Messengers from Imaginary Time

Instantons and Non-Perturbative physics – from quantum mechanics to QCD and experimental searches

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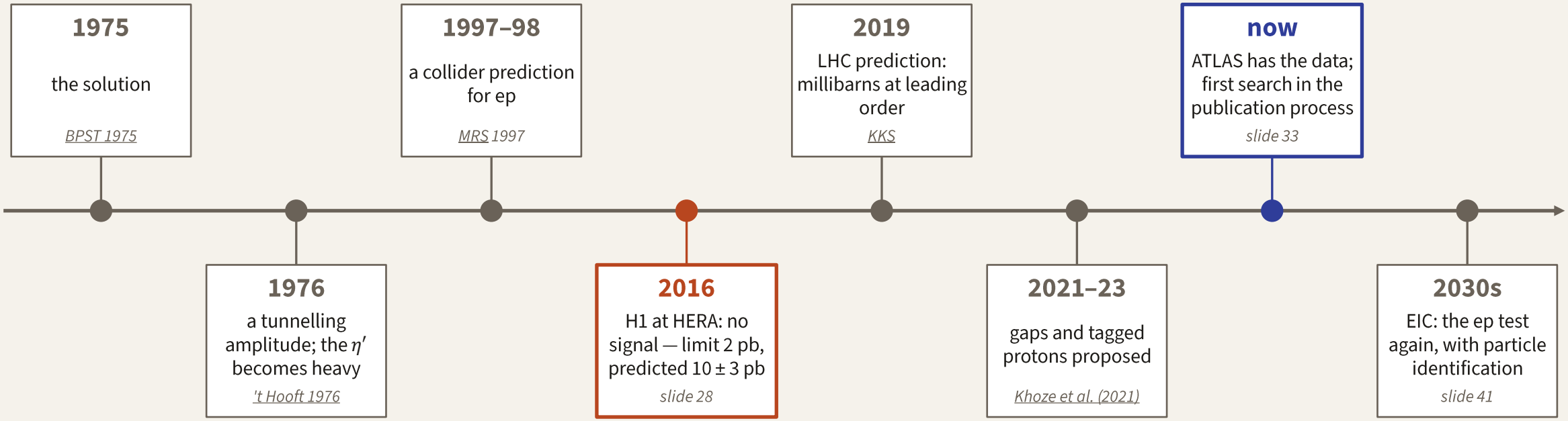
Seminar of the Division of Elementary Particle Physics

Institute of Physics of the Czech Academy of Sciences

Thursday, 12 November 2026

ESTABLISHED

Fifty years in the Standard Model — and never observed



Why care: the same objects stand behind the η' mass (slide 18), the puzzle of $\bar{\theta} < 10^{-10}$ (slide 19), and the only Standard-Model process that changes baryon number (slide 21).

Tonight: predicted at millibarns at the LHC; at HERA the upper limit is five times below the prediction — is the calculation missing something, or the search? **Behind the title:** are these messengers telling us that we live in a complex spacetime?

SYMBOLS ep electron–proton · HERA the ep collider at DESY, H1 one of its detectors · EIC Electron–Ion Collider (Brookhaven) · pb, mb units of cross section, 1 mb = 10^9 pb · $\bar{\theta}$ the CP-violating angle of QCD

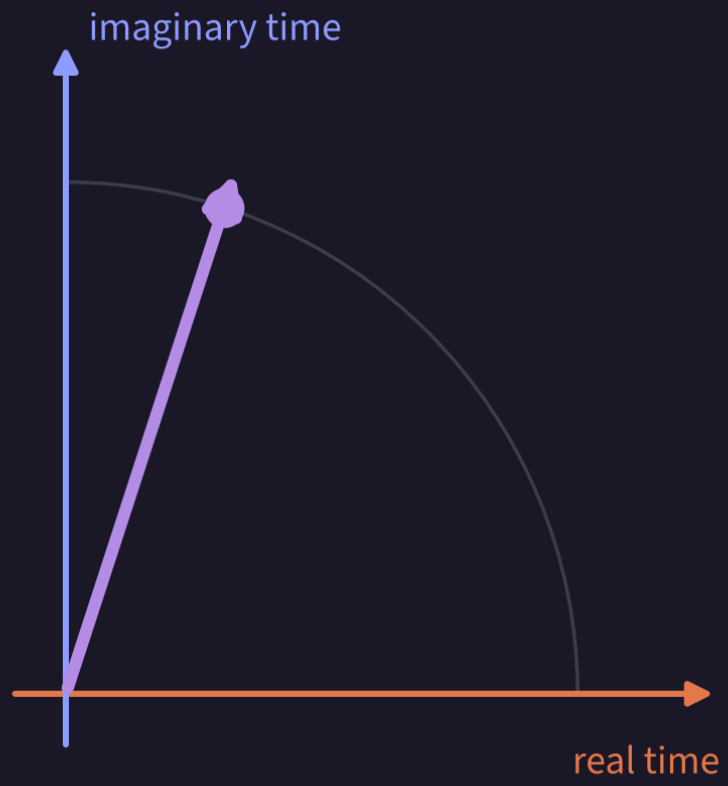
SOURCES Belavin et al. 1975 · 't Hooft 1976 (PRD) · Moch, Ringwald, Schrempp 1997 · Ringwald, Schrempp 1998 · H1 2016 · Khoze, Krauss, Schott 2020 · Khoze, Khoze, Milne, Ryskin 2021 · Taševský et al. 2023 · Taševský, Ryskin 2026

PART I

Imaginary time

What is a particle doing while it tunnels?

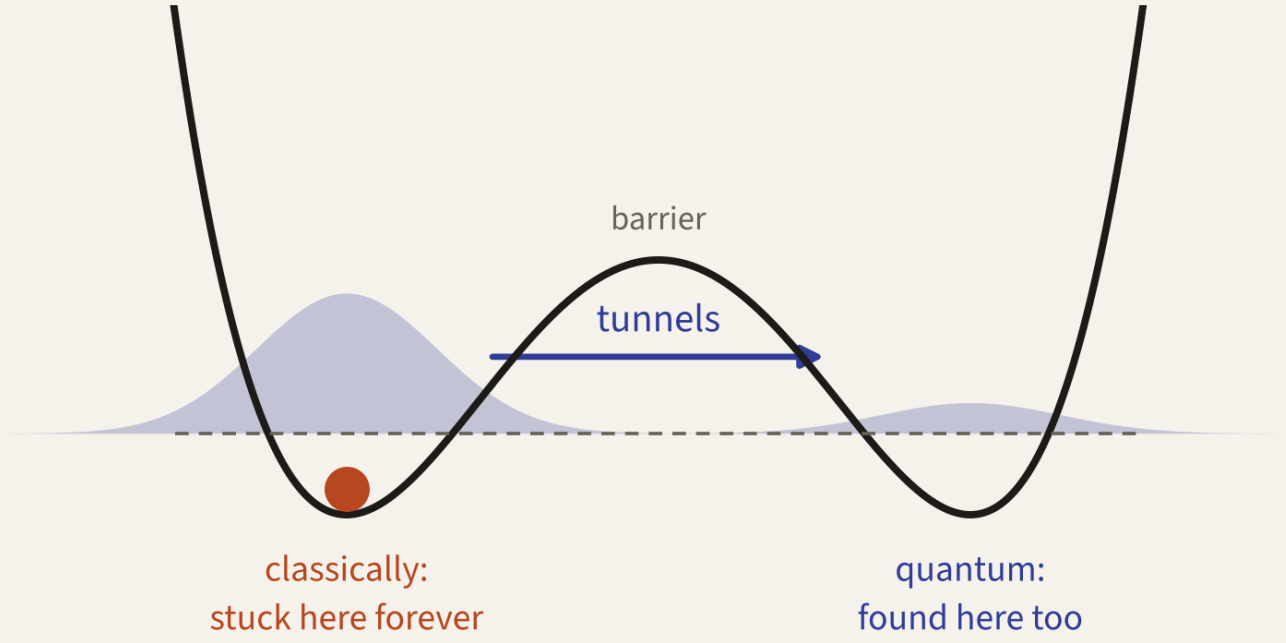
Instantons: predicted in 1975, part of the Standard Model, never observed directly. Is imaginary time only a way to compute the tunnelling amplitude — or part of the world? Keep the question open until the end.

