR shocks

$$n_{\text{CR}} / n_0 \sim 10^{-3} \quad M_A \sim 10^2$$

$$t \sim 10 / \gamma_{\max} \sim 200 \Omega_i^{-1}$$

For realistic parameters resolving
Bell is challenging for PIC

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For realistic parameters resolving
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Saturation due to magnetic tension

$$\frac{|\nabla \times \mathbf{B}|}{\mu_0} \simeq \frac{kB}{\mu_0} \sim J_{\text{CR}}$$

so B-field on scale $L = k^{-1}$ saturates at

$$B \approx \mu_0 J_{\text{CR}}/k$$

$$L \sim r_{g,\text{CR}}$$

$$B^2/B_0^2 \propto \gamma_{\text{CR}} J_{\text{CR}}$$

Numerical setup – periodic box simulation

Niemiec+2008, Stroman+2009

Magnetic field $\mathbf{B}_0 = B_0 \hat{\mathbf{x}}$

Streaming **CRs** (isotropic and monoenergetic in their rest frame) $n_{\text{CR}}/n_0, \mathbf{v}_{\text{CR}} = v_{\text{CR}} \hat{\mathbf{x}}, \gamma_{\text{CR}}$

Background **ions** are at rest (maxwellian) $n_0, v_{th,i}$

Background **electron** drift
(maxwellian at rest) $n_e = n_0 + n_{\text{CR}}, \mathbf{v}_d = \mathbf{v}_{\text{CR}} n_{\text{CR}}/n_e, v_{th,e}$

To improve statistics each CR (and an associated electron) is splitted into several particles with weights

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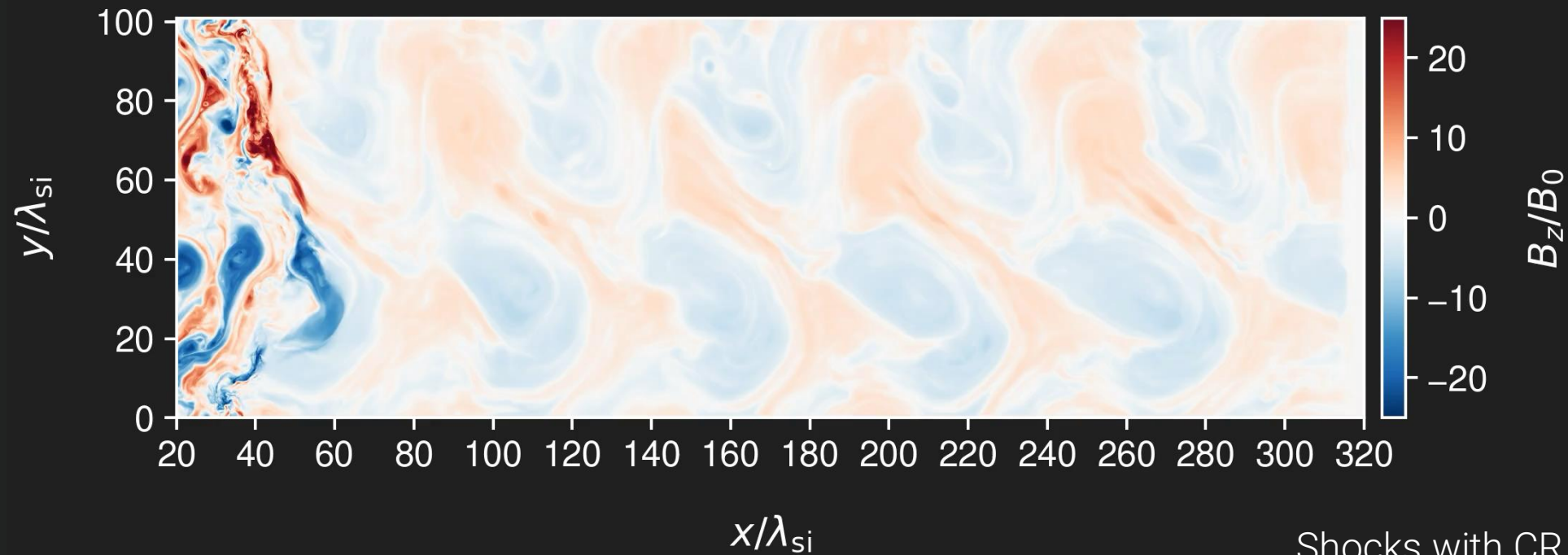
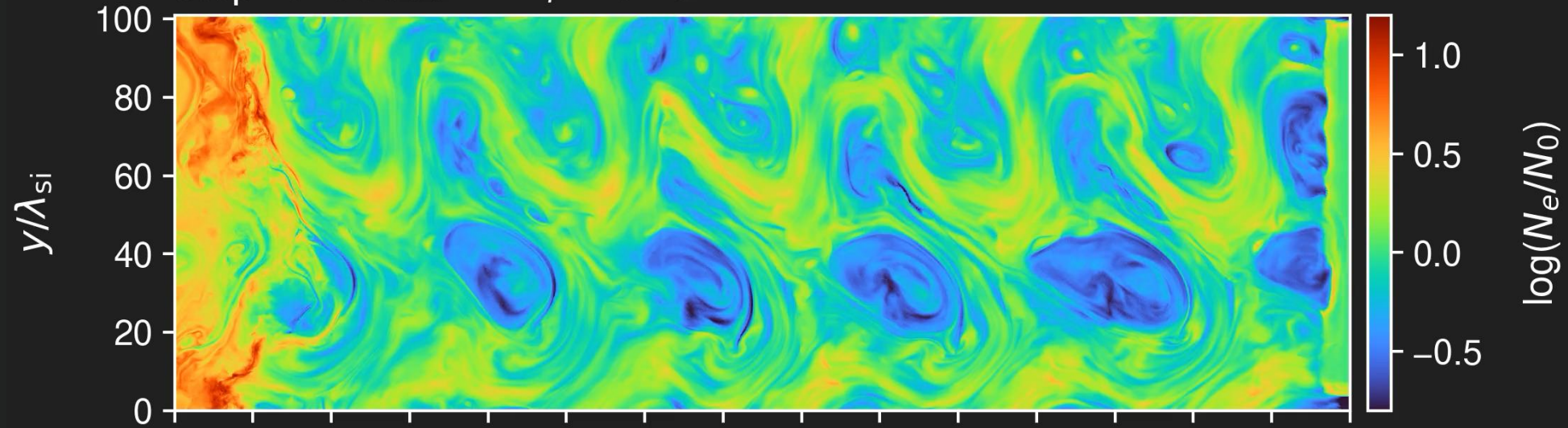
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**WE DO THIS IN THE
DOWNSTREAM FRAME**

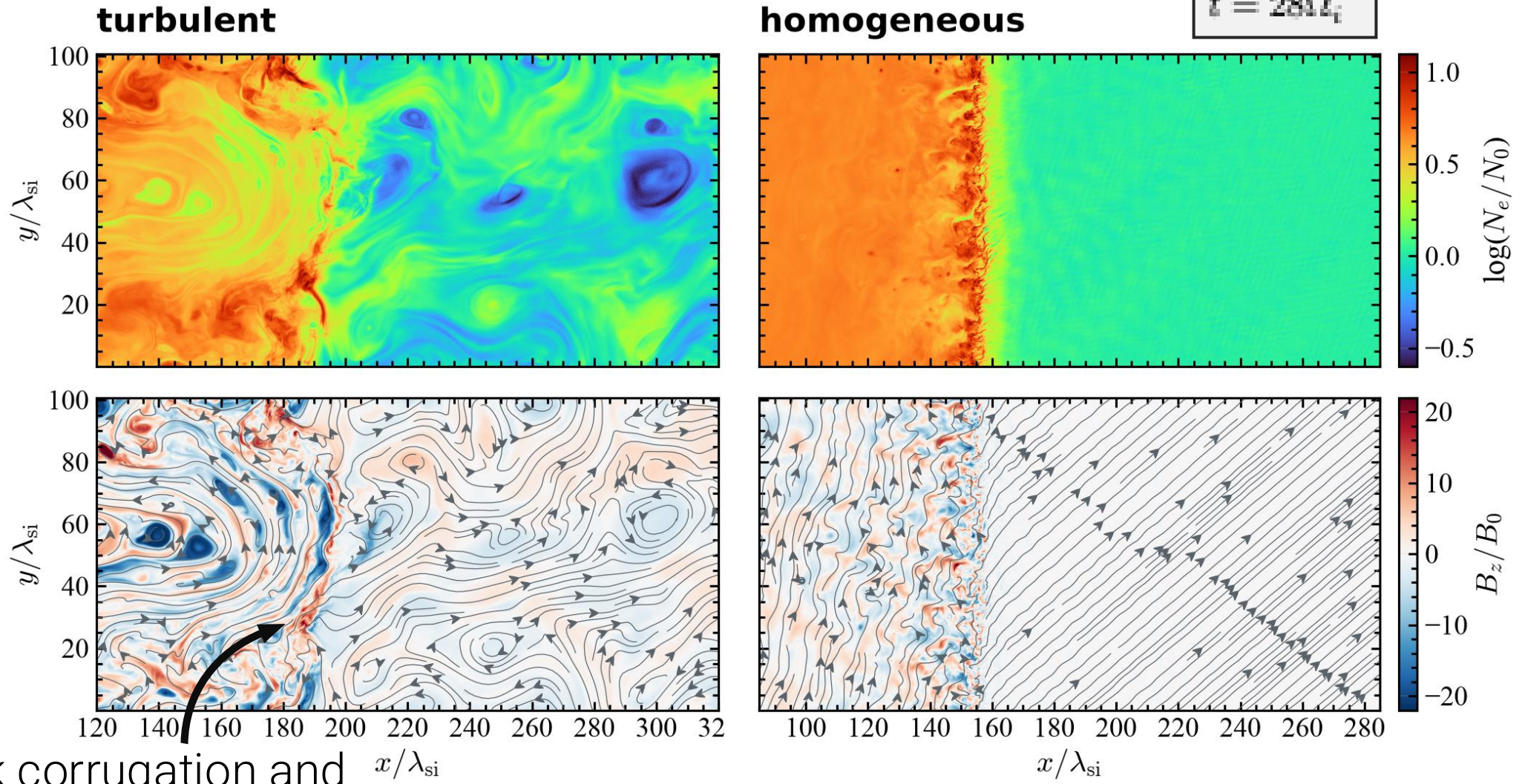
step=0100000 $t\Omega_i = 6.10$

$M_S/M_A \sim 30$ $\theta_{Bn} = 40^\circ$ (in-plane) $\beta = 1$ $m_i/m_e = 100$



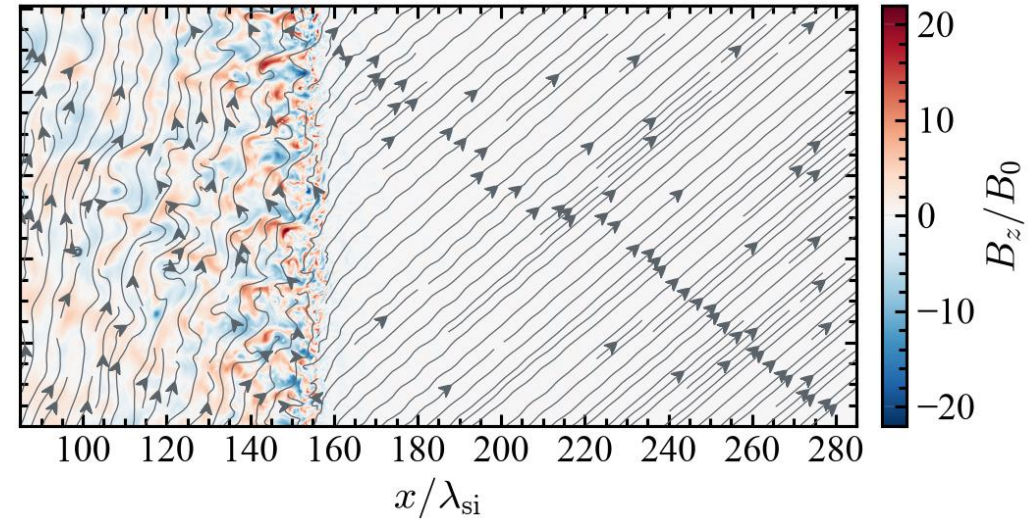
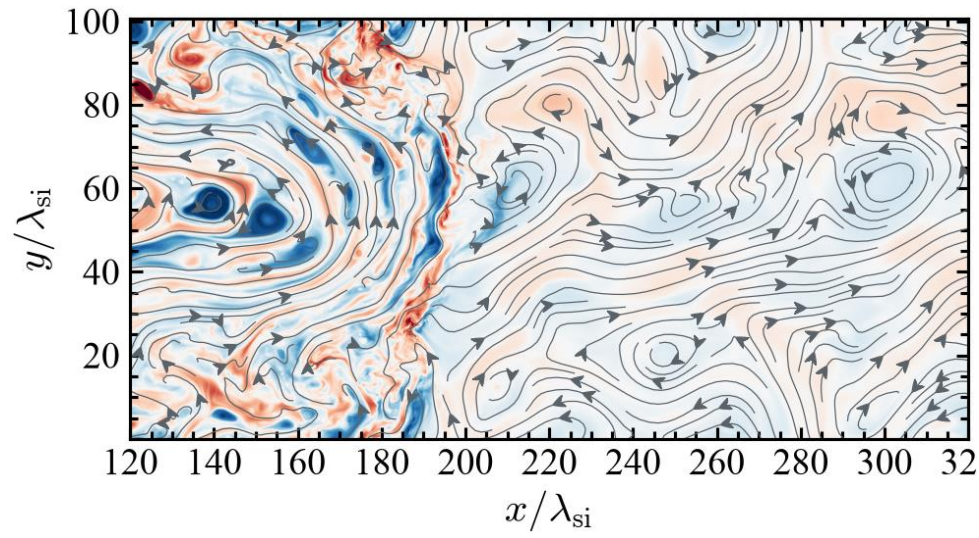
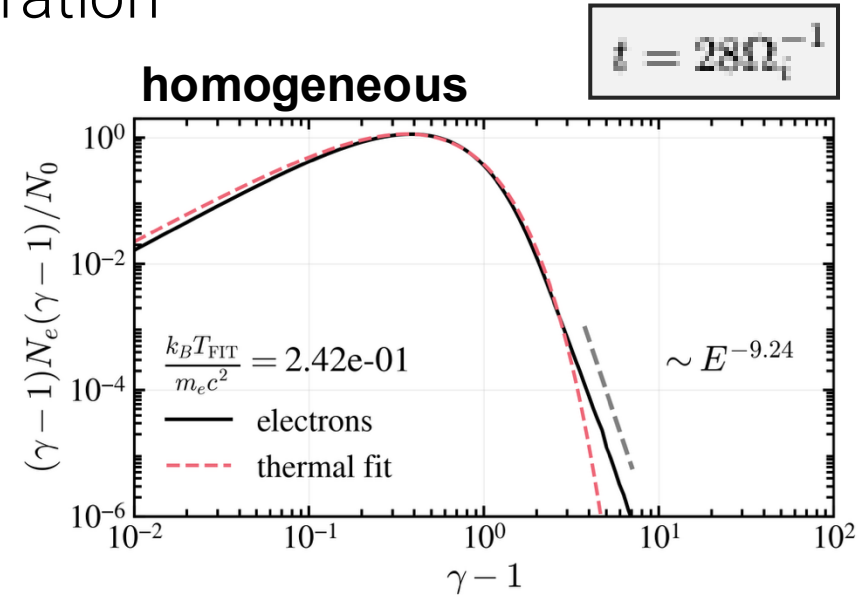
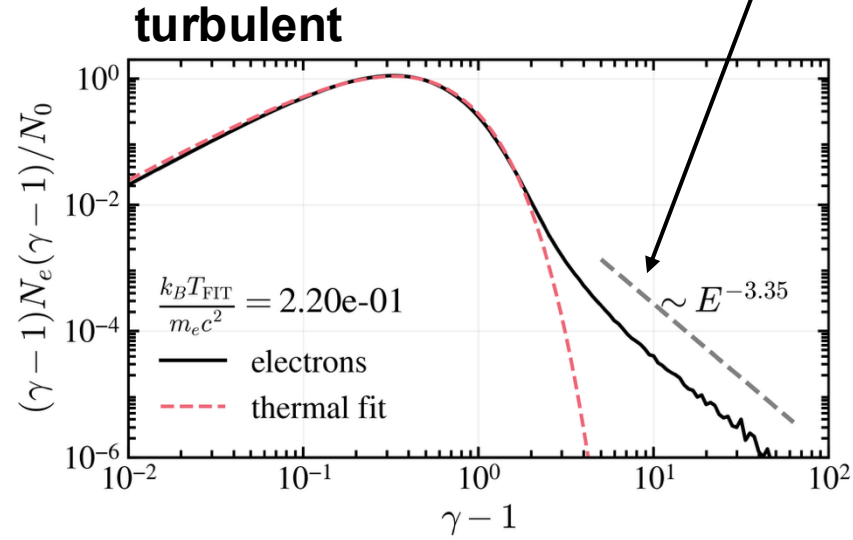
$$M_S/M_A \sim 30 \quad \theta_{Bn} = 40^\circ \quad \beta = 1 \quad m_i/m_e = 100$$