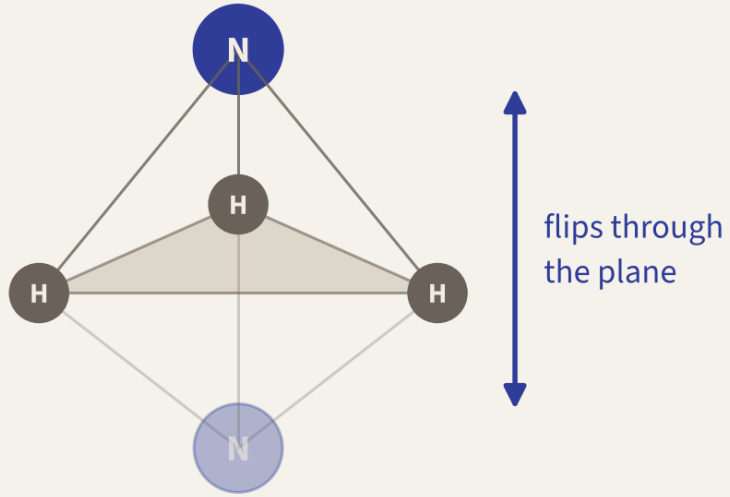


A progress marker only: the arrow turns from imaginary time back to real time as the talk goes on

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A ball that should not get through





ammonia NH_3: 23.87 GHz

Classically forbidden, quantum mechanically routine. What is the particle doing while it tunnels? The measurable trace is an energy splitting; we return to ammonia on slide 11.

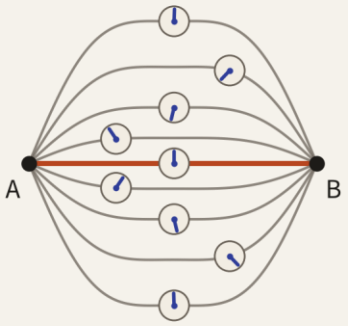
SYMBOLS barrier = a region where the potential energy is higher than the particle's energy · NH_3 ammonia molecule · $\mathrm{GHz} = 10^9$ oscillations per second · QCD quantum chromodynamics, the theory of quarks and gluons · LHC Large Hadron Collider (CERN) · ATLAS one of its two large general-purpose detectors · EIC Electron–Ion Collider (Brookhaven) · B1, B2, … backup slides after the talk

SOURCES Paris Observatory dictionary: ammonia maser

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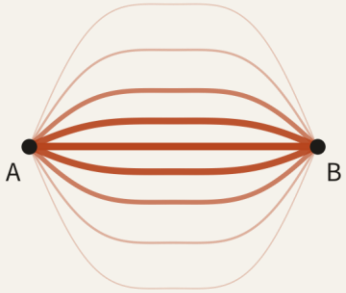
Feynman: the particle tries every path

real time: every path carries a phase (a clock hand)



$e^{iS/\hbar}$: the hands point everywhere and mostly cancel

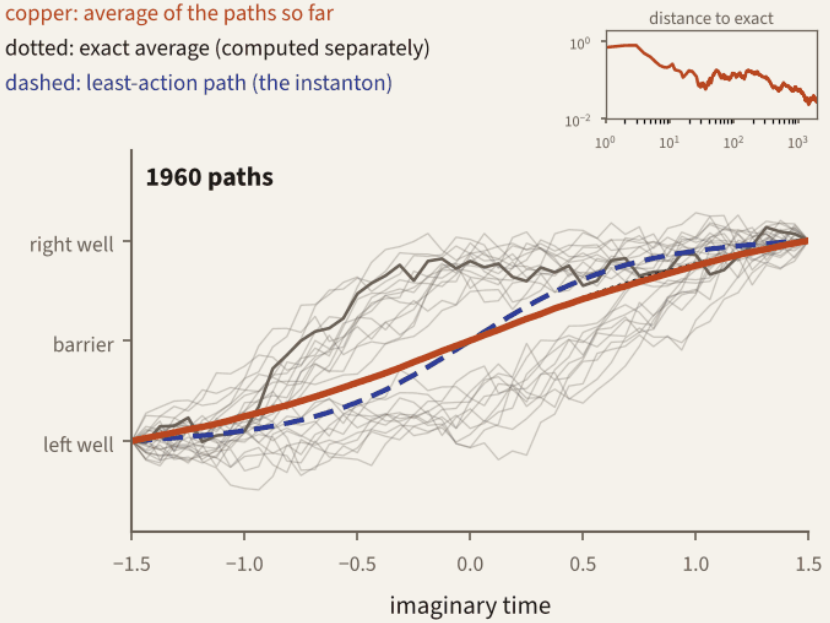
imaginary time: every path carries a weight



$e^{-S_E/\hbar}$: paths near the least-action path dominate

Random paths, weighted by $e^{-S_E/\hbar}$

copper: average of the paths so far
dotted: exact average (computed separately)
dashed: least-action path (the instanton)



Computed (Metropolis), $V = (x^2 - 1)^2$, $\hbar = 0.25$; exact average by transfer matrix

① **Real time:** $\int \mathcal{D}x\, e^{iS/\hbar}$ — complex phases interfere; no single path can be picked out.

② **Imaginary time, $t = -i\tau$:** $\int \mathcal{D}x\, e^{-S_E/\hbar}$ — every weight is positive: a probability distribution over paths.

③ **Sample it:** Metropolis keeps paths with probability $\propto e^{-S_E/\hbar}$, so their plain average is the weighted average $\langle x(\tau) \rangle$ (copper); the exact sum on a grid (dotted) checks it.

④ **The physics is in the sum:** for large T it falls as $e^{-E_0 T/\hbar}$ and gives the levels (the splitting: slide 11). The average is not the instanton (dashed): each path crosses at its own moment.

Which paths dominate? In the semiclassical limit, those near the classical imaginary-time path; with the endpoints in opposite wells, that path is the instanton.

SYMBOLS path = one possible history $x(t)$ between A and B · S action of a path · $e^{iS/\hbar}$ its phase (a clock hand) · $e^{-S_E/\hbar}$ its weight in imaginary time

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Three faces of one idea

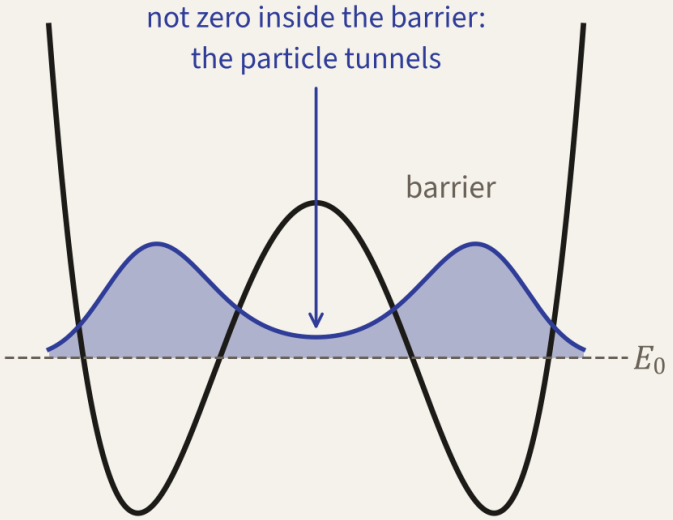
statistical physics: a gas at temperature T



energy levels how often occupied

imaginary time $\hbar/k_BT$ gives these weights

quantum mechanics: a particle in a double well



lowest state: probability $|\psi_0(x)|^2$, drawn at its energy E_0

the tunnelling path runs in imaginary time: weight $e^{-S_0/\hbar}$

gravity: a black hole



dots: Hawking radiation,
thermal at temperature T_H

imaginary time around the horizon
must close: period $\hbar/k_BT_H$

$$t \;\to\; -i\tau \qquad\qquad e^{\,iS/\hbar} \;\to\; e^{-S_E/\hbar}$$

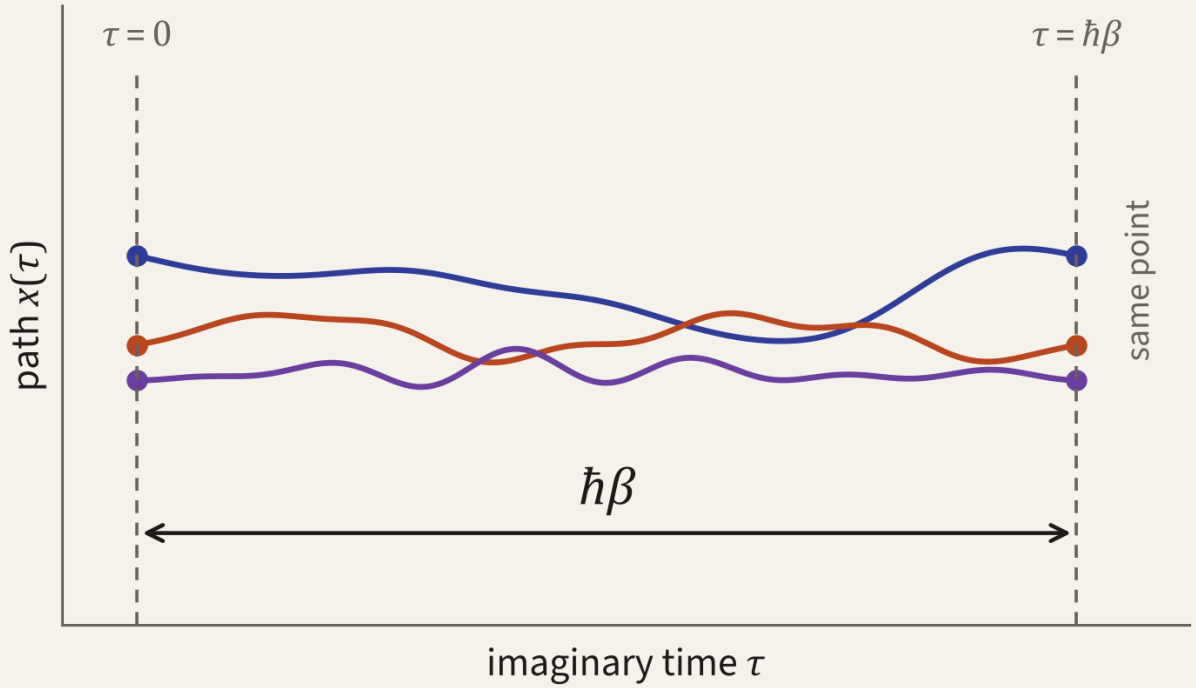
One step, $t \to -i\tau$, behind all three (violet). Three applications of one continuation, not three identical systems. Left and right: a period in imaginary time is a temperature. Middle: the tunnelling path lives in imaginary time.

SYMBOLS E_n energy levels · k_B Boltzmann constant · T temperature · $|\psi_0(x)|^2$ probability of finding the particle at x in the lowest state · E_0 its energy · S_0 cost of the tunnelling path · T_H Hawking temperature of a black hole · τ imaginary time

SOURCES Hawking 1975 · Gibbons, Hawking 1977

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Temperature sets an imaginary-time period



Thermal physics adds up all states, each with its Boltzmann factor: $Z = \sum_n e^{-E_n/k_BT}$.

Each factor is quantum evolution $e^{-iE_nt/\hbar}$, run for an imaginary time $t = -i\hbar/k_BT$.

The sum is a trace of the evolution operator: in paths, each path returns to its starting point, and we integrate over that point. Paths close; imaginary time becomes a circle of length $\hbar/k_BT$.

Without equilibrium there is no temperature and no circle: time then runs along an open contour in the complex plane.

Here the temperature sets the period. On the next slide geometry fixes the period, and with it a temperature.

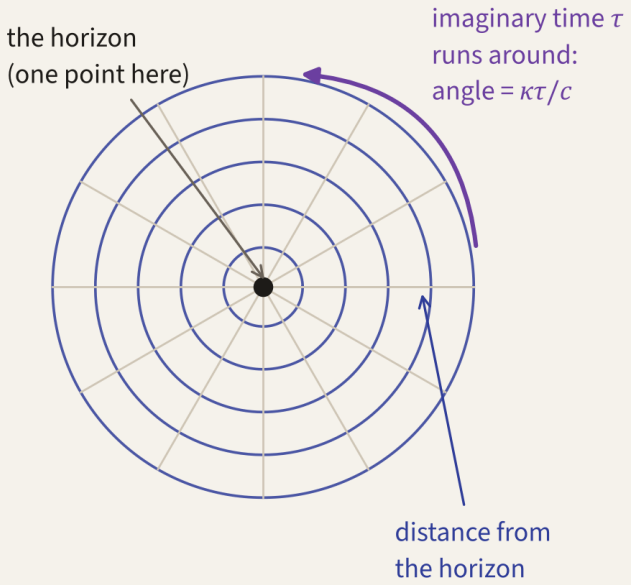
IN DETAIL $Z = \mathrm{Tr}\, e^{-\beta H} = \oint \mathcal{D}x\; e^{-S_E[x]/\hbar}, \qquad x(0) = x(\hbar\beta)$ the trace: each state back to itself, all states summed

SYMBOLS Z partition function = the sum over all states of e^{-E_n/k_BT} · $\beta = 1/k_BT$ · k_B Boltzmann constant · trace = sum over all states of 'start in a state, end in the same state' · for the staff: H Hamiltonian, $|n\rangle$ its states, S_E Euclidean action · Σ sum over

SOURCES Hawking 1975 · Gibbons, Hawking 1977

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Black holes have a temperature for the same reason



imaginary time closes after $2\pi c/\kappa$:
smooth, no tip



any other period: a wedge is missing,
the plane becomes a cone with a sharp tip

Not the black hole in space. Two directions only: distance from the horizon (outwards) and imaginary time (around). The horizon is the centre point.

Imaginary time is an angle, $\kappa\tau/c$: the arrow is not a motion around the black hole.

Smooth only if the angle closes after one turn: a fixed period $2\pi c/\kappa$, so a temperature, as on slide 7.

About 6×10^{-8} K for a black hole of one solar mass.

An imaginary-time period is a temperature; here smoothness fixes it: black holes glow. For tunnelling we need something else: an open path connecting two configurations.

IN DETAIL

$$T_H = \frac{\hbar\kappa}{2\pi c\, k_B} = \frac{\hbar c^3}{8\pi G M k_B} \approx 6 \times 10^{-8}\,\mathrm{K} \times \frac{M_\odot}{M}$$

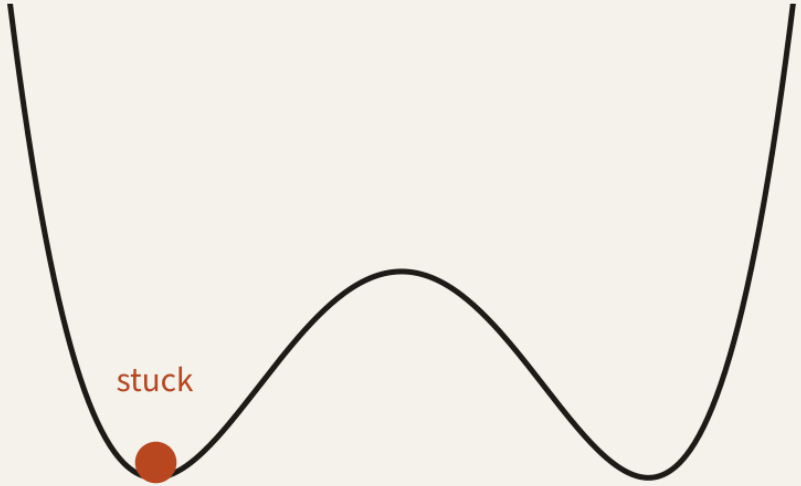
SYMBOLS κ surface gravity (how strongly the horizon pulls) · τ imaginary time · c speed of light · T_H Hawking temperature · G Newton's constant · M mass of the black hole · M_{sun} mass of the Sun · $\pi = 3.14159\ldots$

SOURCES Hawking 1975 · Gibbons, Hawking 1977

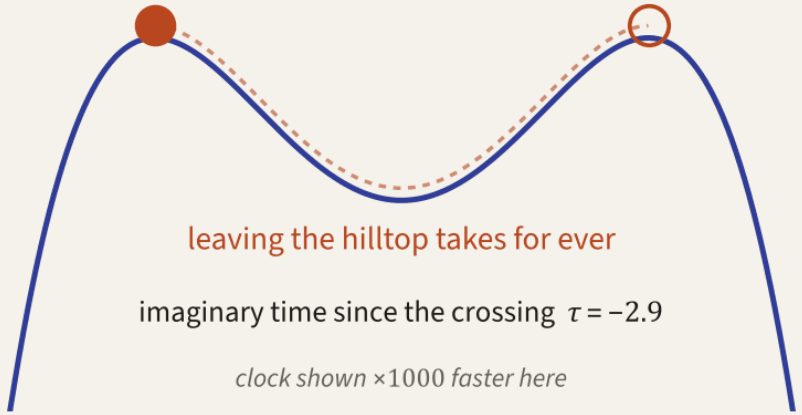
A barrier becomes a valley

$$m\frac{d^2x}{dt^2} = -V'(x) \quad \overset{t=-i\tau}{\Rightarrow} \quad m\frac{d^2x}{d\tau^2} = +V'(x)$$