$$t = 28\Omega_i^{-1}$$

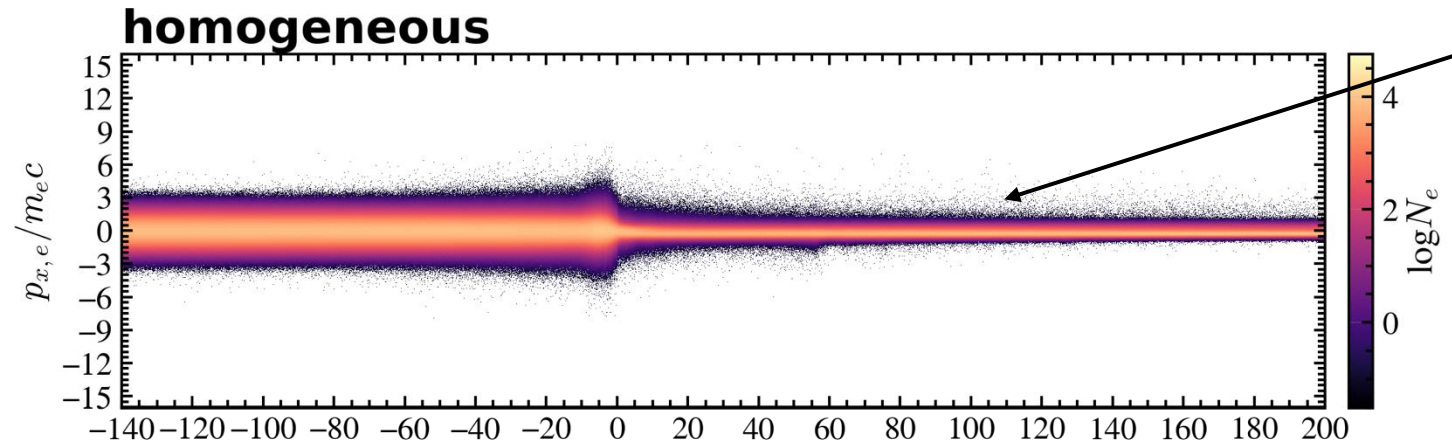


Shock corrugation and
local obliquity variations

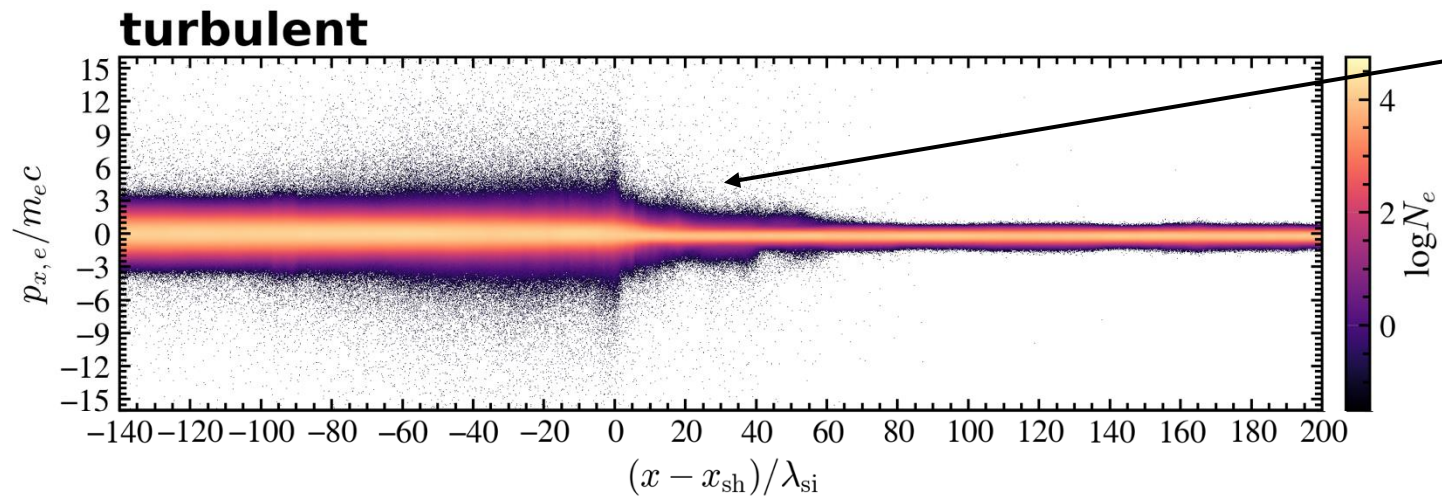
Efficient acceleration



Electron phase-space distribution

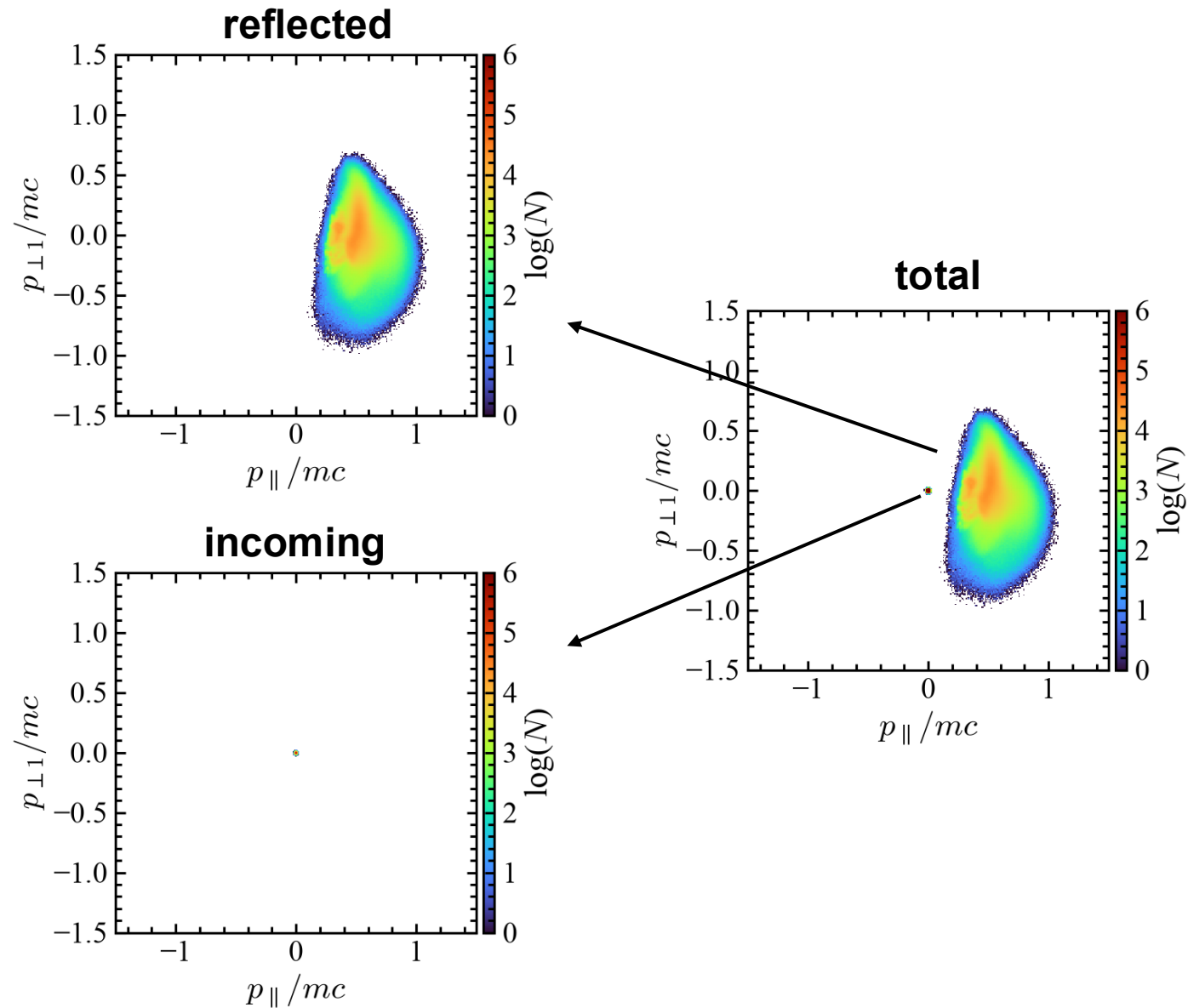


Reflected electrons move along the upstream B-field

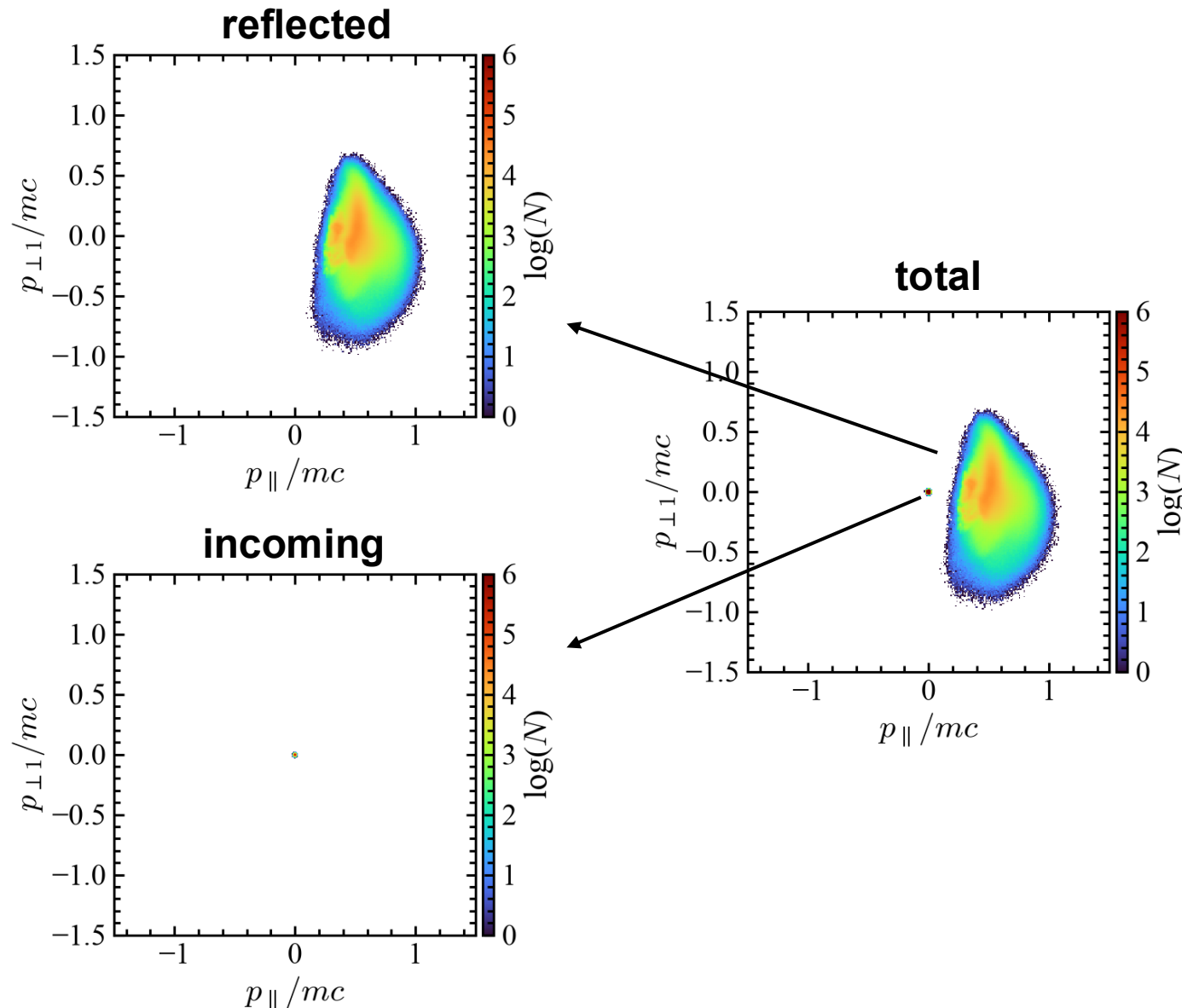


Electrons are confined close to the shock
→ no instabilities driven by reflected electrons

Distribution separation



Distribution separation



For each particle store

$$x_{\min} \equiv \min[x(t)]$$

At timestep t_0 check if

$$x_{\min} > x_{\text{sh}}(t_0)$$

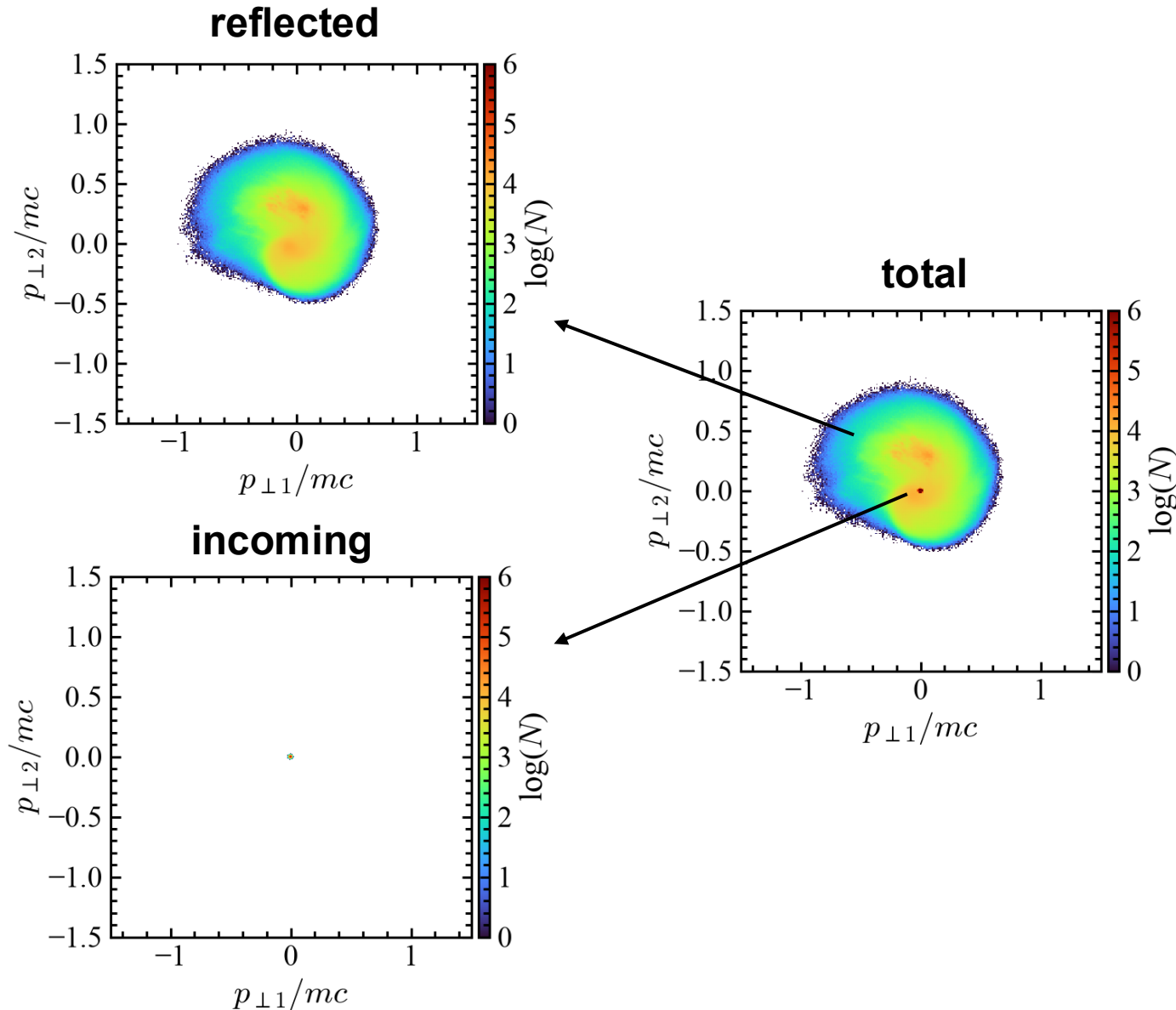
Particle has not reached
the shock → **incoming**

$$x_{\min} < x_{\text{sh}}(t_0)$$

Particle has been behind
the shock → **reflected**

(Includes particles scattered
in the upstream)

Distribution separation



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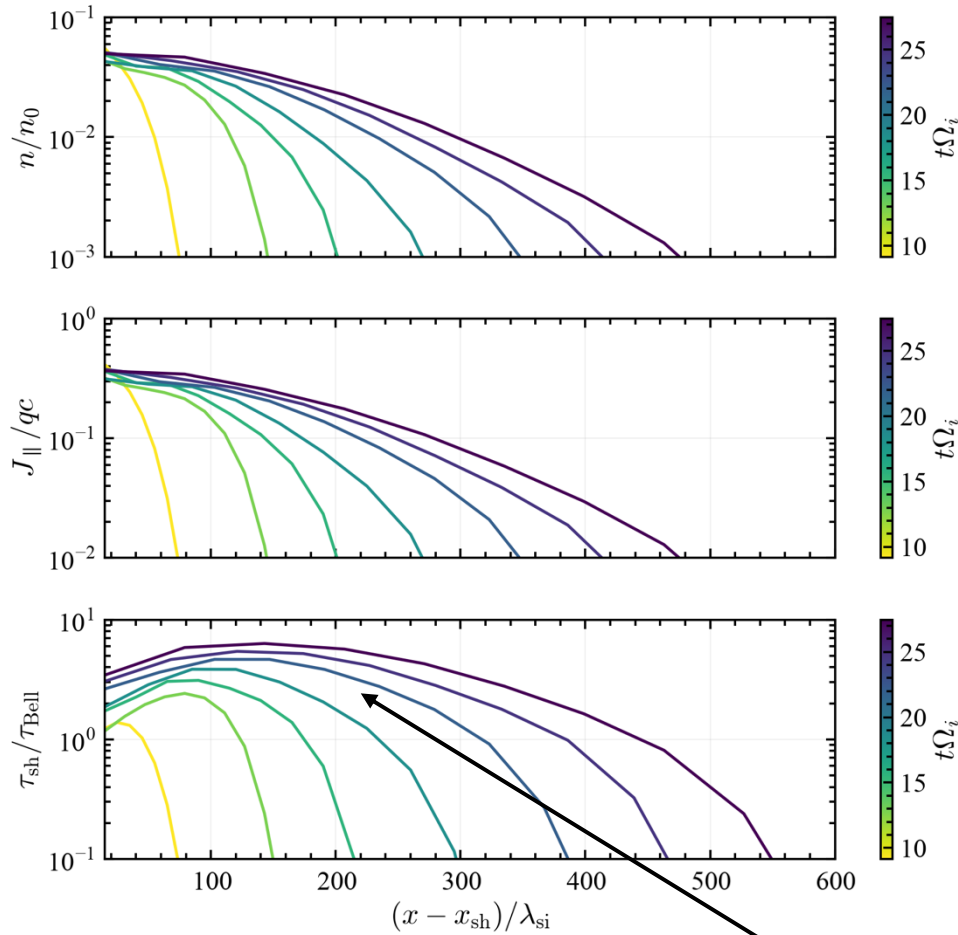
Upstream ions

$$\tau_{\text{sh}} \equiv L/v_{\text{sh}}$$

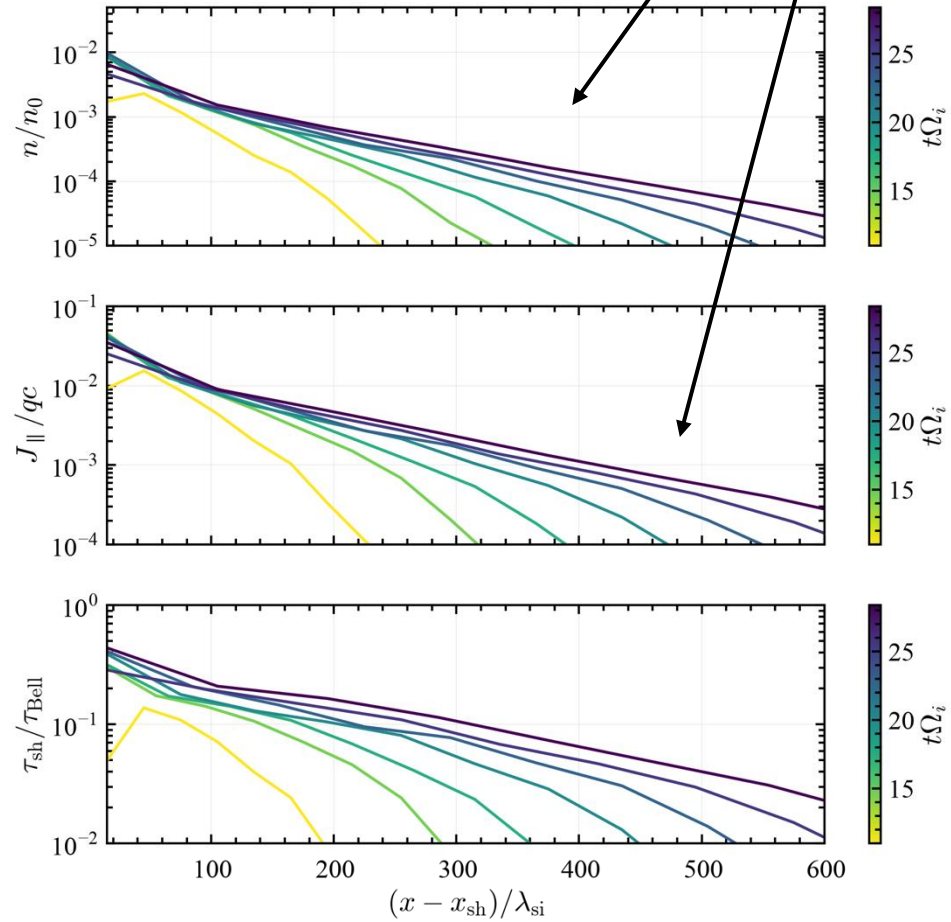
$$\tau_{\text{Bell}} \equiv \gamma_{\text{max}}^{-1}$$