real time: the potential V



imaginary time: upside down, $-V$

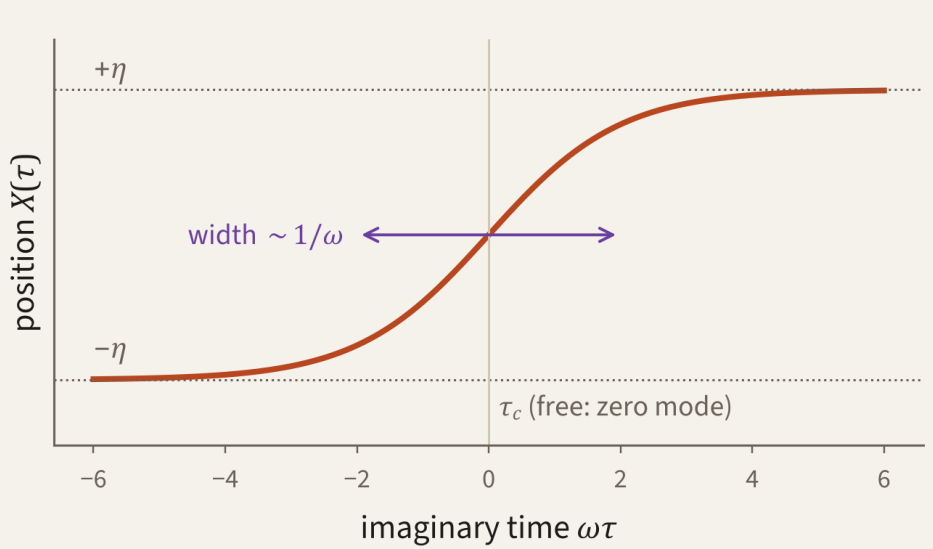


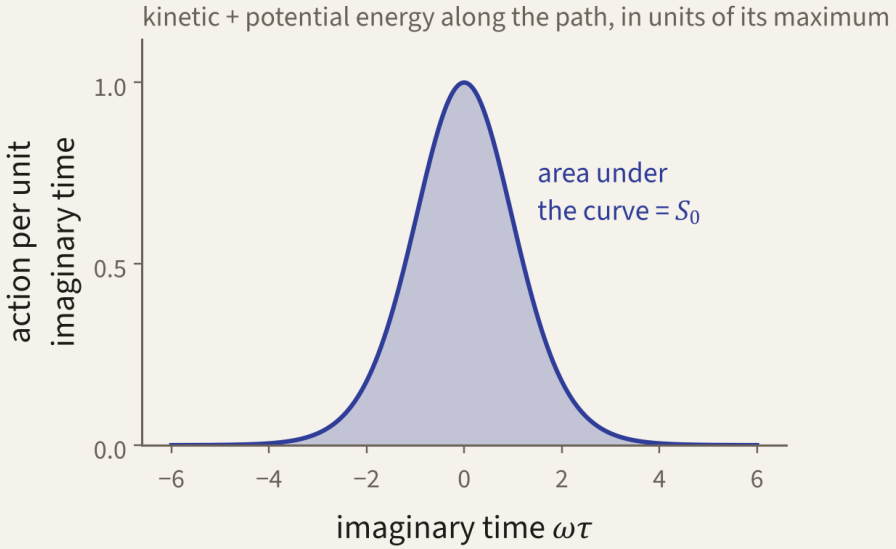
The potential has not changed: the imaginary-time equation reads like mechanics in $-V$. Its special path leaves one hilltop in the infinite past and reaches the other in the infinite future: that path is the instanton.

SYMBOLS x position · m mass · $V(x)$ potential energy · $V' = dV/dx$ (minus the force) · τ imaginary time · valley = the barrier seen upside down: in imaginary time the particle moves in $-V$

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The instanton of the double well





WKB, the semiclassical method of the quantum-mechanics course: inside the barrier ψ falls off like $\exp\left(-\frac{1}{\hbar}\int\sqrt{2m(V-E)}\,dx\right)$. At the bottom of the well ($E \to 0$) the instanton's action S_0 gives the same factor $e^{-S_0/\hbar}$, now as the action of a path. Why no power series can produce it: slide 12.

The instanton: one well, a crossing of finite width in imaginary time, the other well. It turns the under-barrier exponent into the action of a path; slide 11: what it does to a measurable energy.

IN DETAIL

$$S_0 = \int_{-\eta}^{\eta}\sqrt{2mV(x)}\,dx = \frac{4}{3}\sqrt{2\Lambda}\,\eta^3 \qquad (m=1)$$

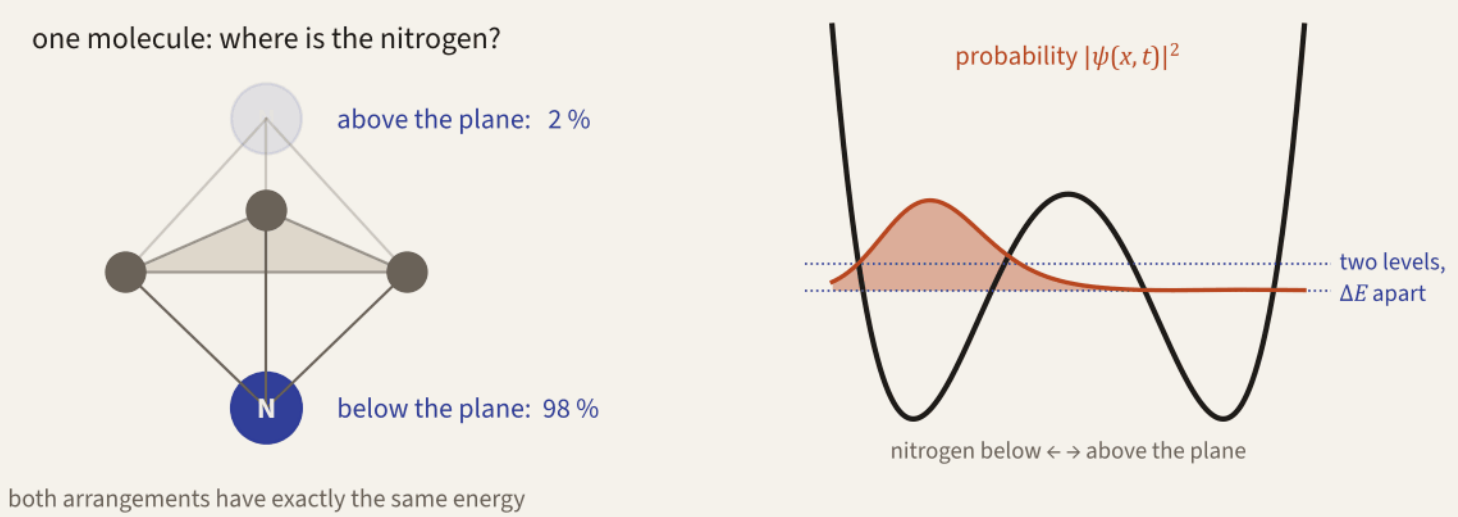
the WKB exponent at zero energy — for $V = \Lambda(x^2-\eta^2)^2$

SYMBOLS $X(\tau)$ position along the crossing · τ imaginary time · S_0 the cost (action) of the crossing · WKB (Wentzel–Kramers–Brillouin) the semiclassical approximation of the quantum-mechanics course · E energy · for the staff: Λ coupling, $\pm\eta$ the two minima, m mass, ω frequency of small vibrations

SOURCES Loubal, Sýkora, Kos 2026

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Ammonia: tunnelling you can measure



Two positions → two coherent superpositions → two levels: symmetric and antisymmetric. The line between them: measured 1934 (Cleeton, Williams); the first maser, 1954 (Gordon, Zeiger, Townes).

Why so small? About 10^{-3} of one vibration (0.79 vs 950 cm^{-1}). The splitting is measured; the instanton action sets its factor $e^{-S_0/\hbar}$.

Honest: for one molecule, WKB gives the same leading exponential. The instanton picture carries over to fields, where the configuration is a whole function of space, not one coordinate.

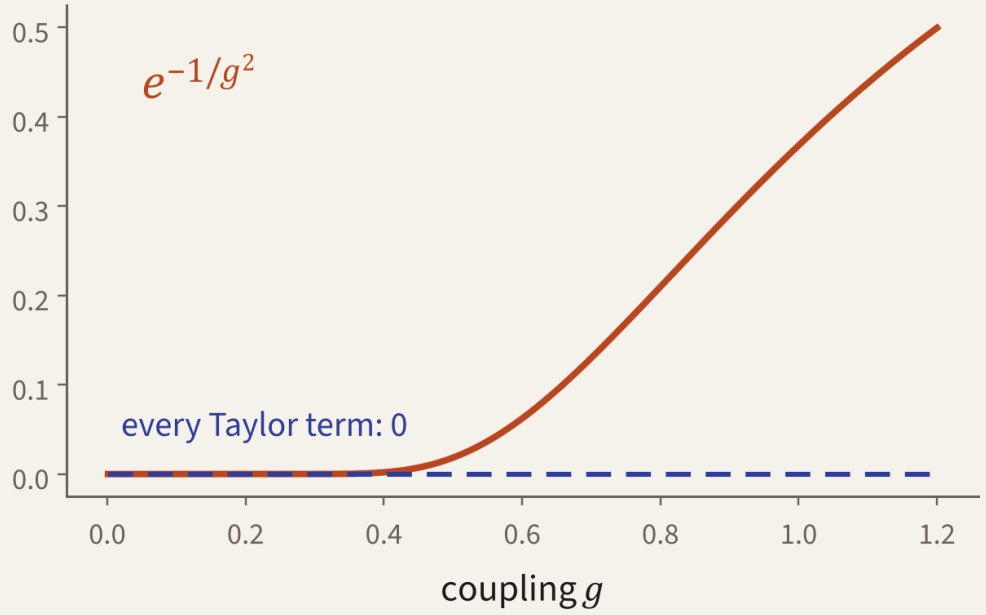
IN DETAIL $$E_{\pm} = \frac{\hbar\omega}{2} \mp \hbar K\, e^{-S_0/\hbar}$$ a dilute gas of instantons and anti-instantons