Bell turbulence → reflected ion number density and current decreases

homogeneous



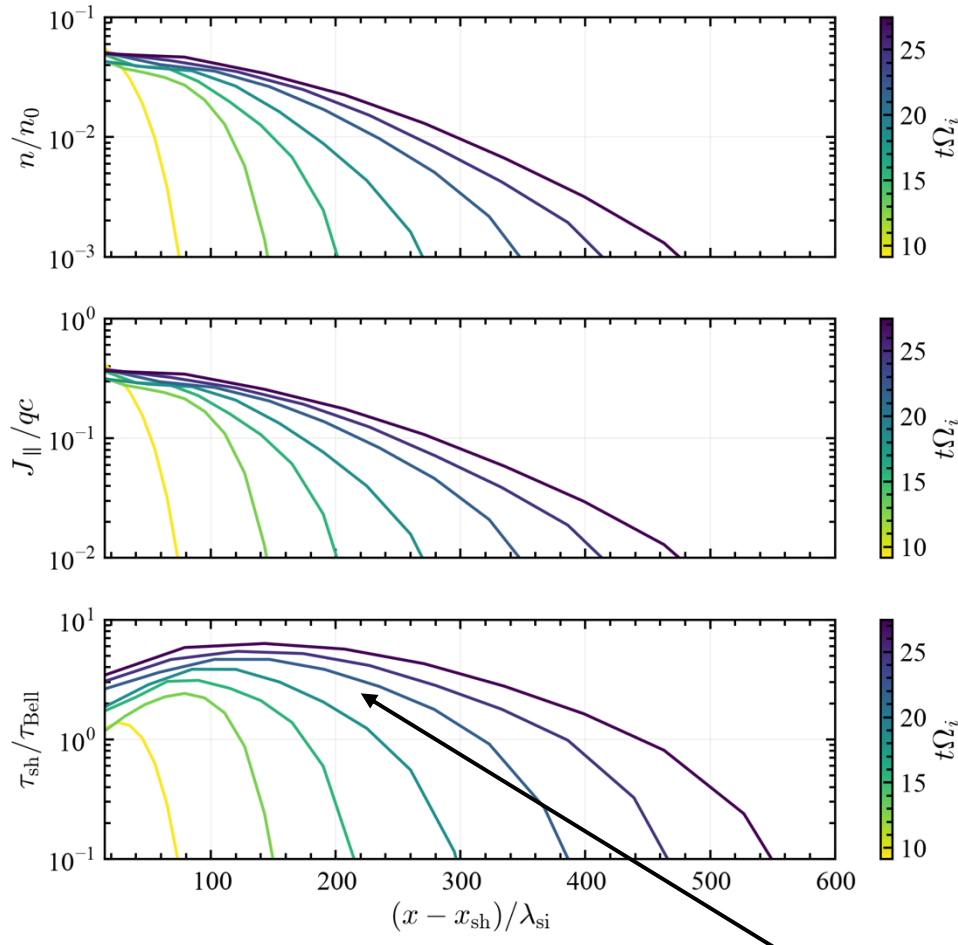
turbulent



Roughly 5 e-folding times

Upstream ions

homogeneous



Roughly 5 e-folding times

$$\frac{\gamma_{\text{max}}}{\Omega_i} = k_{\text{max}} \lambda_{\text{si}} \simeq \frac{1}{2} M_A \left(\frac{n_{\text{CR}}}{n_0} \right)$$

$$\gamma_{\text{max}}/\Omega_i = k_{\text{max}} \lambda_{\text{si}} \approx 0.75$$

Not really Bell regime

$$B^2/B_0^2 \propto \gamma_{\text{CR}} J_{\text{CR}}$$

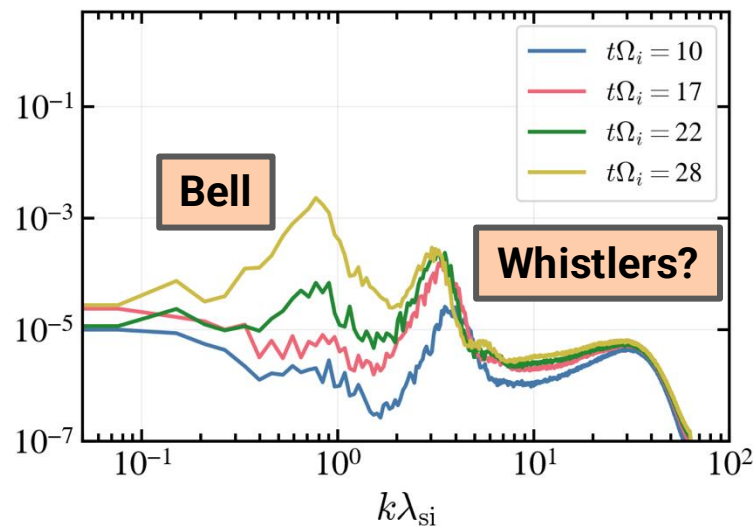
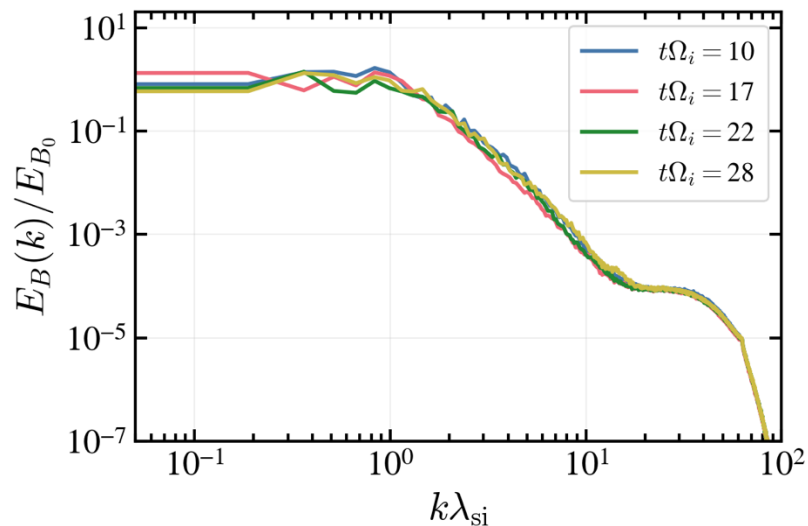
$$\gamma_{\text{CR}} \approx 1.2$$

Magnetic spectrum

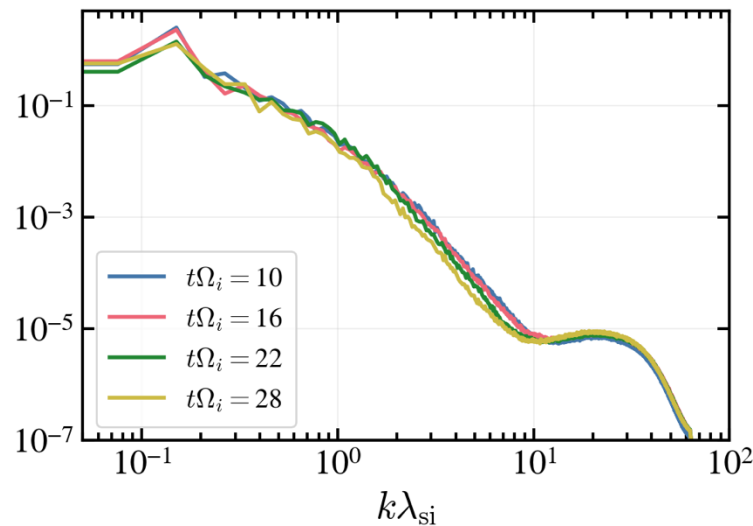
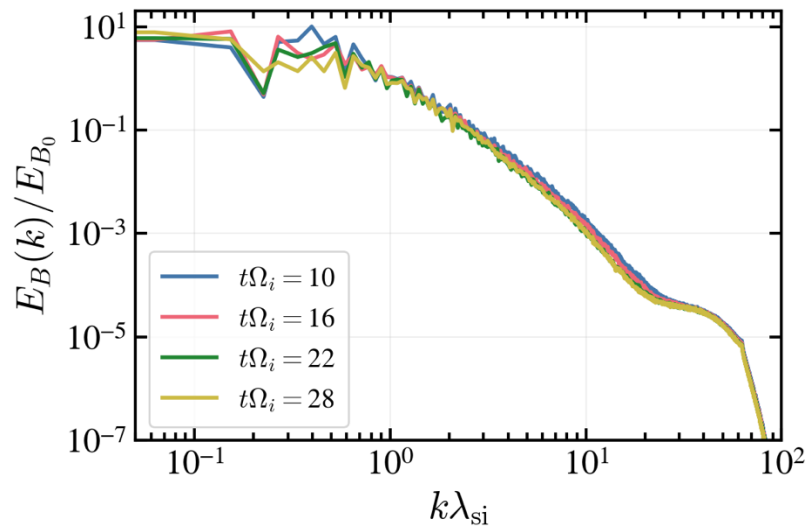
behind the shock

ahead of the shock

homogeneous



turbulent



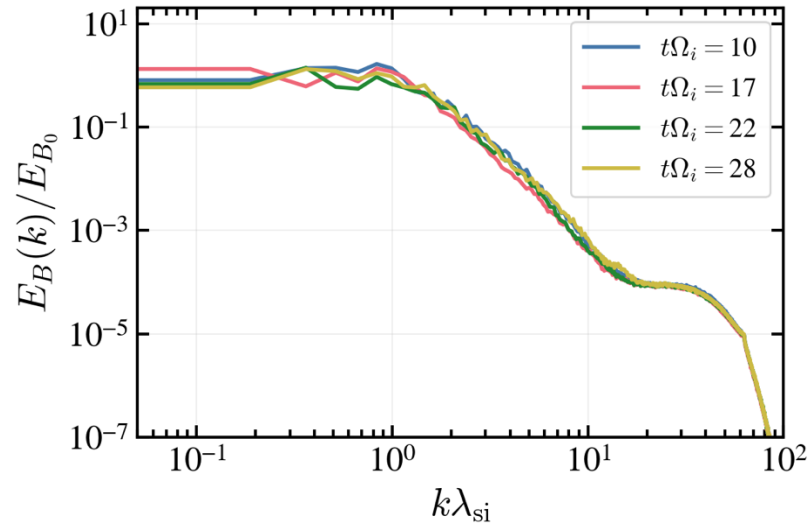
- Broad spectrum ahead of the shock for turbulent simulation
- Broad spectrum behind the shock in both cases, stronger for the turbulent simulation

$$\delta B/B_0 \sim 3 \text{ vs. } \delta B/B_0 \sim 1$$

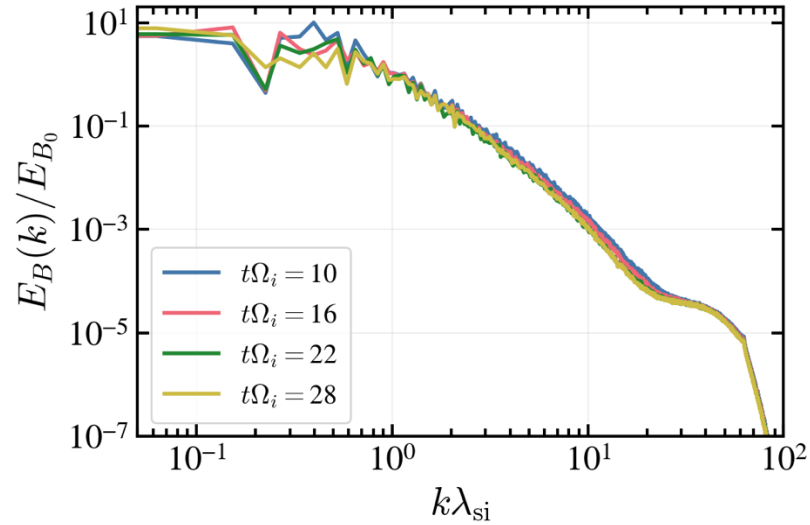
Magnetic spectrum

behind the shock

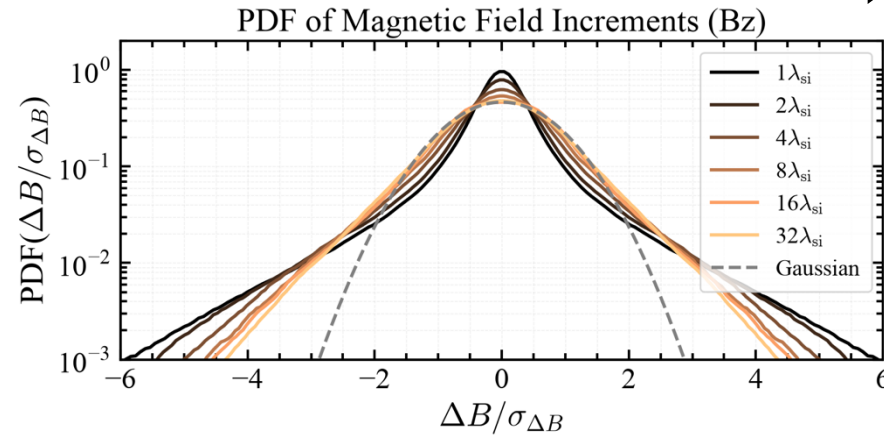
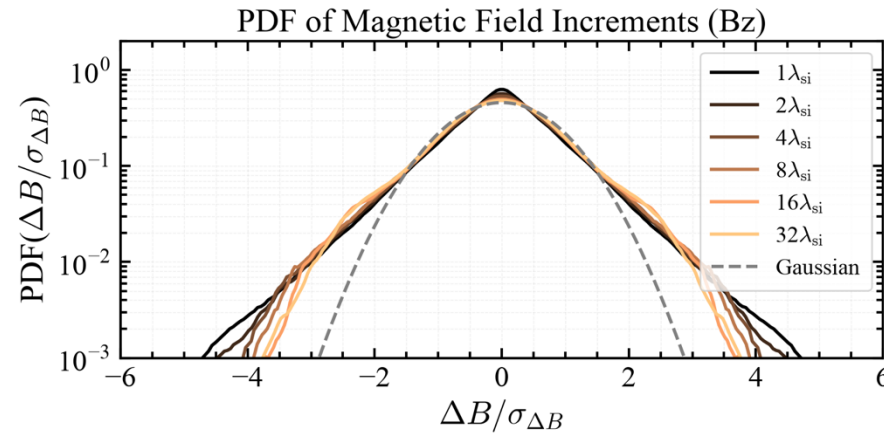
homogeneous



turbulent



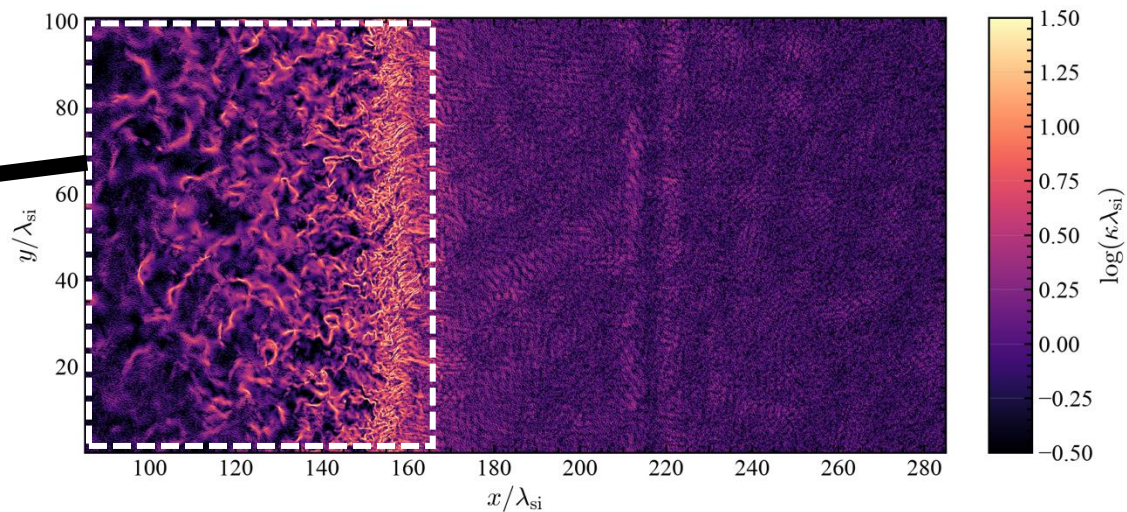
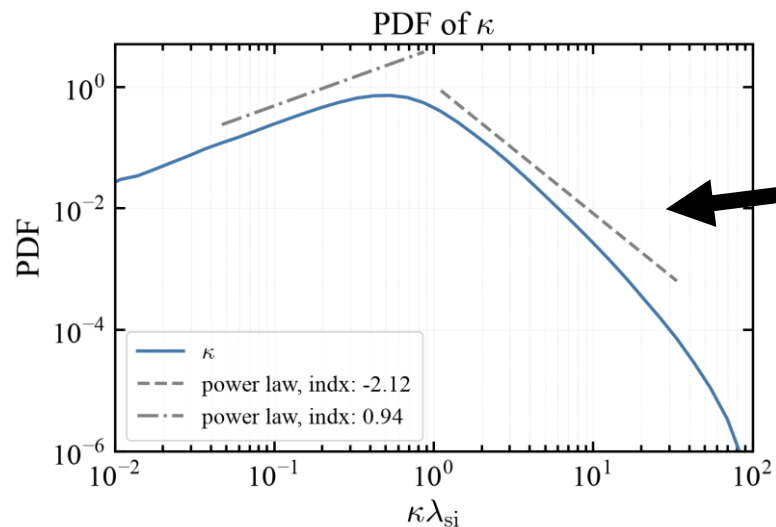
B-field is intermittent!