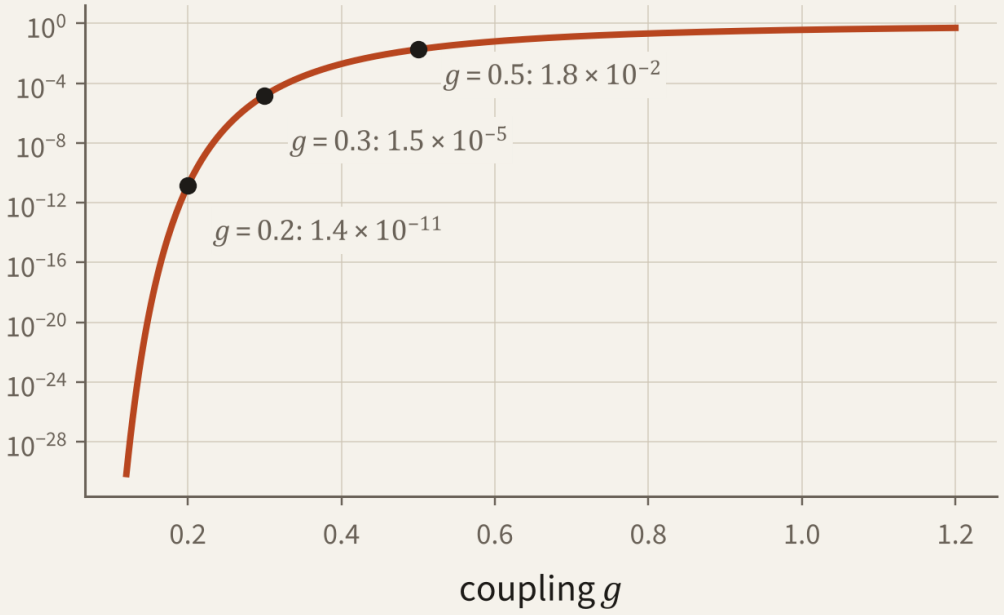
SYMBOLS maser = microwave amplification by stimulated emission of radiation, the laser's older sibling · GHz $= 10^9$ oscillations per second · λ wavelength · cm^{-1} energy in spectroscopists' units · for the staff: $E_{\pm}$ the two lowest levels, K fluctuation prefactor

SOURCES Cleeton, Williams 1934 · Gordon, Zeiger, Townes 1954 · Rioux, LibreTexts · Paris Observatory dictionary: ammonia maser

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Invisible to perturbation theory





$$\frac{d^n}{dg^n} e^{-1/g^2}\bigg|_{g=0} = 0 \qquad \text{for every } n$$

$$e^{-8\pi^2/g^2} = e^{-2\pi/\alpha_s} \approx 2 \times 10^{-14} \qquad (\alpha_s = 0.2)$$

Perturbation theory expands around one well. Tunnelling adds e^{-1/g^2}: every derivative at $g = 0$ vanishes, although the function is not zero for $g \neq 0$. It is missing at every finite order.

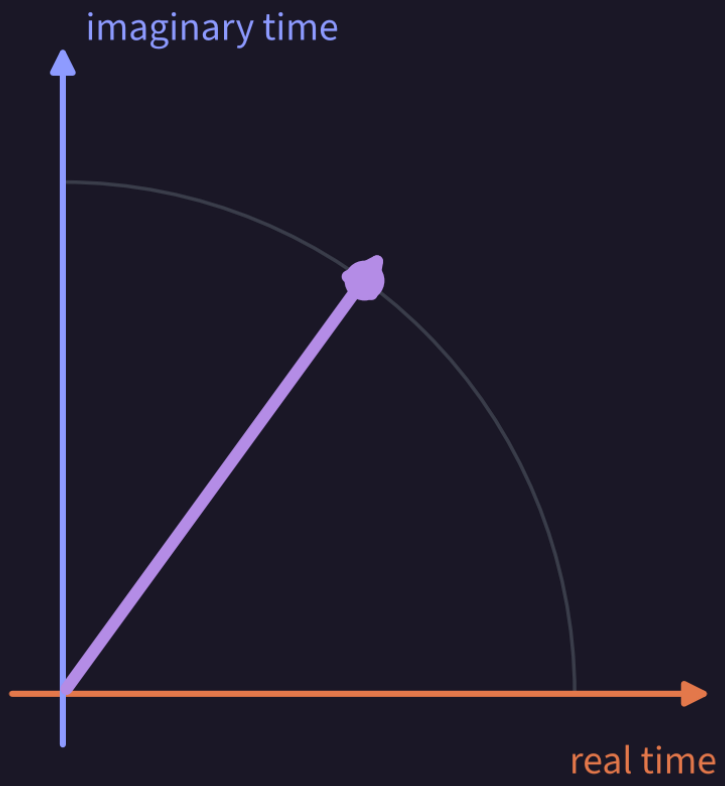
SYMBOLS g coupling: in our double well a small g means a high, wide barrier, $S_0 \propto 1/g^2$ · perturbation theory = expanding in powers of g around one well · QCD quantum chromodynamics, the theory of quarks and gluons · α_s strong coupling

PART II

Topology

How does tunnelling shape the forces of nature?

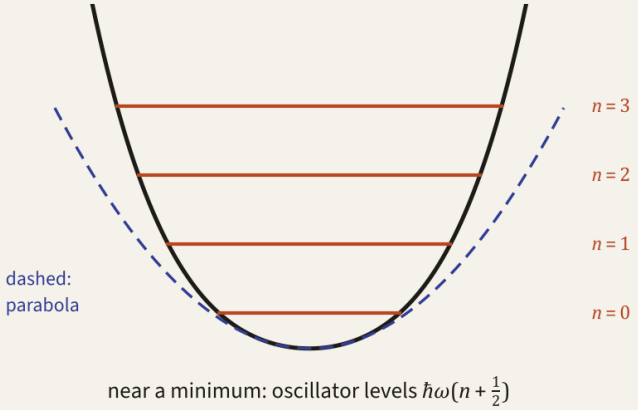
We know how a path between two endpoints contributes. What decides which endpoints exist? For a particle we drew the wells by hand; for gauge fields, topology helps organise the vacua and the paths between them.

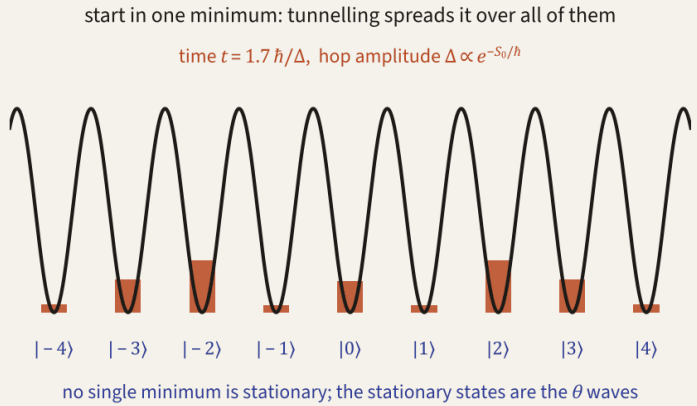


Progress marker

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What is the vacuum, and what is a particle?





illustrative potentials; right: exact tight-binding evolution, probabilities $J_n(2\Delta t/\hbar)^2$

A particle is a quantum of small oscillation around a minimum: levels $\hbar\omega(n+1/2)$; in a field, n counts particles.

Several minima: tunnelling mixes them (two wells: slide 11).

Many equivalent minima: the vacuum is a Bloch wave, $|\theta\rangle = \sum_n e^{in\theta}|n\rangle$ — for gauge fields, the θ vacuum.

The vacuum is not ‘nothing’: the minima, and the tunnelling between them, determine what the particles are.

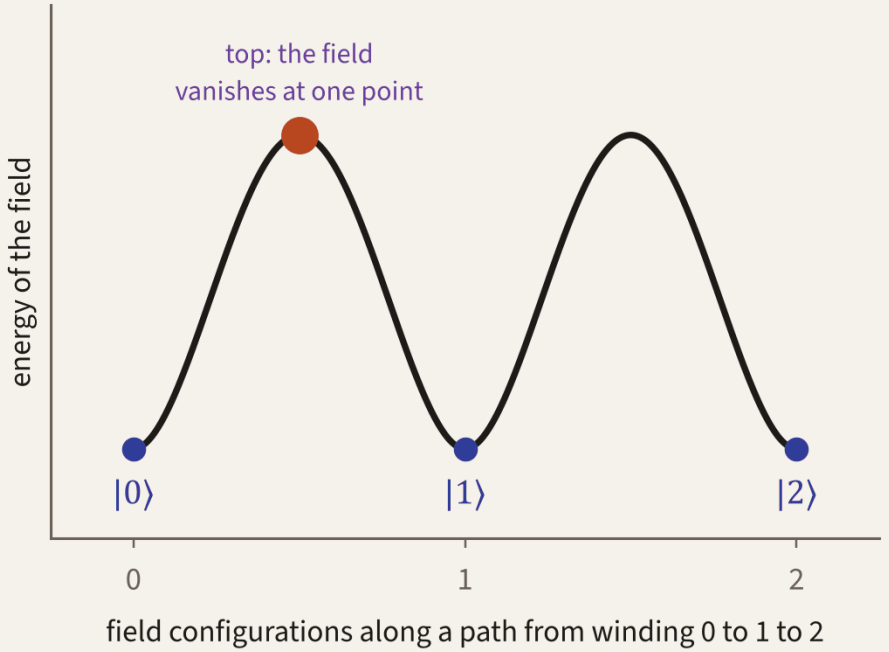
Dilute gas of hops $|n\rangle \to |n \pm 1\rangle$: $E(\theta) = \hbar\omega/2 - 2\hbar K\, e^{-S_0/\hbar}\cos\theta$