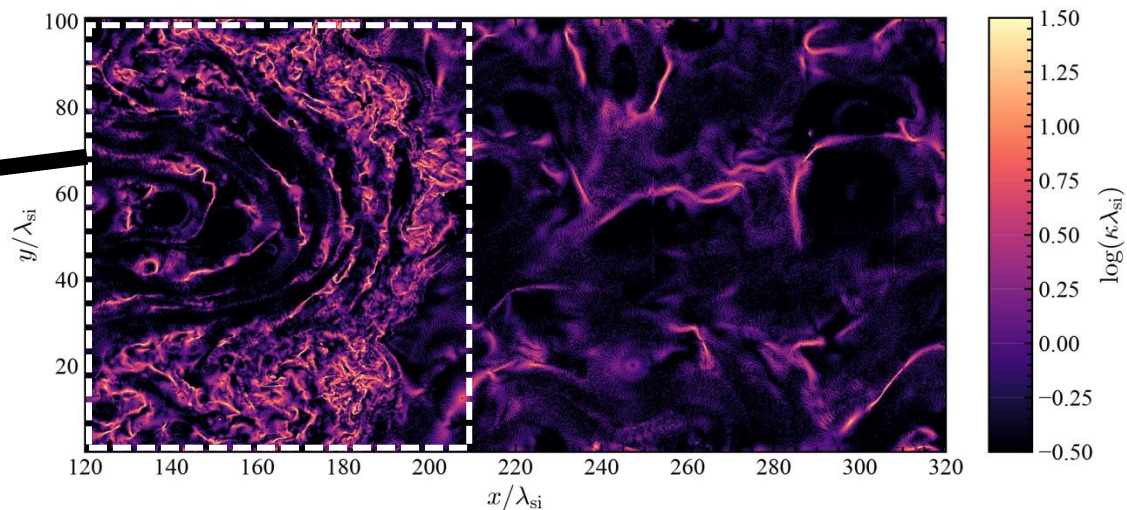
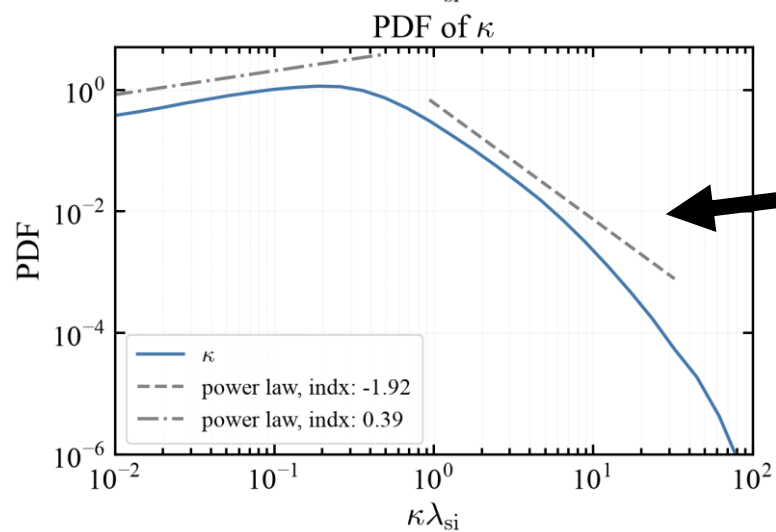
Curvature properties

$$\kappa = |\mathbf{B} \cdot \nabla \mathbf{B}| / B^2$$

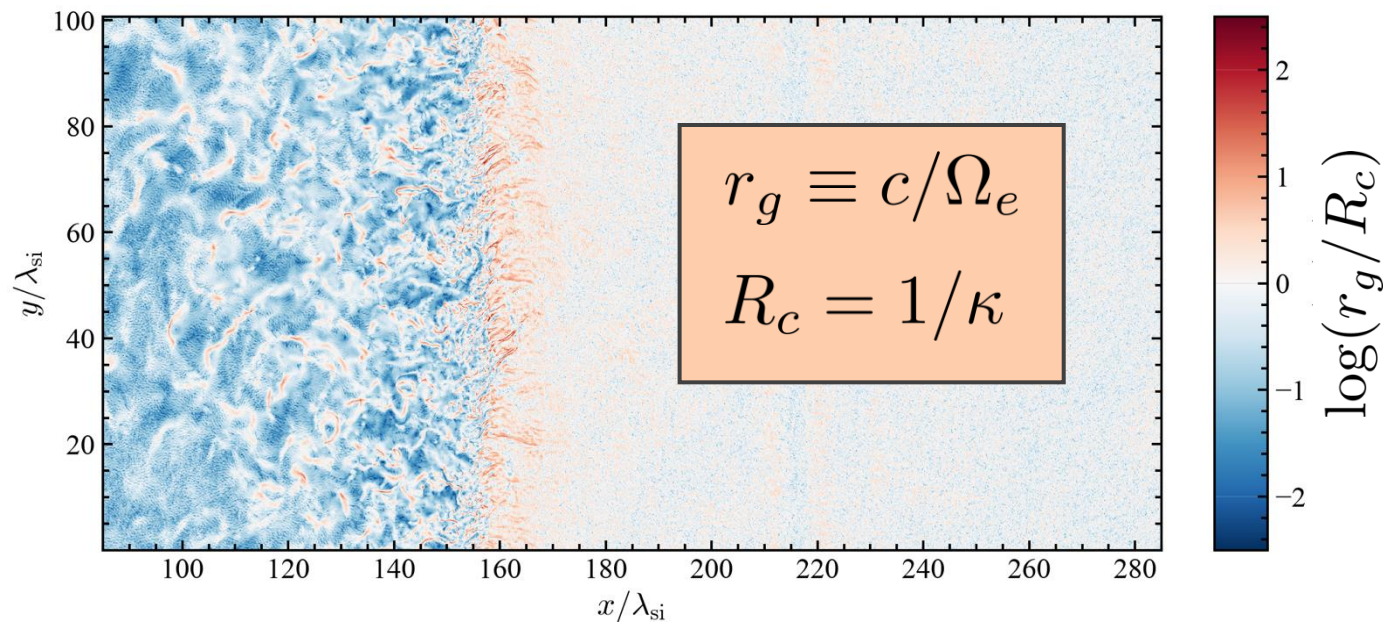
homogeneous



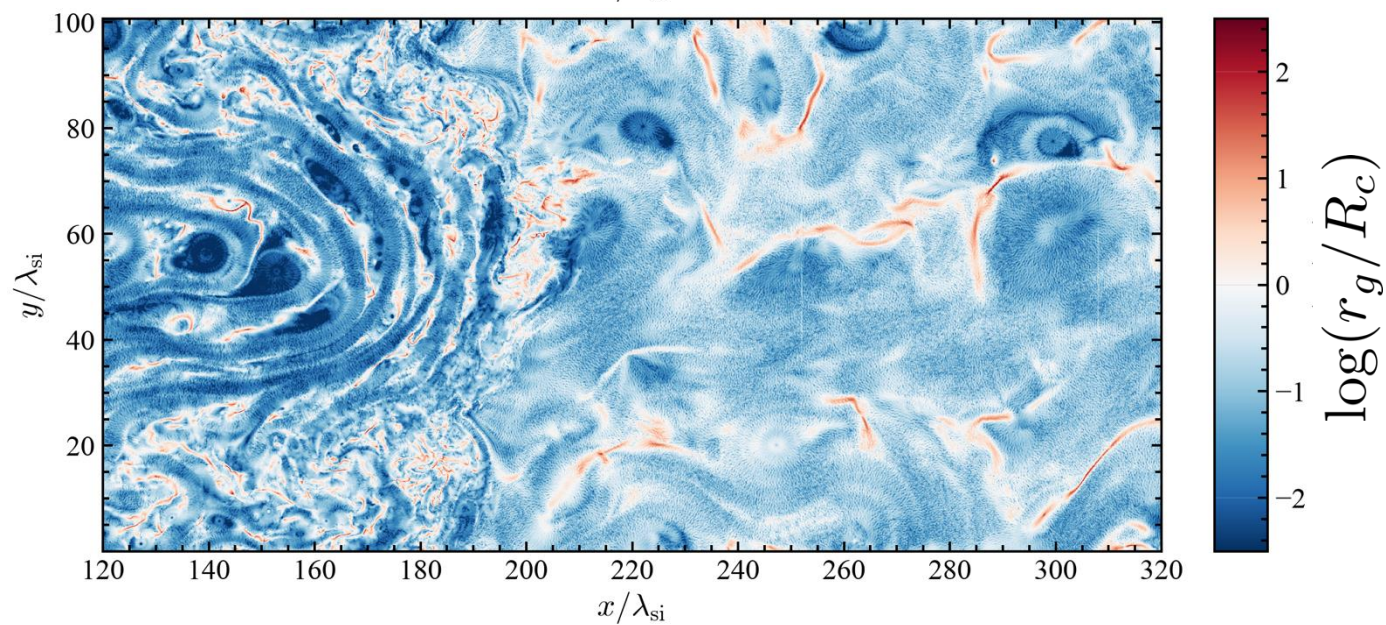
turbulent



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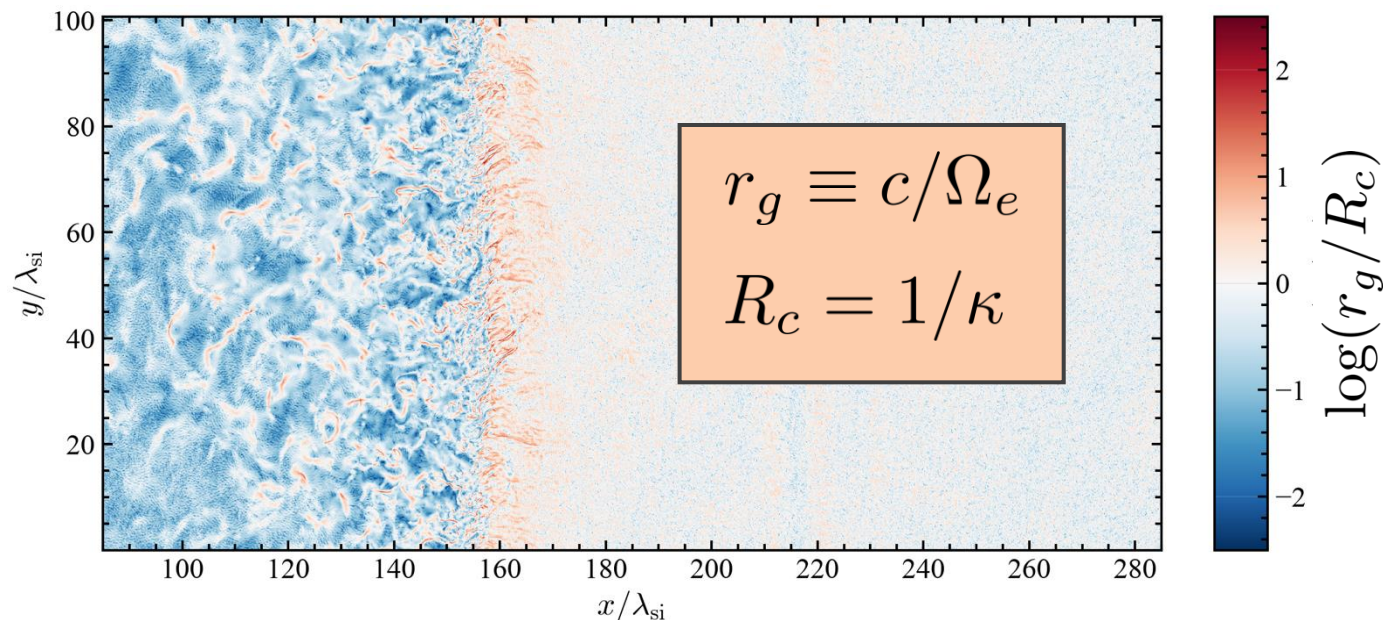


Suprathermal electrons
might encounter regions
where the **guiding-center
approximation breaks down**

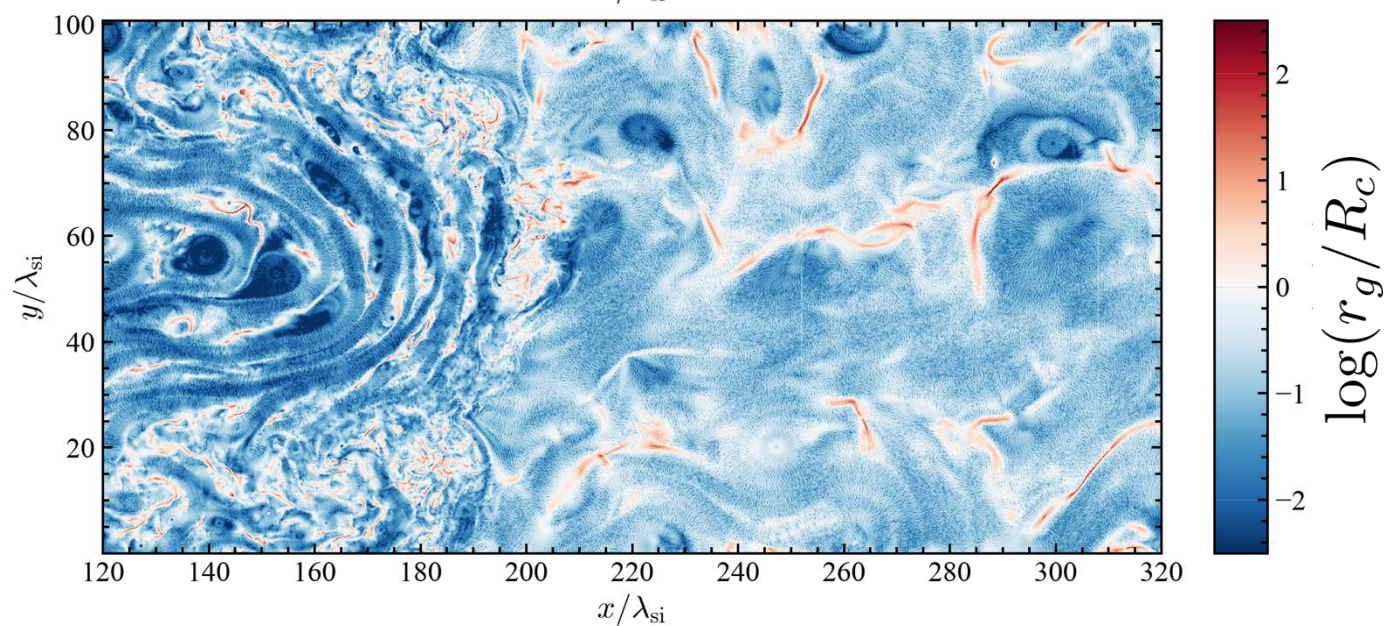


Nonadiabatic interaction
with the shock

homogeneous



turbulent



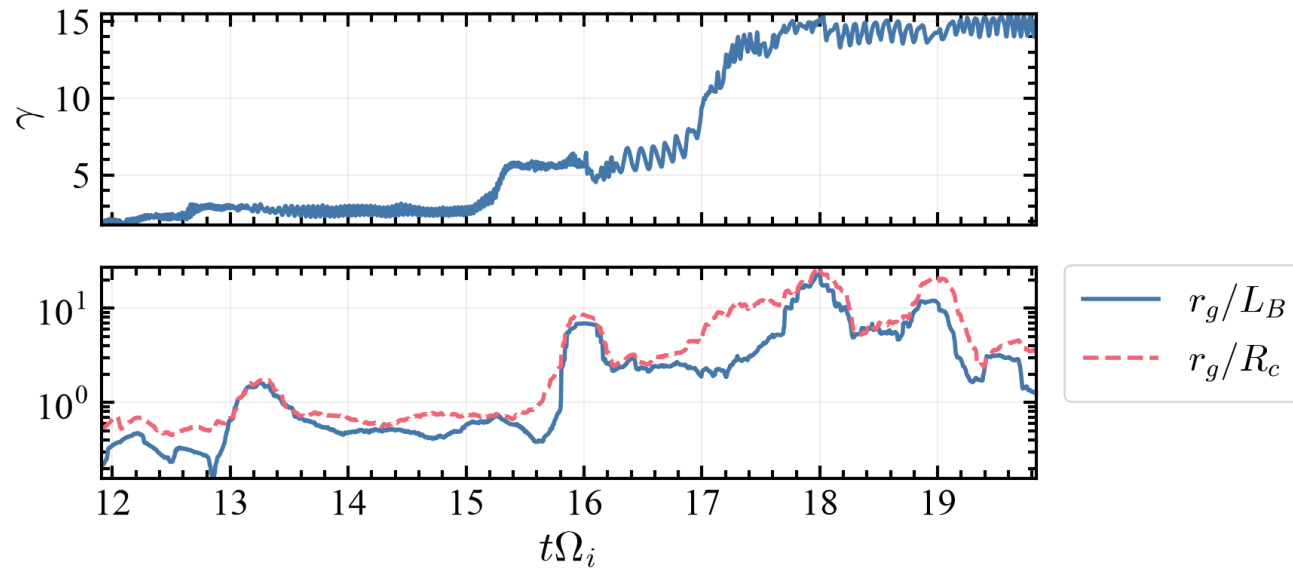
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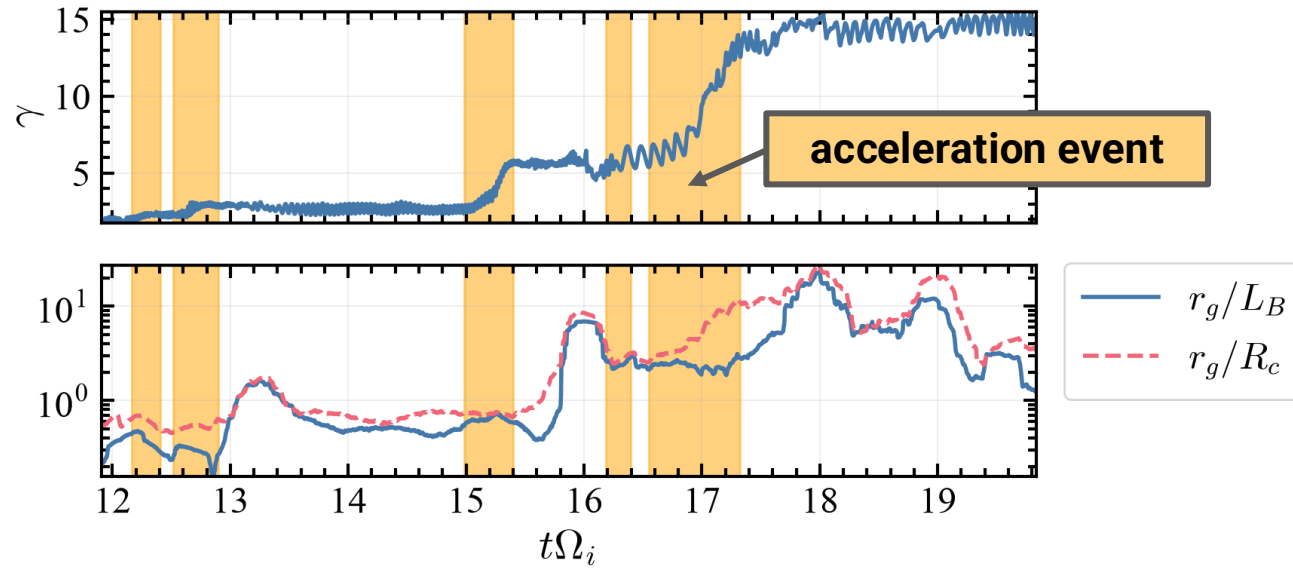
Nonadiabatic interaction
with the shock