SYMBOLS normal mode = an independent pattern of small oscillation · $\hbar\omega$ one quantum of a mode · $|n\rangle$ state sitting in the n-th minimum · θ Bloch angle = vacuum angle · K fluctuation prefactor · θ vacuum = the superposition $\Sigma\, e^{in\theta}|n\rangle$ of all the vacua $|n\rangle$ · Δ hop amplitude between neighbouring minima, $\propto e^{-S_0/\hbar}$ · J_n Bessel function

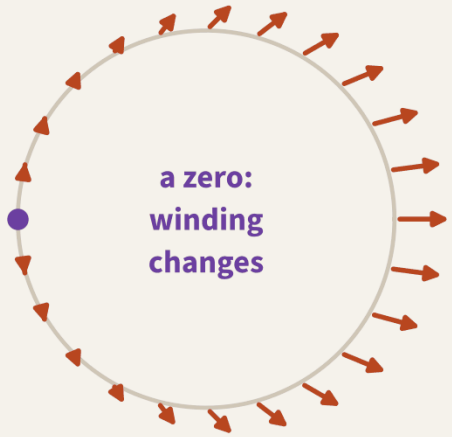
SOURCES Jackiw, Rebbi 1976 · Callan, Dashen, Gross 1976

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Gauge fields wind too



field configurations along a path from winding 0 to 1 to 2



the field on a ring in space
(arrow = its phase and size)

From the particle to the field (animation: a toy field on a ring):

position x → a whole gluon field in space

$V(x)$ → the energy of that field

two wells → vacua with winding …, −1, 0, 1, …

instanton → tunnelling to the next vacuum

Sphaleron: the top; it returns on slide 21, where heat lets fields hop over.

The integer labels carry over; the coordinate does not. Each point is a whole field configuration, and the drawing is a schematic path through a vastly larger space.

IN DETAIL $\left|\theta\right\rangle = \sum_{n\in\mathbb{Z}} e^{in\theta} \left|n\right\rangle$ the true vacuum: every winding, with the phase θ

SYMBOLS gauge field = the field that carries a force (photons for electromagnetism, gluons for the strong force) · configuration = the whole field in space at one moment · N_{CS} winding number of a configuration · $|n\rangle$ lowest-energy field with winding n · sphaleron = the field at the top of the barrier

SOURCES Callan, Dashen, Gross 1976 · Jackiw, Rebbi 1976

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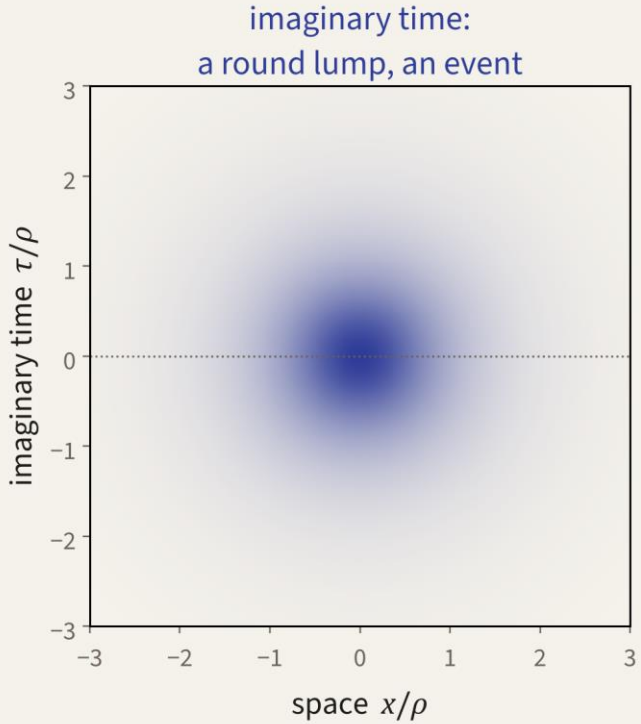
The instanton of the strong force

$$A^a_\mu(x) = \frac{2}{g}\,\frac{\eta^a_{\mu\nu}\,(x-z)_\nu}{(x-z)^2+\rho^2}$$

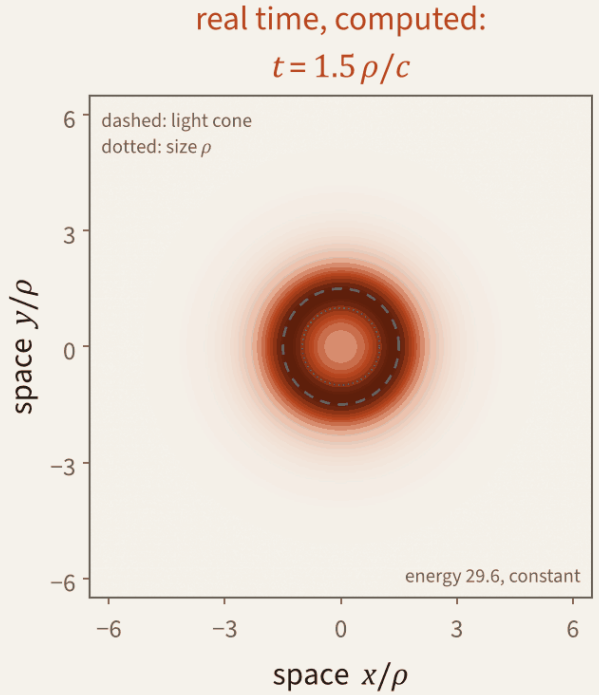
In words: a gluon field centred at z, falling off with distance, with one free size ρ. Sliced at fixed imaginary time, it interpolates between the vacua of slide 15.

The right picture is not the real-time continuation of the left one. It is a related construction (Shuryak–Zahed): at fixed size ρ, the field at the barrier top, energy $3\pi^2/(g^2\rho)$, purely magnetic and at rest at $t = 0$, then evolved by the ordinary real-time equations (evaluated by our script).

Through the barrier (left; zero energy, not zero action) versus evolving from its top (right): the distinction returns on slide 21.



action density of the BPST formula



energy density in a plane, normalised at each time

► full version with the energy check: backup B43

IN DETAIL $S_{\text{inst}} = \dfrac{8\pi^2}{g^2} = \dfrac{2\pi}{\alpha_s}$ for every size ρ the classical theory has no length scale

SYMBOLS A gluon field · z its centre · ρ its size · x a point of four-dimensional spacetime · g strong coupling · for the staff: η 't Hooft symbol (regular gauge) · BPST Belavin, Polyakov, Schwartz, Tyupkin (the 1975 solution)

SOURCES Belavin et al. 1975 · 't Hooft 1976 (PRD) · Shuryak, Zahed 2021

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One instanton is a family: collective coordinates and moduli



one instanton of SU(3): 4 + 1 + 7 = 12 = $4N$ collective coordinates (cartoon)

Symmetry → zero modes: every symmetry the solution breaks gives a deformation at no cost: a collective coordinate.

Integrated exactly: $\int d^4z\, d\rho\, d\Omega$ — the origin of $D(\rho)$ and of the size integral (slide 25).

Moduli space: k instantons of SU(N) form a space of dimension $4kN$, built explicitly by ADHM (1978). On D-branes the instanton is itself a D-brane (Douglas 1995); moduli-space integrals give exact results (Nekrasov 2002).

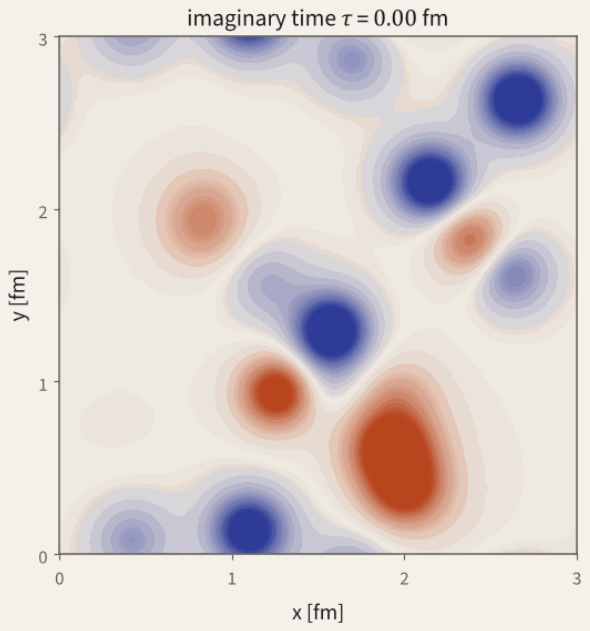
One instanton of QCD is a 12-parameter family. Its position and size are sampled by a generator; its colour orientation becomes colour factors.