Stochastic acceleration in
strong and intermittent
turbulence

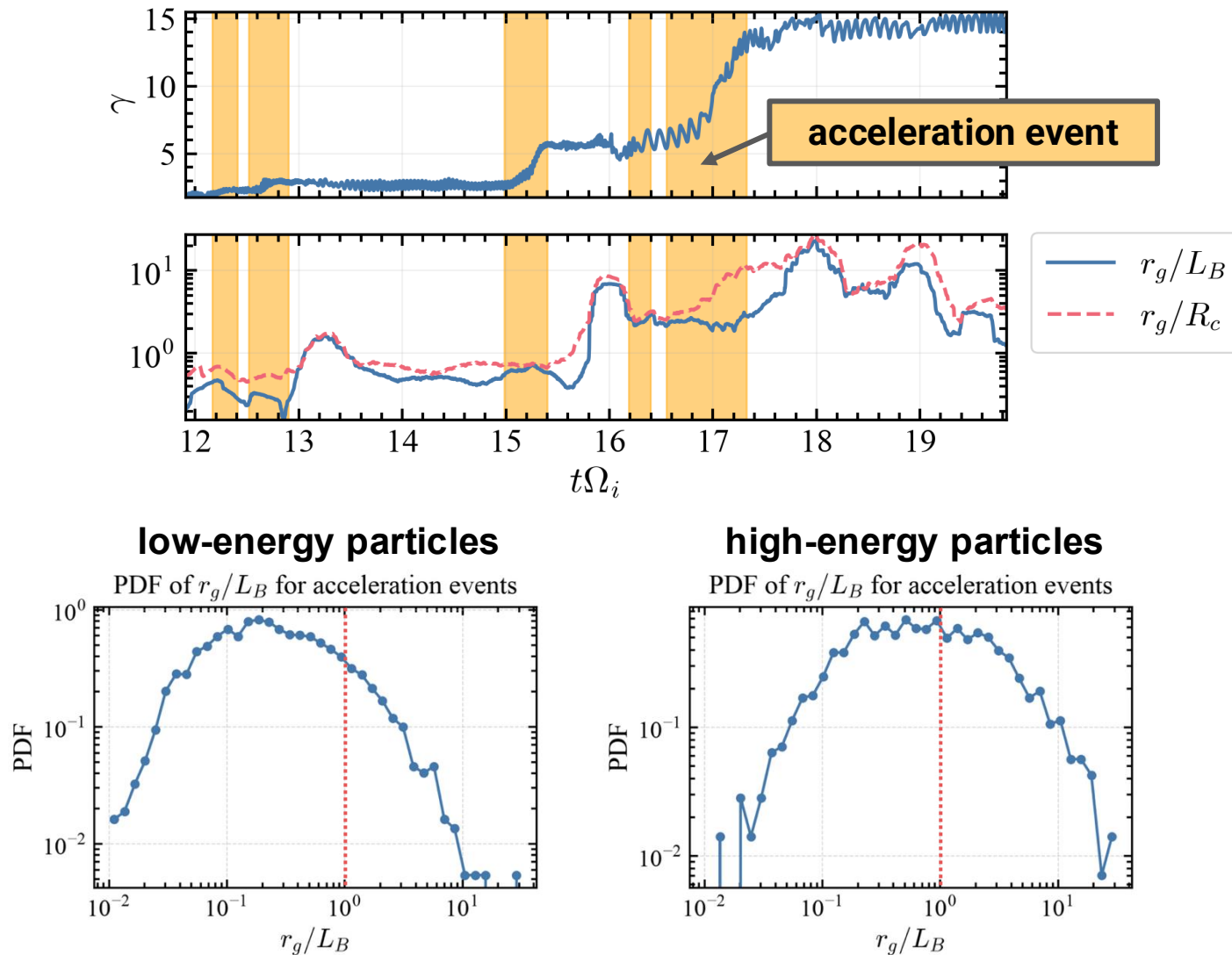
Electron acceleration mechanism



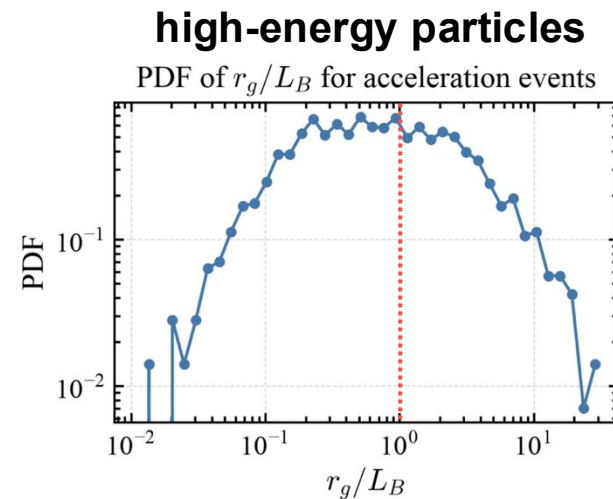
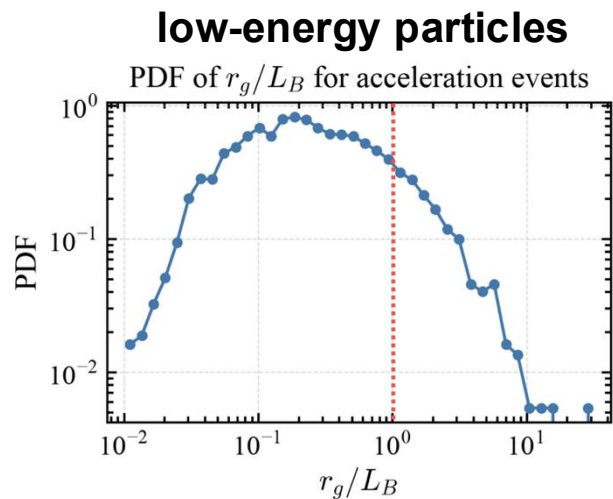
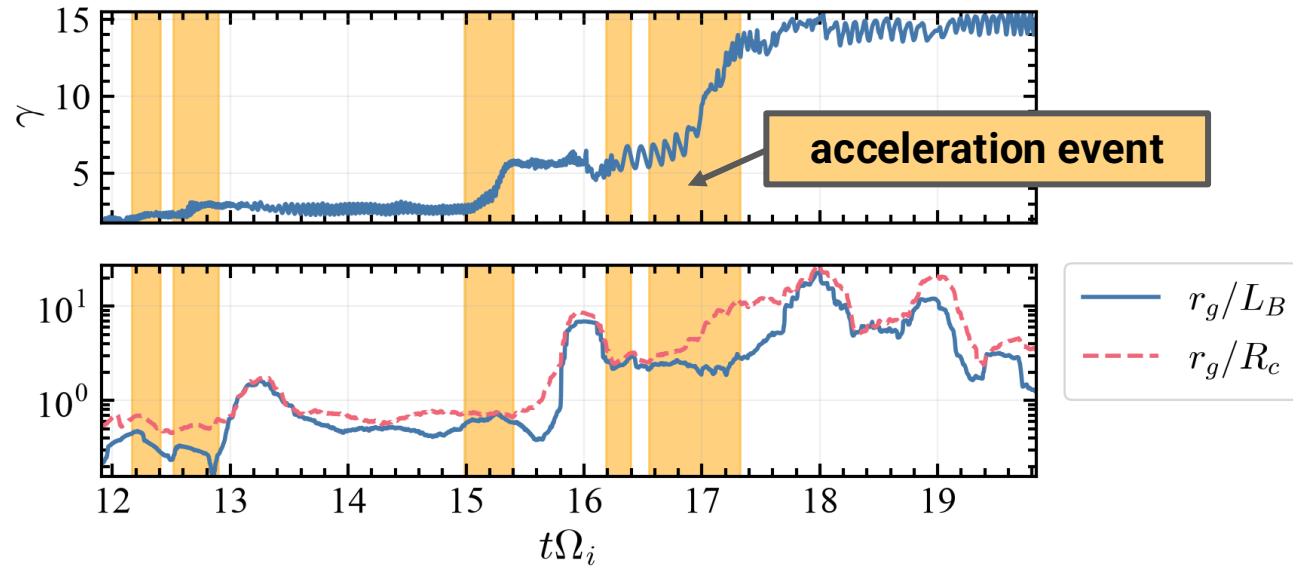
Electron acceleration mechanism



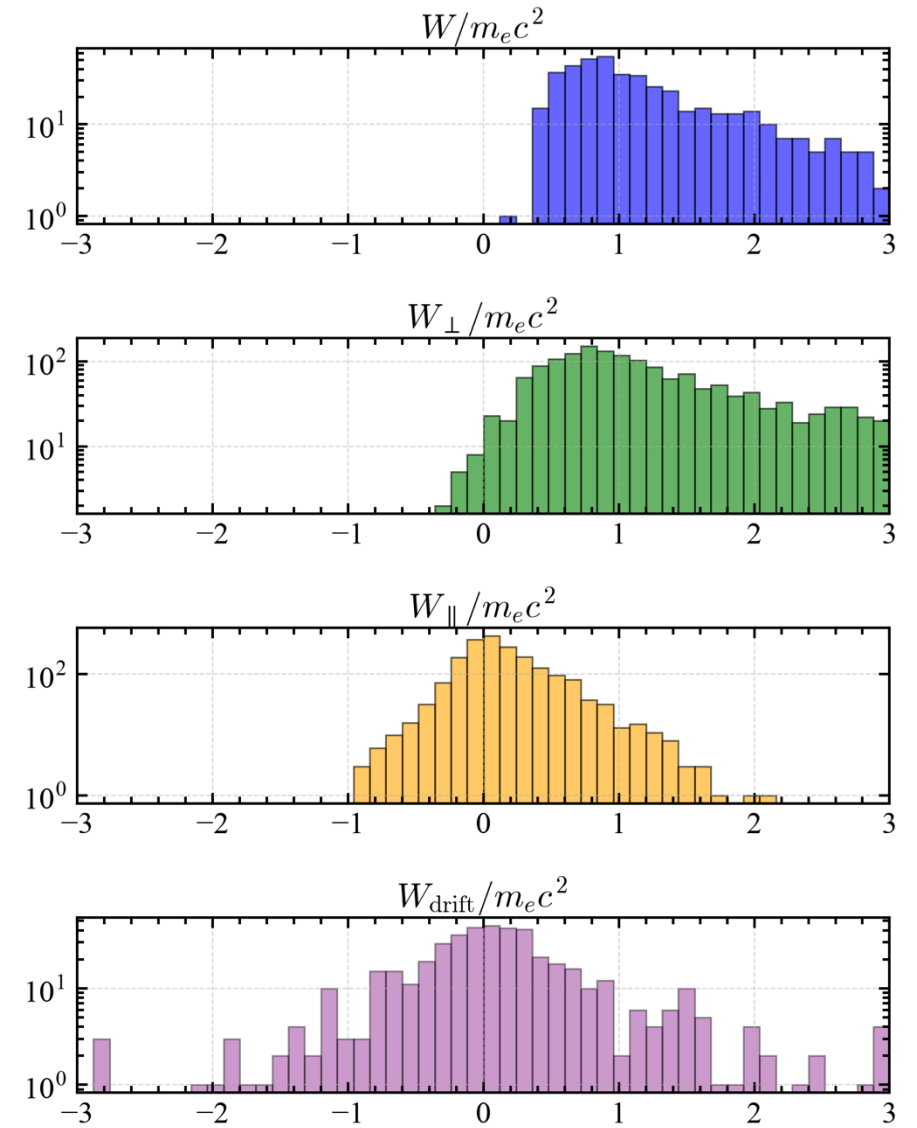
Electron acceleration mechanism



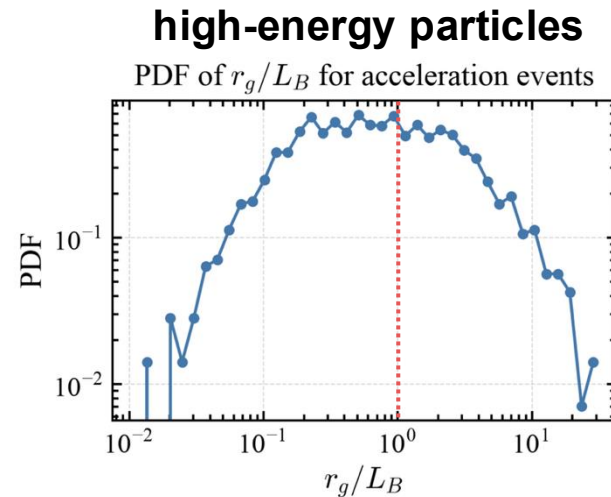
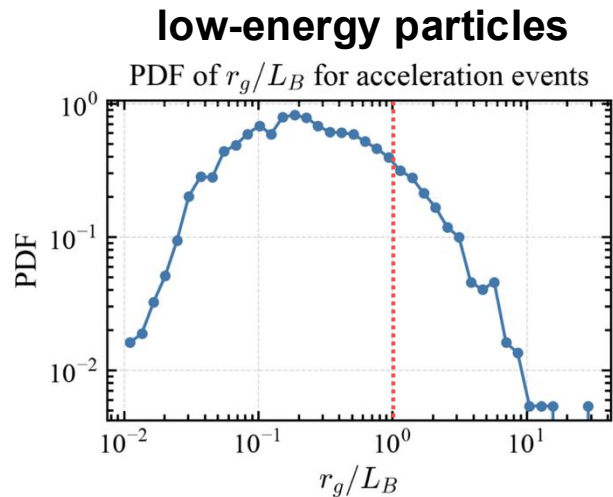
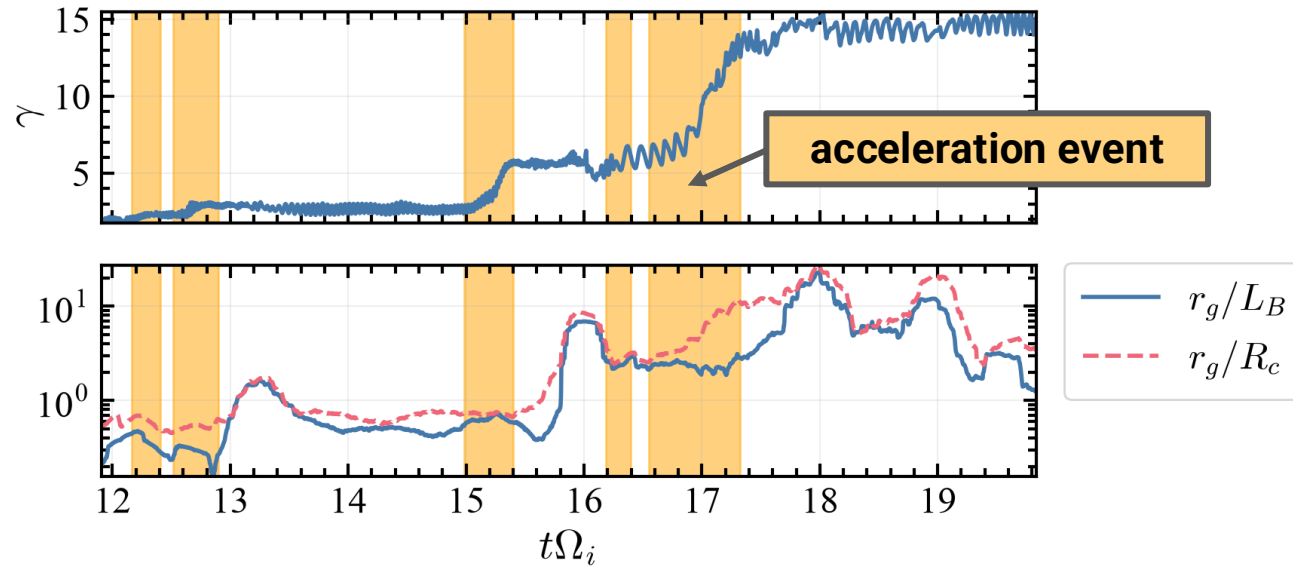
Electron acceleration mechanism



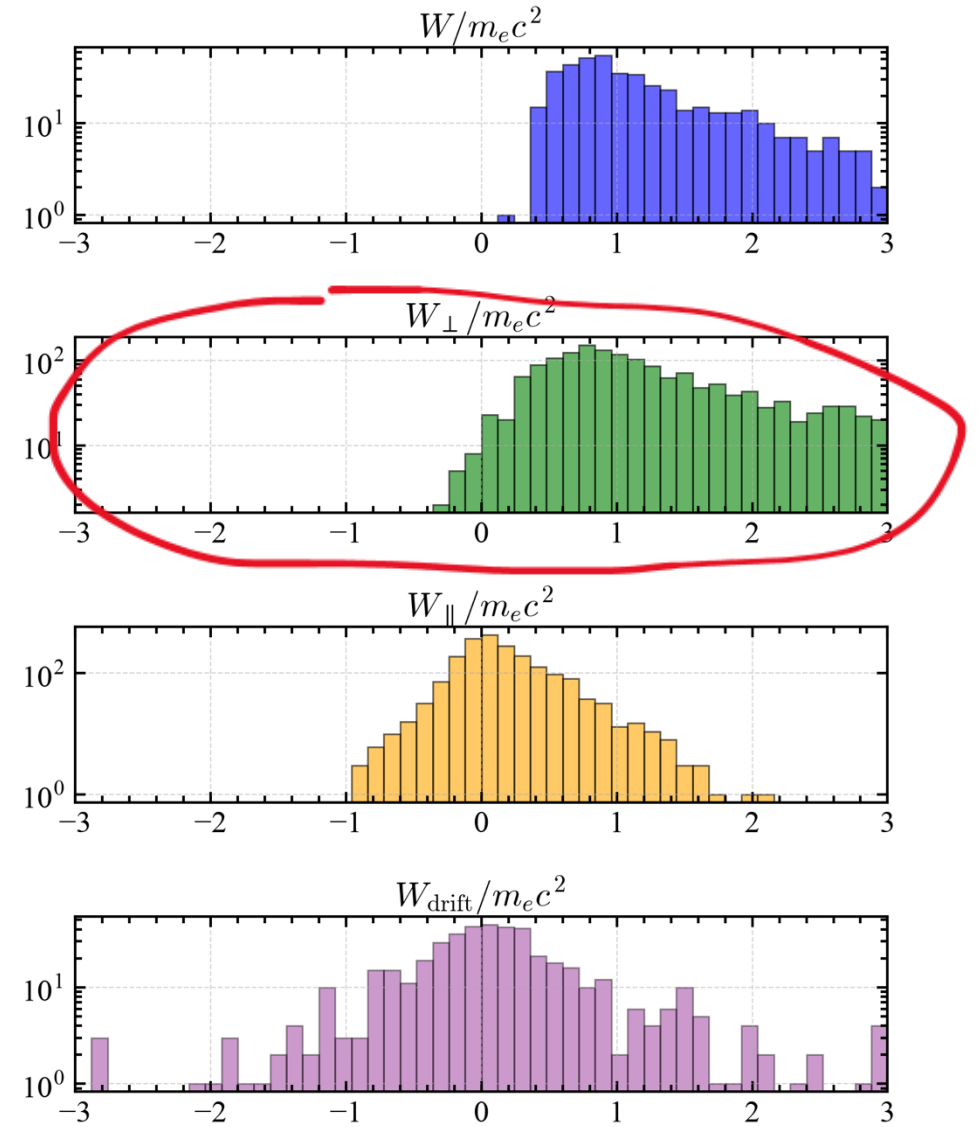
PDFs for acceleration events

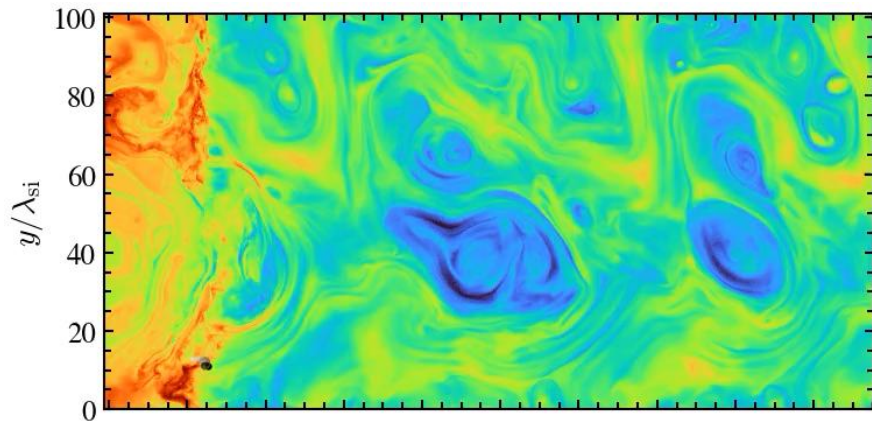
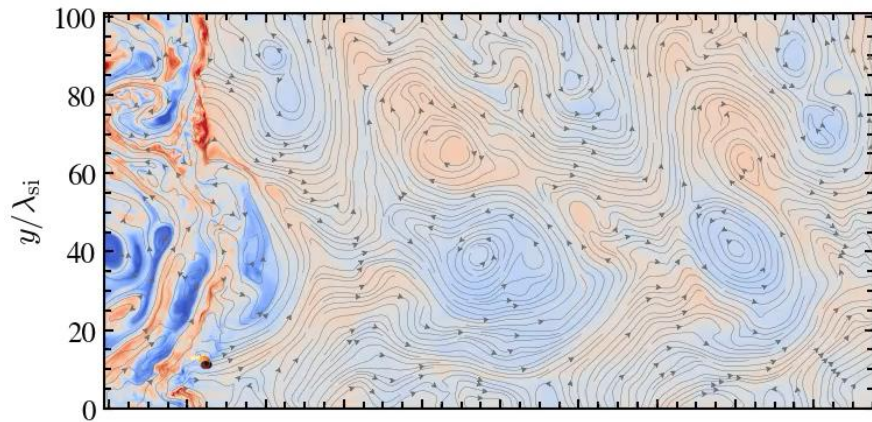
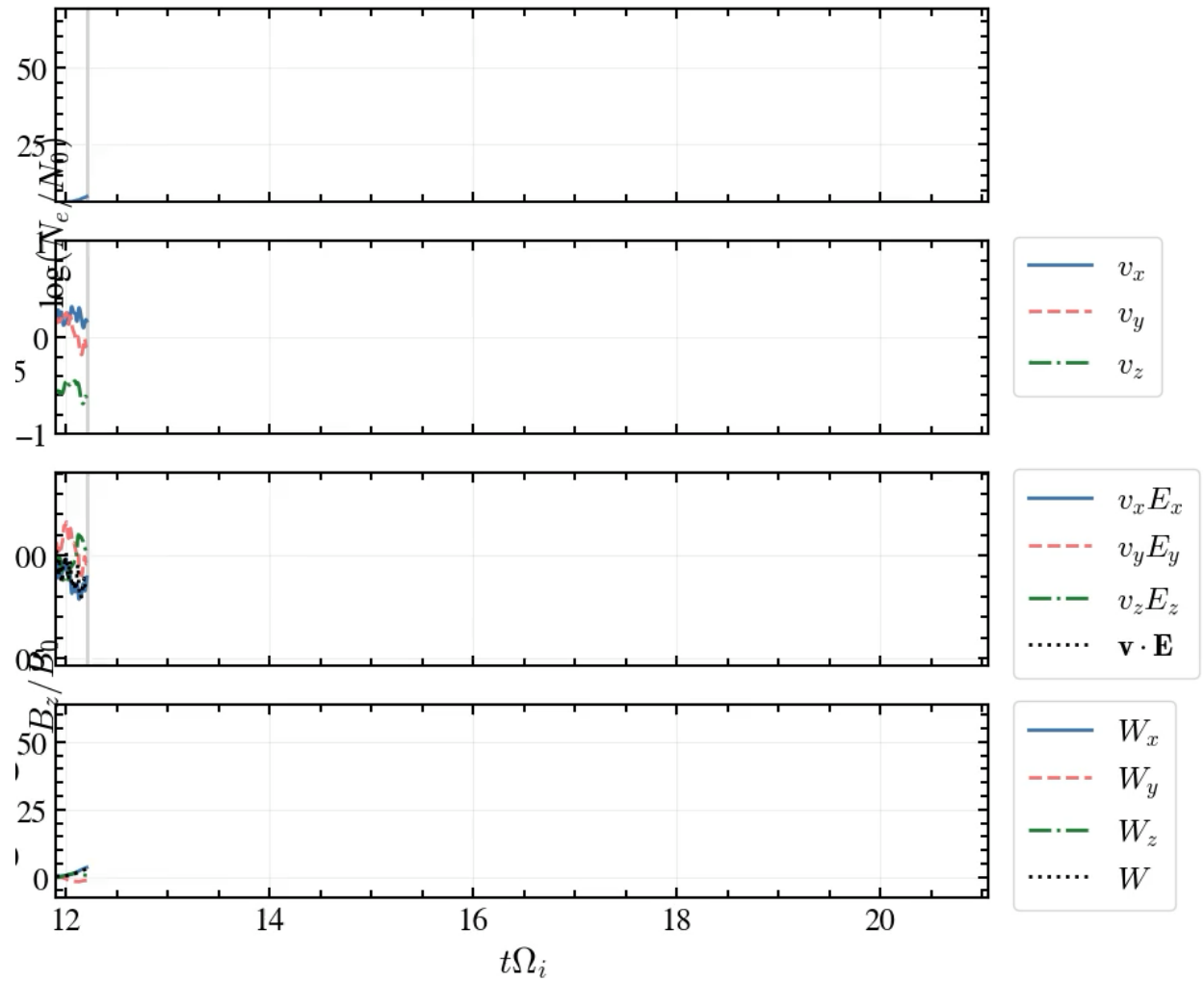
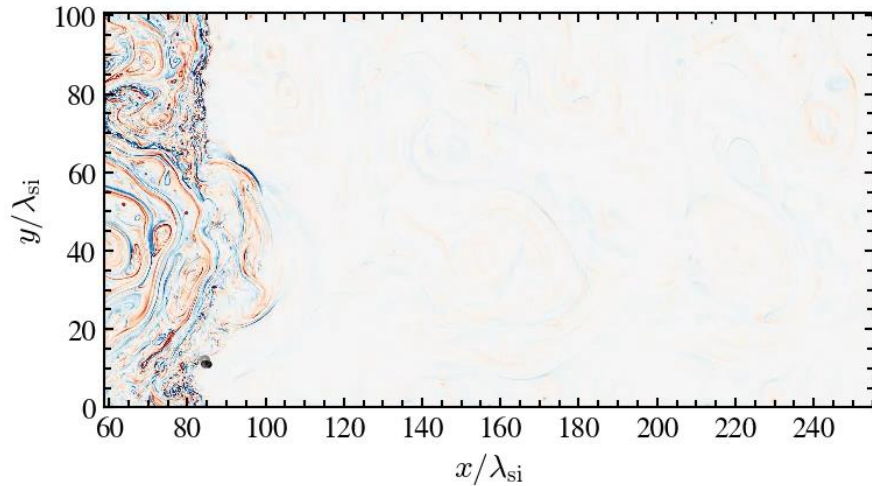


Electron acceleration mechanism



PDFs for acceleration events



N_e

 B_z

 J_z


Summary

How to include large-scale effects in kinetic simulations?

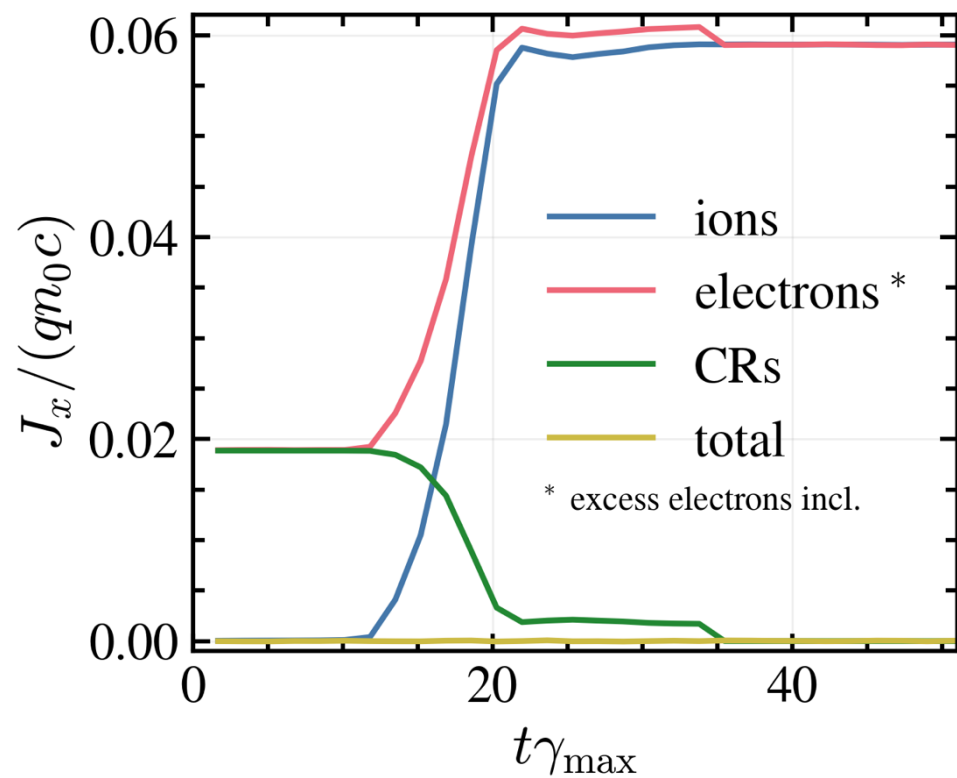
Bell turbulence leads to strong shock front corrugations and local variations in the obliquity.

Efficient electron acceleration due to strong turbulence near the shock front and reflection in the upstream Bell turbulence.

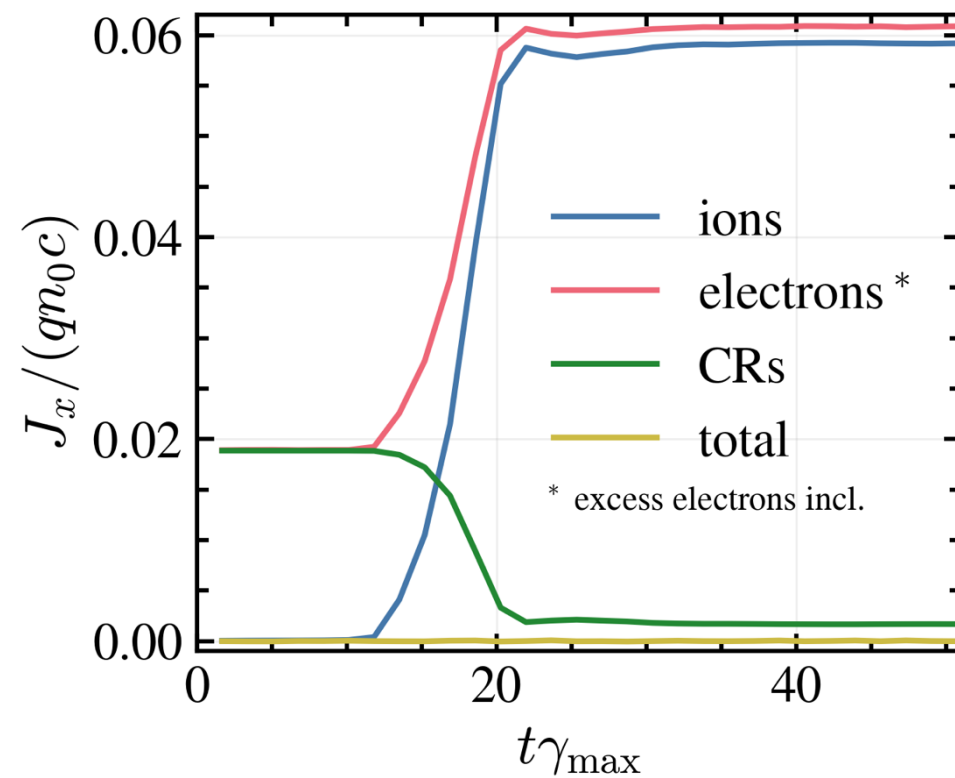
Standard drift picture does not adequately describe electron dynamics.