SYMBOLS collective coordinate = a parameter of a family of solutions with the same action · zero mode = a deformation that costs no action · moduli space = the space of all solutions · ADHM Atiyah–Drinfeld–Hitchin–Manin · D-brane = a surface on which strings end · SU(N) the symmetry group of N colours (SU(3) for the strong force) · Ω colour orientation

SOURCES Dorey et al. 2002 · Atiyah et al. 1978 · Douglas 1995 · Nekrasov 2002

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Why instantons matter: mass — and what they do not explain



Mass: the Higgs gives the u and d quarks a few MeV, about 1% of the proton's 938 MeV; most of the proton's mass arises from QCD dynamics. The instanton-liquid model addresses an important part of this physics: chiral symmetry breaking through overlapping quark zero modes.

The η': without the anomaly, a ninth light meson; the axial anomaly makes it heavy — through instantons, in the picture of 't Hooft 1976.

Not confinement: small, dilute lumps give no string tension; on the lattice, removing centre vortices removes confinement.

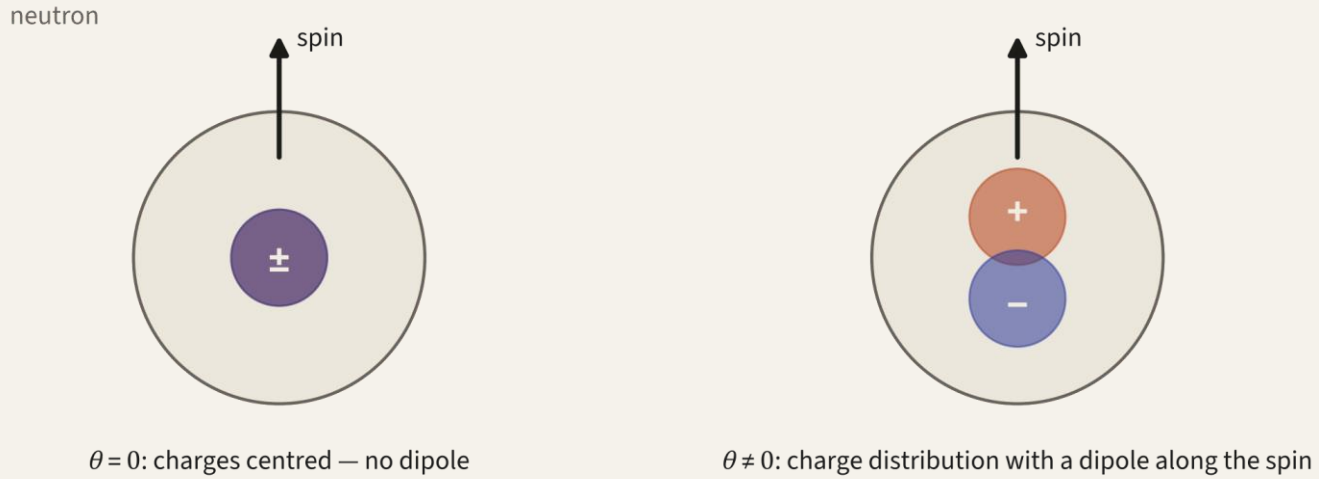
Instantons illuminate chiral symmetry breaking and the η' mass — but are not, by themselves, an explanation of confinement. The liquid is a model; the animation is an illustration, not lattice data.

SYMBOLS chiral symmetry = left- and right-handed quarks transform independently · condensate = a non-zero vacuum average of quark–antiquark pairs · η' a heavy neutral meson · string tension = the constant force between a distant quark and antiquark · centre vortex = another topological object of the gluon field · anomaly = a classical symmetry broken by quantum effects · instanton liquid = a model: the vacuum as many instantons

SOURCES Schäfer, Shuryak 1998 · 't Hooft 1976 (PRL) · de Forcrand, D'Elia 1999

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Fingerprint 1: the strong force has its own θ



$\theta = 0$: charges centred — no dipole

$\theta \neq 0$: charge distribution with a dipole along the spin

Like the Bloch angle of slide 14: each winding n carries a phase $e^{in\bar{\theta}}$. $\bar{\theta}$ is a parameter of the theory, not a property of one collision.

Why the neutron? A mirror turns winding n into $-n$, so a generic nonzero $\bar{\theta}$ breaks mirror symmetry: the neutron's charge distribution would get an electric dipole along its spin.

No dipole detected: the neutron-EDM upper limit implies $|\bar{\theta}| \lesssim 10^{-10}$.

$\bar{\theta}$ could take any value, yet $|\bar{\theta}|$ is below 10^{-10}. Why? The strong CP problem. An axion would make $\bar{\theta}$ a field that relaxes to zero by itself, and might be the dark matter.

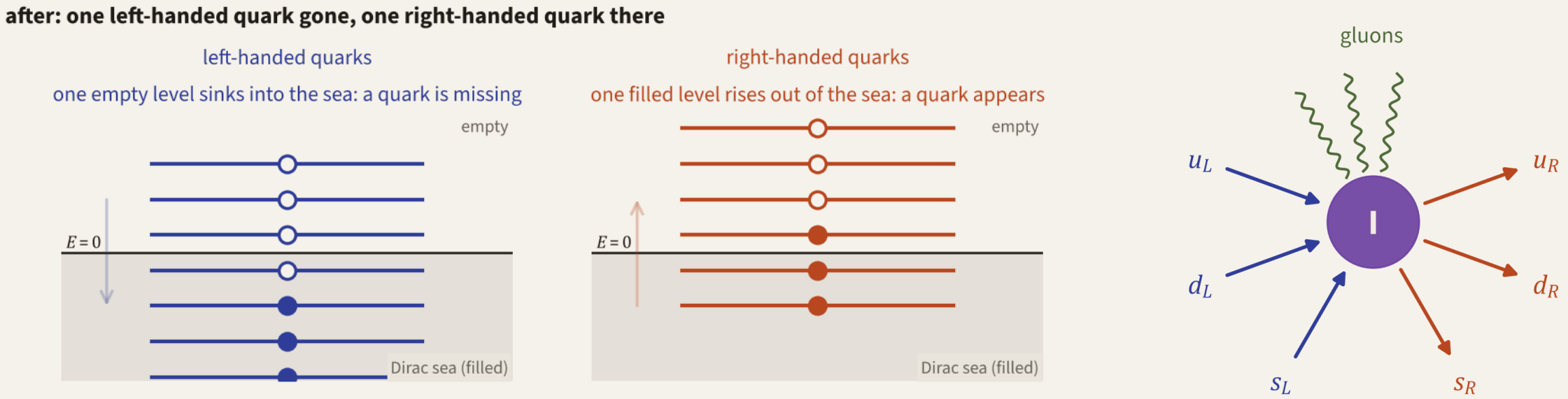
IN DETAIL $$d_n \sim 10^{-16}\,\bar{\theta}\ e\,\mathrm{cm}, \quad d_n^{\mathrm{exp}} = (0.0 \pm 1.1 \pm 0.2) \times 10^{-26}\,e\,\mathrm{cm} \ \Rightarrow\ |\bar{\theta}| \lesssim 10^{-10}$$

SYMBOLS fingerprint = a measurable consequence of the windings, not a sighting of one instanton · $\bar{\theta}$ the vacuum angle of the strong force (the Bloch angle of the gluon vacua) · d_n electric dipole moment of the neutron · e elementary charge · axion = a hypothetical particle · CP charge conjugation times parity (particle ↔ antiparticle, mirror)

SOURCES Abel et al. 2020 · Peccei, Quinn 1977 · Weinberg 1978 · Wilczek 1978

ESTABLISHED DEEPER DIVE ›

An instanton flips quarks



Schematic of the level flow, one light flavour; the same for u, d and s at once. The quark number is unchanged, the handedness is not.