Acceleration: SDA (?) $\rightarrow$ STA $\rightarrow$ DSA

CRs removed after the saturation



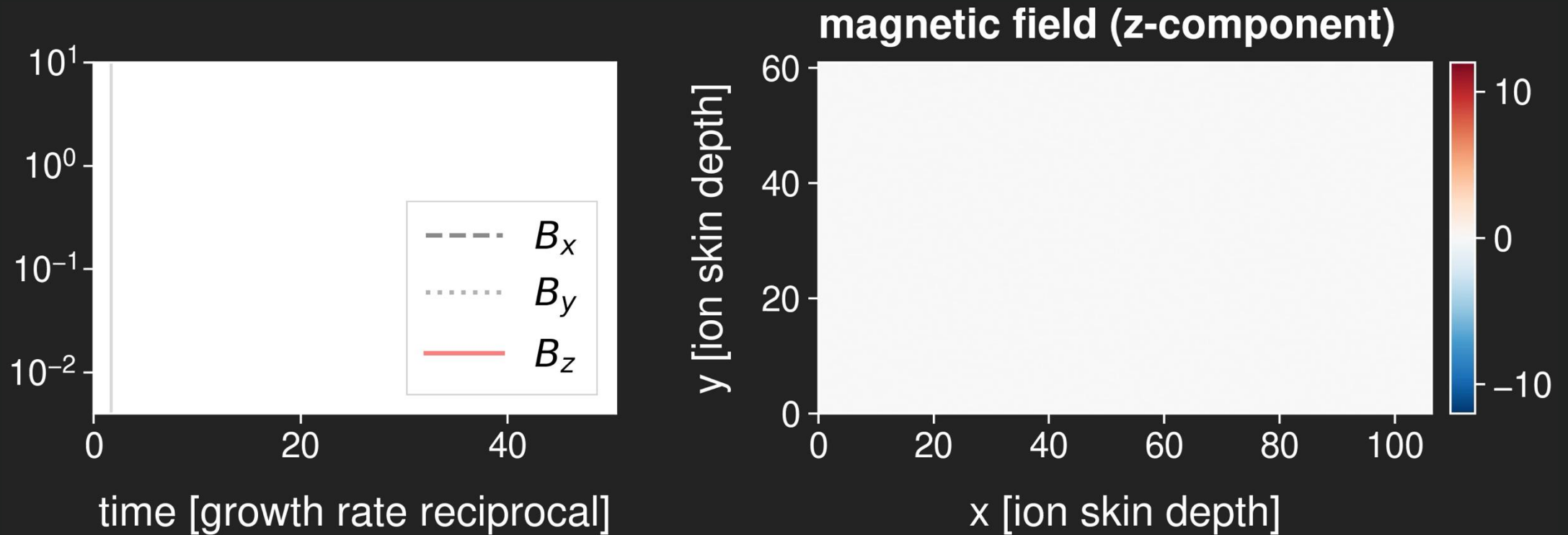
With CRs



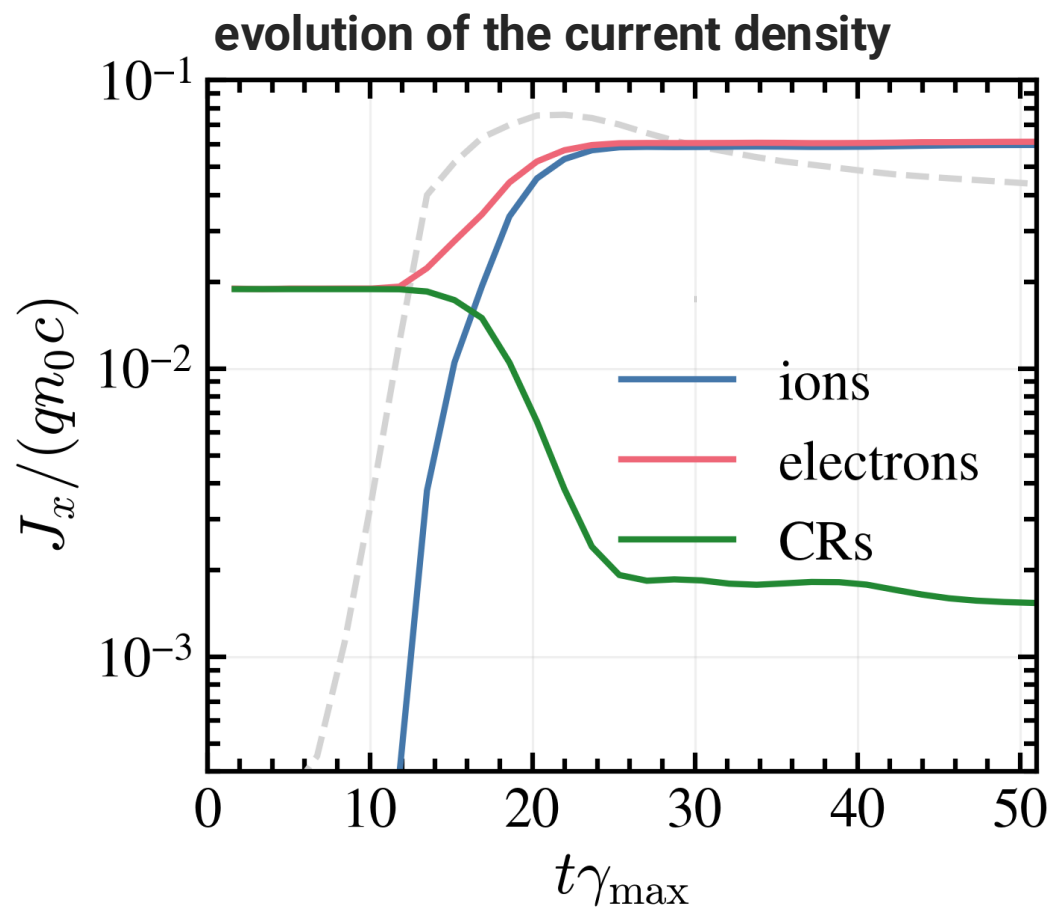
PIC simulations of the Bell instability

e.g., Riquelme and Spitkovsky 2009, Stroman et al. 2009

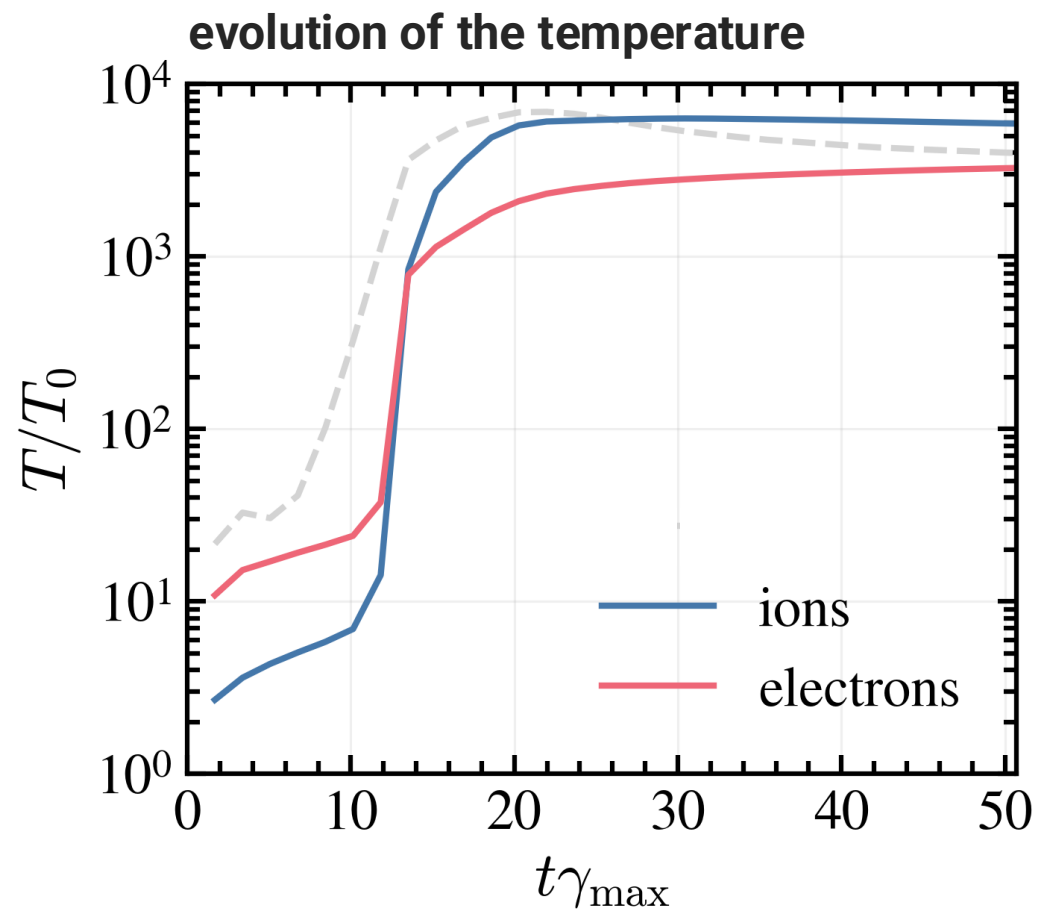
When CRs are streaming with a velocity larger than the Alfvén speed, the background plasma reacts to reduce the speed → growth of the non-resonant modes Bell 2004



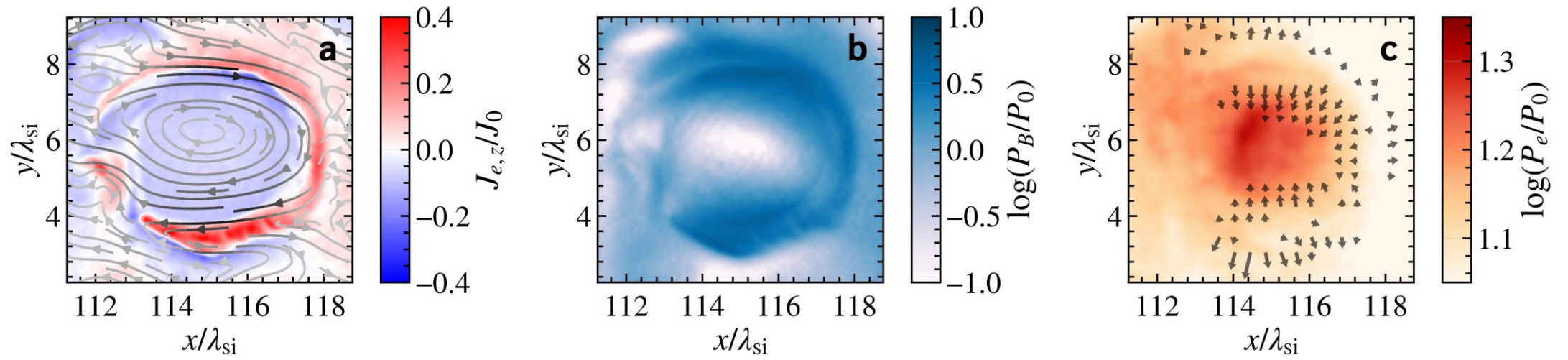
$$\gamma_{\text{CR,drift}} = 3 \quad n_{\text{CR}}/n = 0.02 \quad v_{\text{th,e}}/c = 0.01 \quad v_A/c = 0.01$$



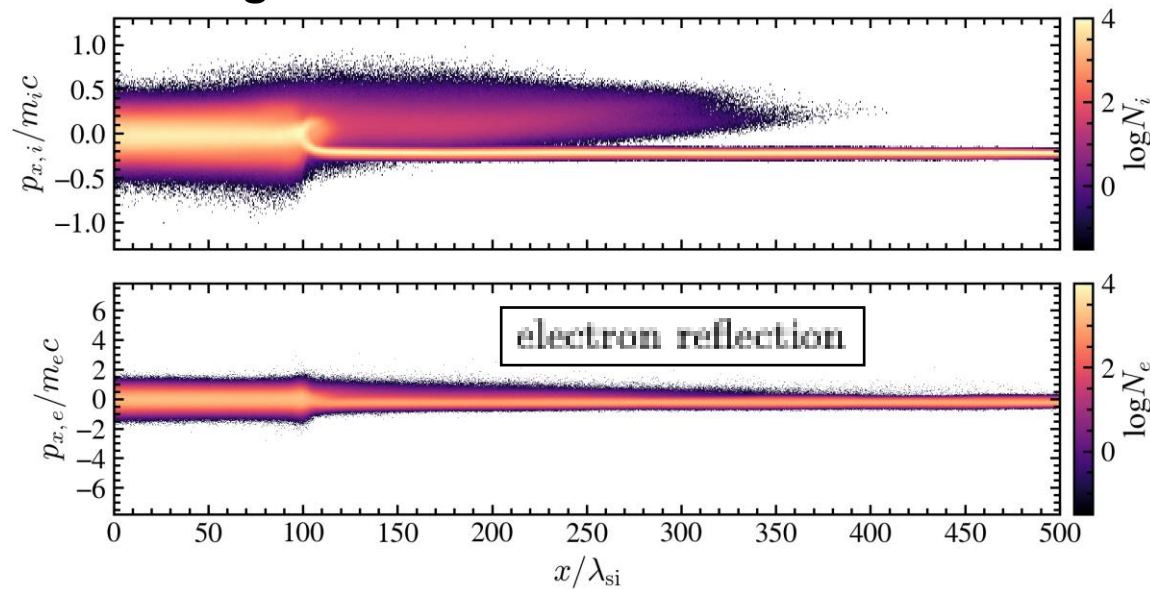
relative drift between CRs
and the background plasma is reduced



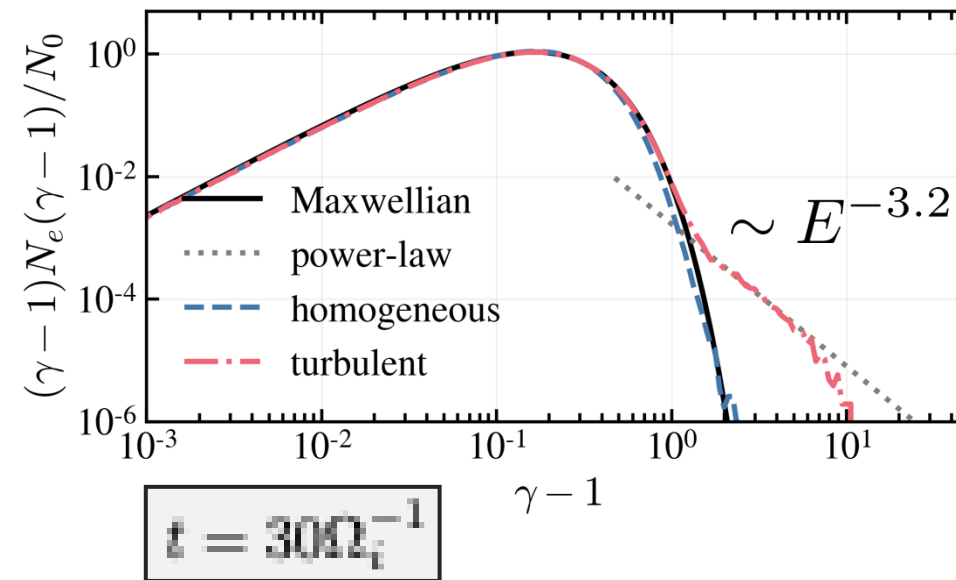
particle heating



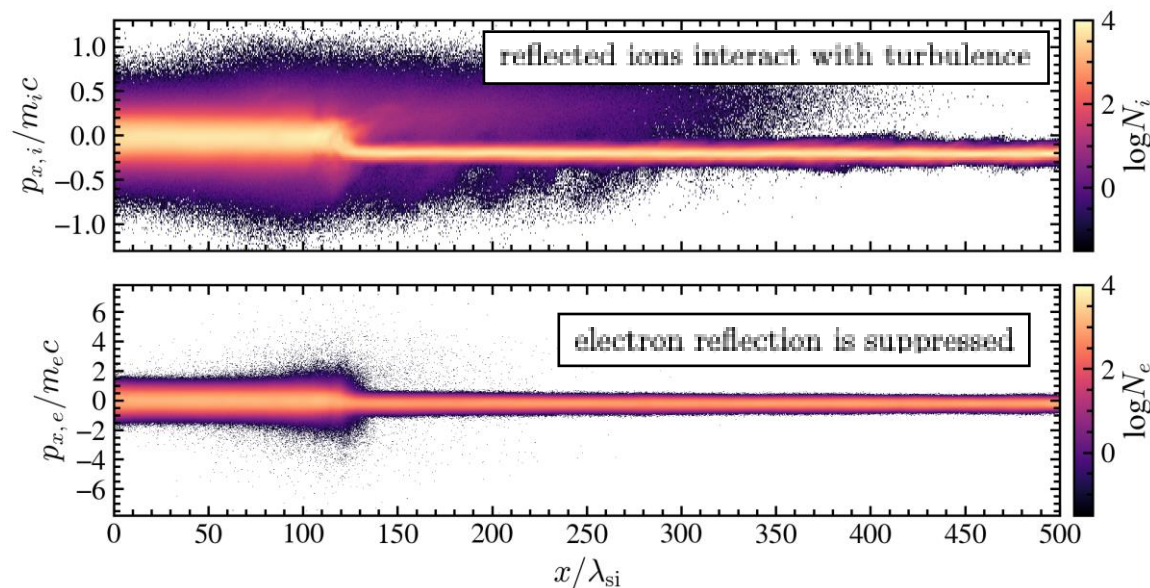
homogeneous



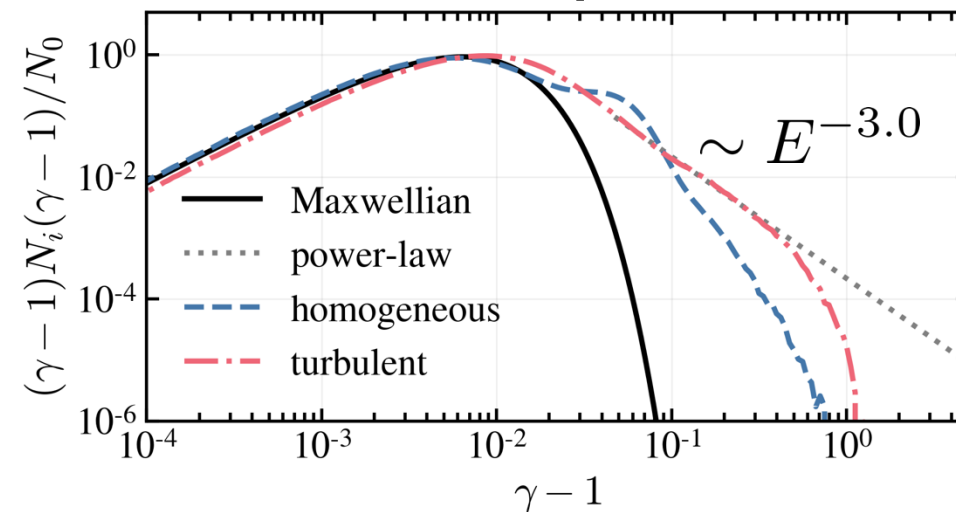
electron downstream spectra

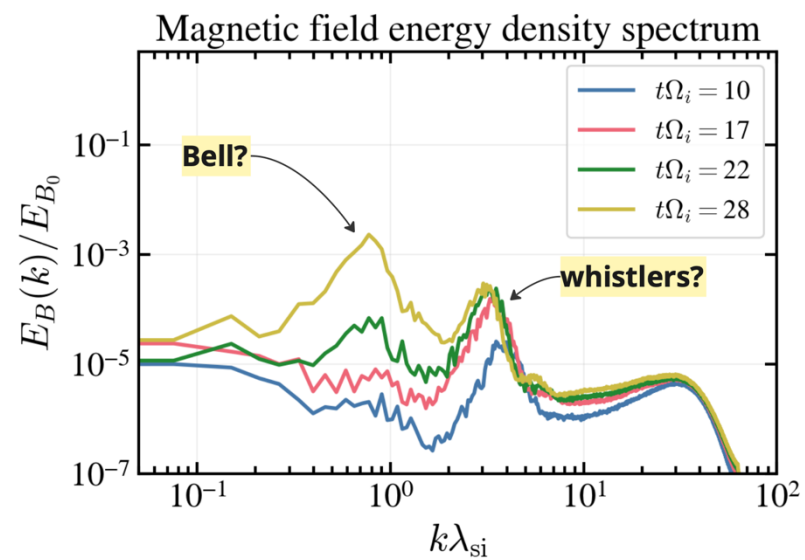
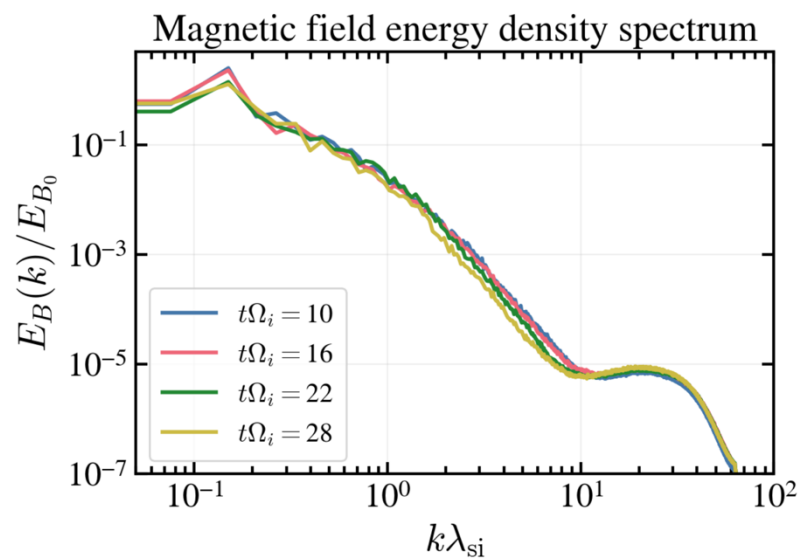
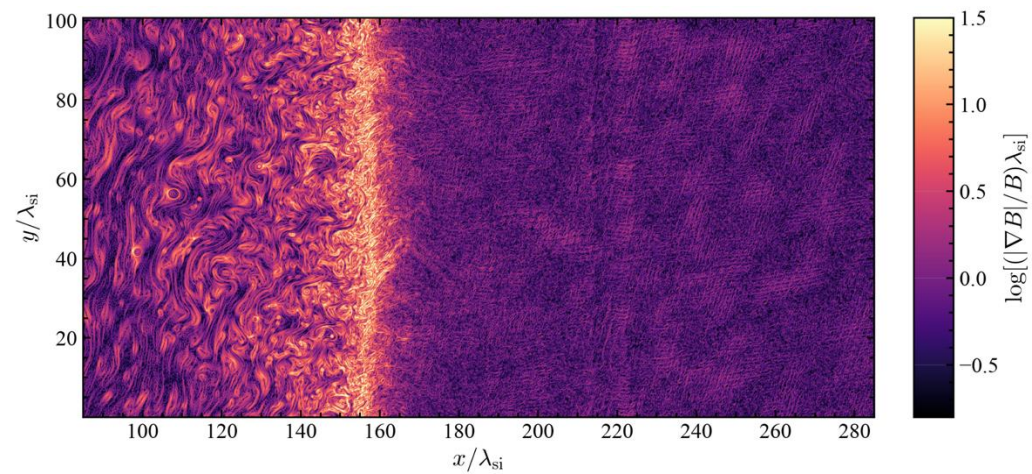
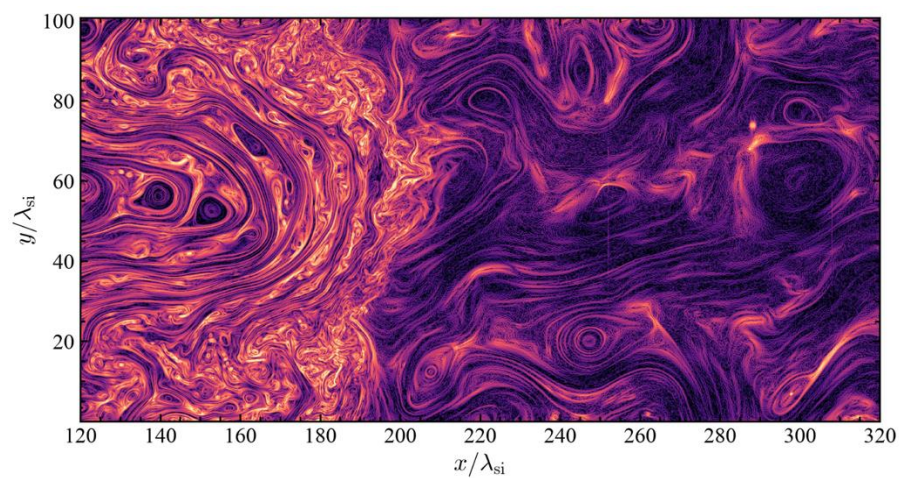


turbulent



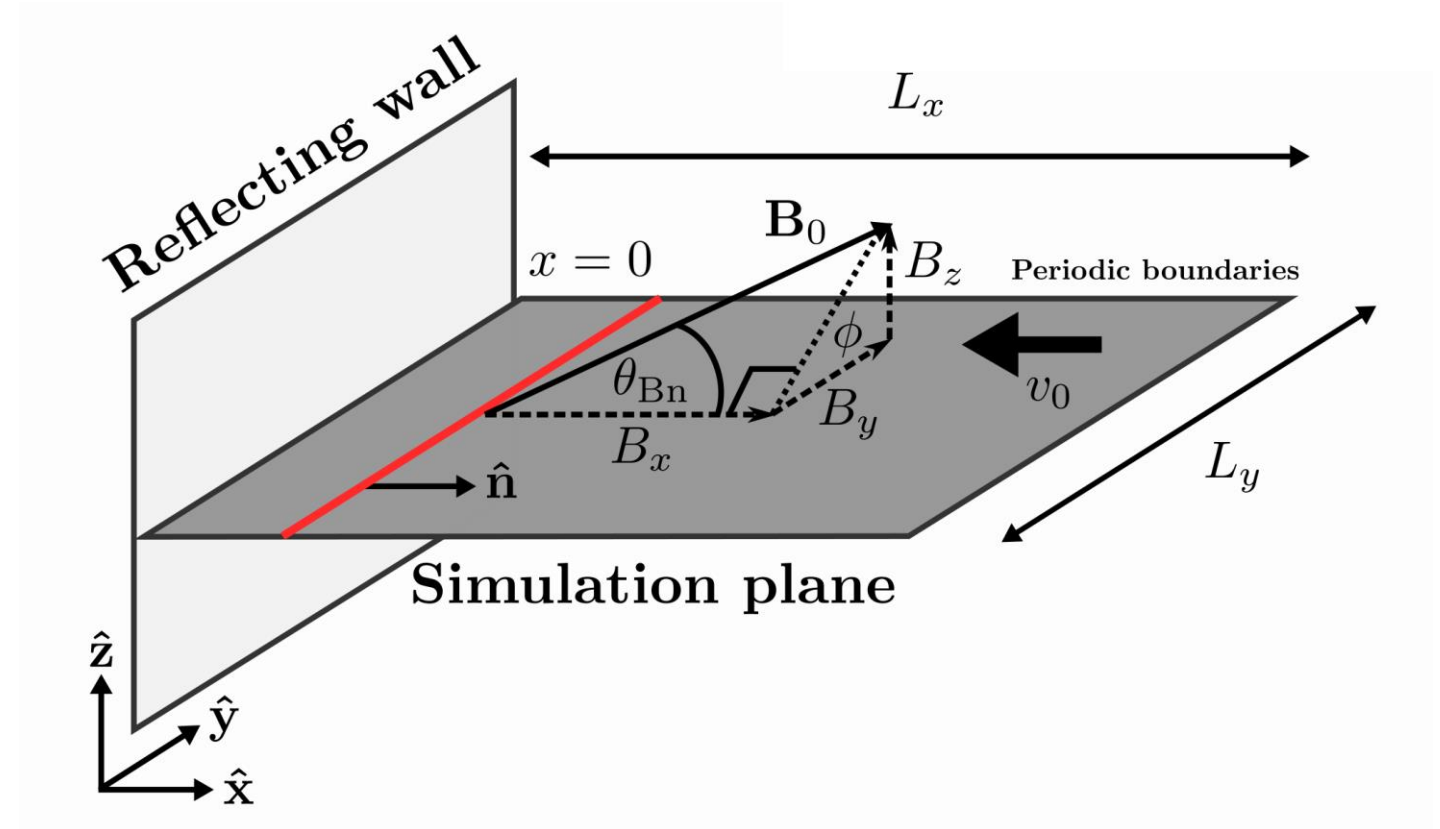
ion downstream spectra

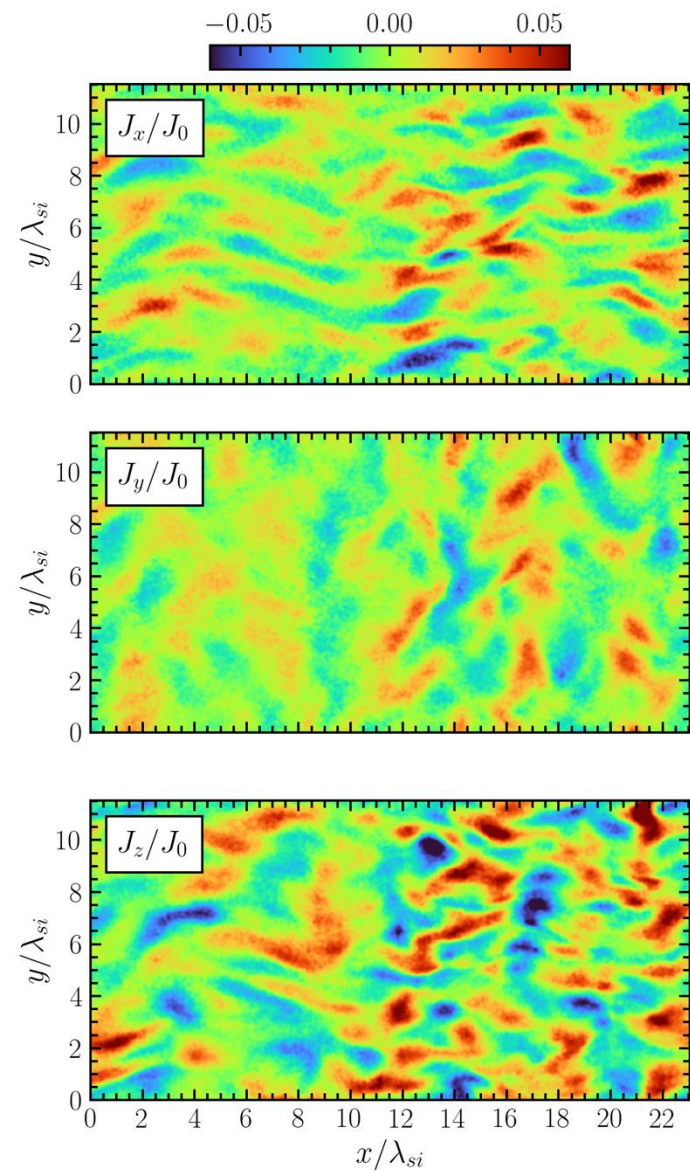
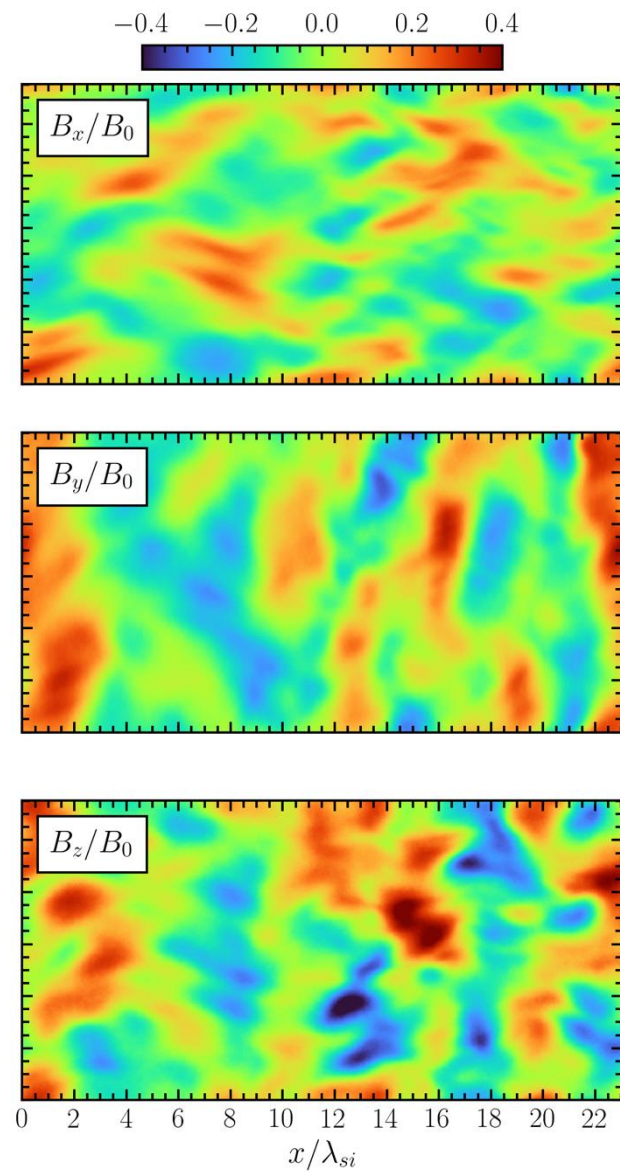




THATMPI code

- 2D3V configuration
- Reflecting wall setup

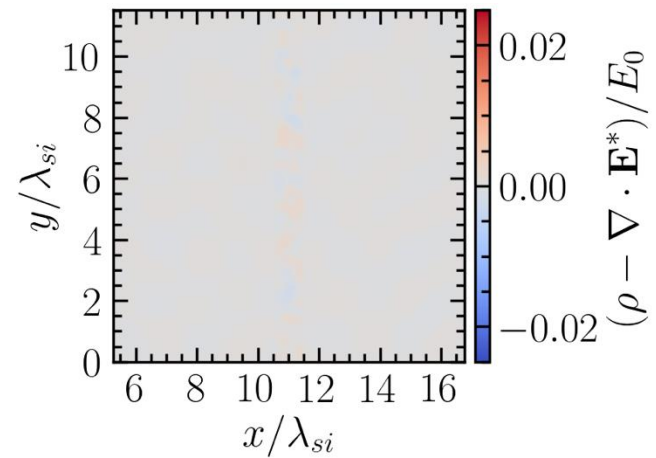
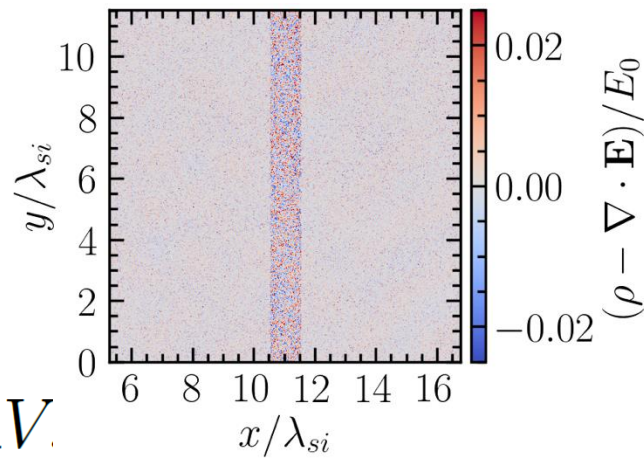




$$\delta\rho = \rho - \nabla \cdot \mathbf{E}$$

$$\rho - \nabla \cdot \mathbf{E}^* = 0 \quad \mathbf{E}^* = \mathbf{E} + \delta\mathbf{E}$$

$$\delta\mathbf{E}(\mathbf{x}) = \frac{1}{4\pi\epsilon_0} \int \delta\rho(\mathbf{x}') \frac{\mathbf{x} - \mathbf{x}'}{|\mathbf{x} - \mathbf{x}'|^3} dV$$

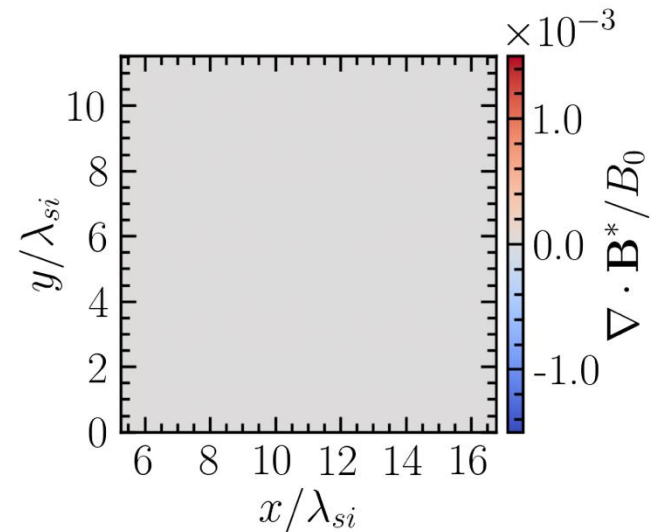
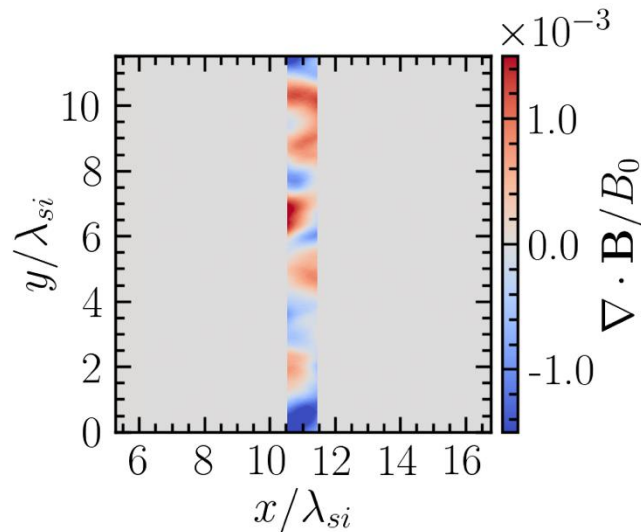


$$\nabla \cdot \mathbf{B} \neq 0$$

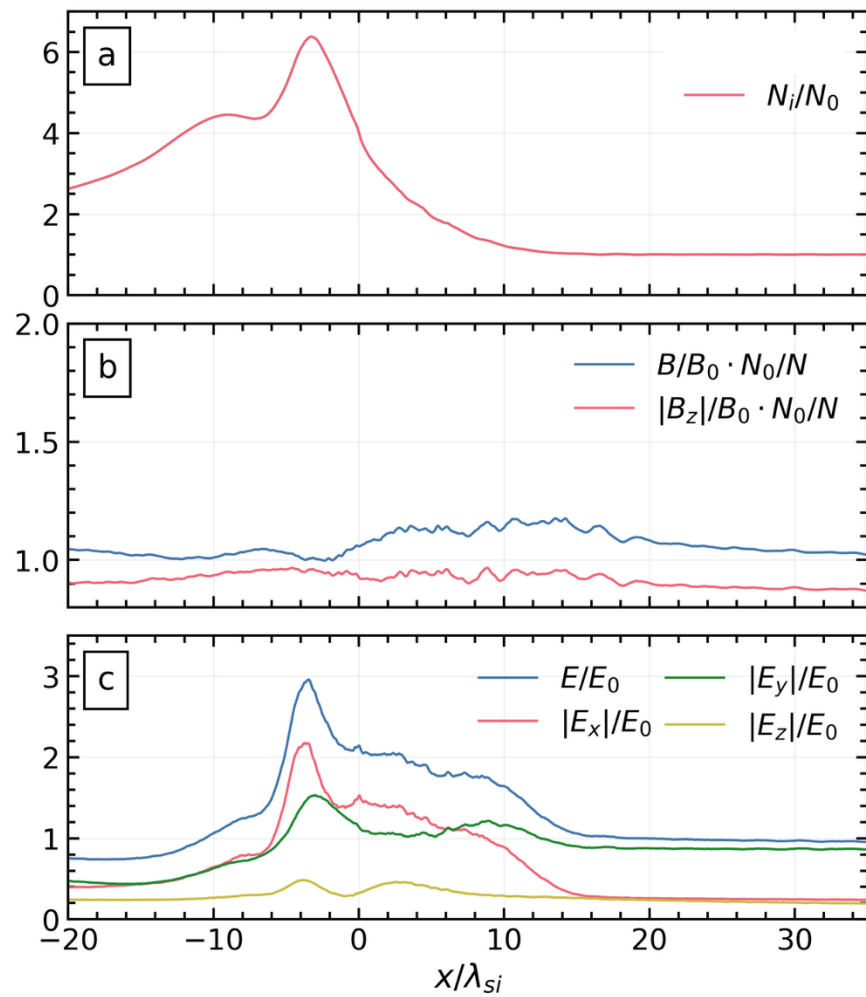
$$\mathbf{B} = \nabla \times \mathbf{A} + \nabla\phi$$

$$\nabla^2\phi = \nabla \cdot \mathbf{B}$$

$$\mathbf{B}^* = \mathbf{B} - \nabla\phi$$



homogeneous



turbulent