Why a vertex: in an instanton background the massless Euclidean Dirac operator has one normalisable zero mode of definite chirality per flavour (index theorem). These give two legs per flavour: for u, d, s a six-quark 't Hooft interaction. **It changes axial charge (right- minus left-handed) but conserves net quark number, hence baryon number.**

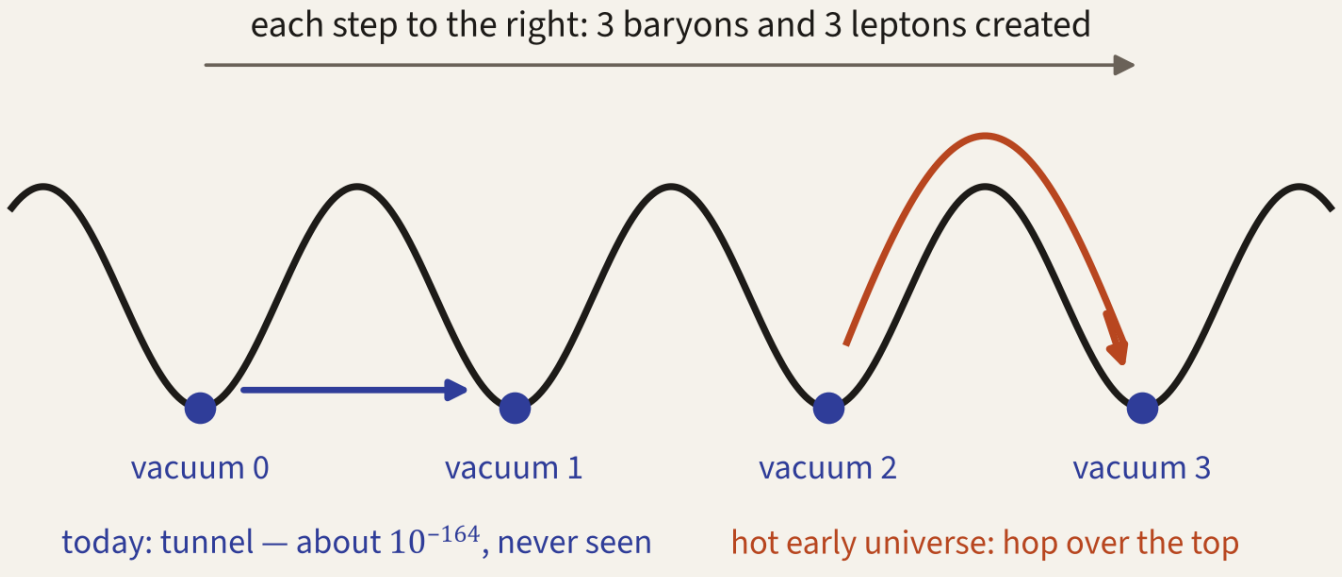
IN DETAIL $n_L - n_R = Q$ (per massless flavour), $\Delta Q_5 = 2N_f\, Q$ Atiyah–Singer index theorem; the axial anomaly

SYMBOLS Dirac sea = the filled negative-energy states · chirality = handedness; for massless quarks the helicity · zero mode = a solution of the Euclidean Dirac equation, not a zero-energy quark · u, d, s quarks · for the staff: n_L, n_R zero modes, Q winding, Q_5 total handedness, N_f light flavours · anomaly = a symmetry of the classical theory broken by quantum effects (here the axial charge) · index theorem = counts the zero modes from the winding · baryon number B = (quarks − antiquarks)/3

SOURCES 't Hooft 1976 (PRD) · 't Hooft 1976 (PRL) · Adler 1969 · Bell, Jackiw 1969

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Fingerprint 2: how topology can help make matter



This slide, the weak-force instanton: it acts on all left-handed particles, quarks and leptons alike. Levels rise out of the sea and none dives: each step creates one particle in every left-handed doublet.

Per step: $\Delta B = \Delta L = 3$ (9 quarks, 3 leptons): per generation one quark of each colour and one lepton.

Contrast, slide 20 (strong-force instanton): a gluon field acts on both handednesses; one level dives as another rises, so quarks only flip handedness and B is unchanged.

Energy: in the early universe from the hot plasma (hopping over the top); today only a collision could supply it, and the step requires tunnelling: $\exp(-4\pi/\alpha_W) \approx 10^{-164}$ for $\alpha_W \approx 1/30$; never seen.

What it is good for: the only Standard-Model process that changes B (leptogenesis: a candidate origin of our baryons), and why colliders hunt the strong instanton: $4\pi/\alpha_W \approx 380$ vs $4\pi/\alpha_s \approx 30$–110.

IN DETAIL $\Delta B = \Delta L = N_{\text{gen}}\,\Delta N_{\text{CS}} = 3\,\Delta N_{\text{CS}}$ $B - L$ is conserved

SYMBOLS weak force = the force of radioactive decay; it acts only on left-handed particles · baryon = proton, neutron (3 quarks) · lepton = electron, neutrino · $\alpha_W \approx 1/30$ weak coupling · $N_{\text{gen}} = 3$ generations · sphaleron = the field at the top of the barrier · for the staff: N_{CS} winding number · Δ change of · leptogenesis = a lepton excess made first, then partly converted into baryons

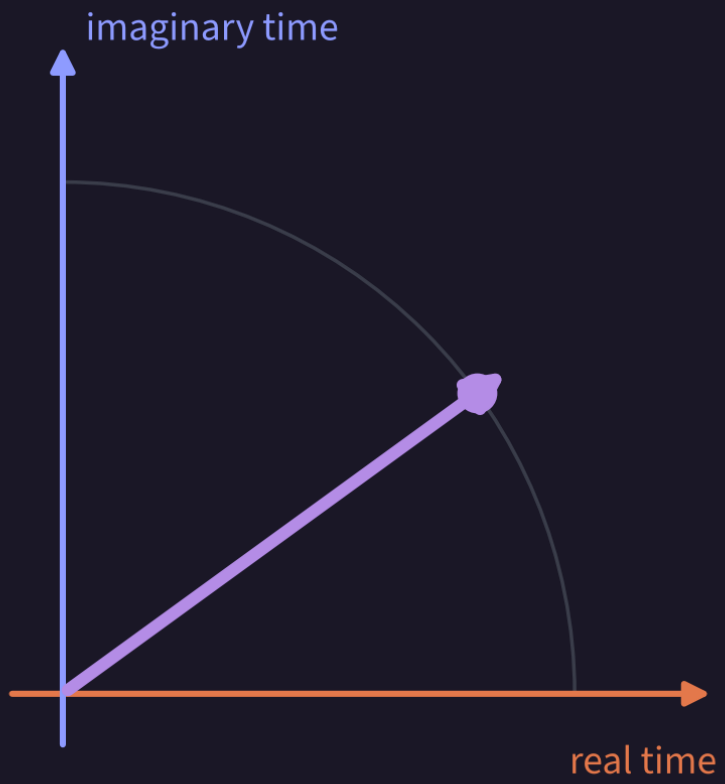
SOURCES 't Hooft 1976 (PRL) · Klinkhamer, Manton 1984 · Kuzmin, Rubakov, Shaposhnikov 1985 · Fukugita, Yanagida 1986

PART III

Back to real time

What does tunnelling look like in real time?

Back to the double well of Part I, where we know the instanton exactly.
Can we get a real-time description from it?



Progress marker

Our results and the Cherman–Ünsal picture

1 **Regular real-time solutions in a family**

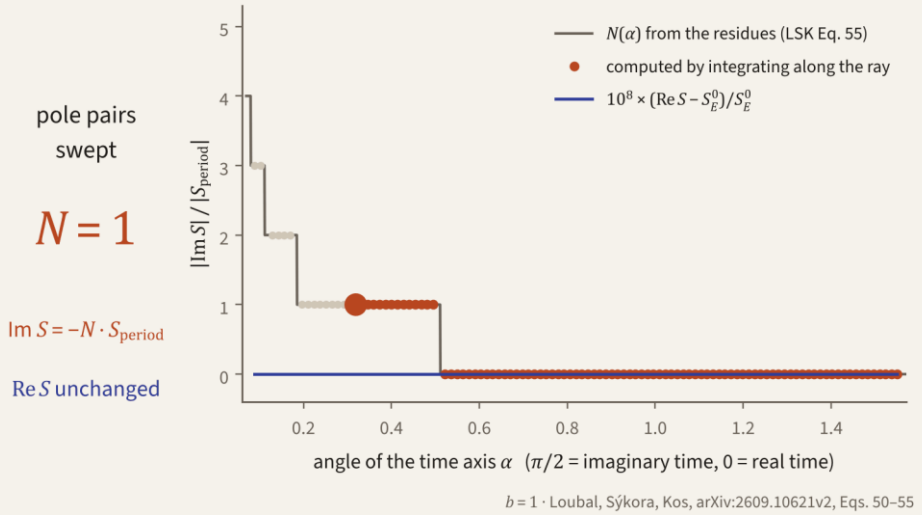
the double well deformed into a triple well: the solution stays bounded; only at the double well itself does it blow up (a ‘comb’)

2 **The suppression does not depend on the angle**

Re S fixed for every angle (10 digits); Im S climbs in steps — a winding number

3 **A periodic real-time direction**

a complex-time cylinder; finite energy closes it into a torus



Cherman–Ünsal (2014): instantons live in the real-time path integral as complex solutions — for the double well a singular comb. **In our family the picture survives, the singularity does not: the suppression stays, the phase is topological.**

For the question behind the title: in our family the instanton continued to real time is complex and regular, and Re S does not move. Whether this holds beyond the family, and which solutions contribute, is open.

SYMBOLS Re S, Im S real and imaginary parts of the action · α angle of the time axis · comb = infinitely many blow-ups · thimble = the integration path through a complex saddle · torus = a doughnut-shaped surface, periodic in two directions

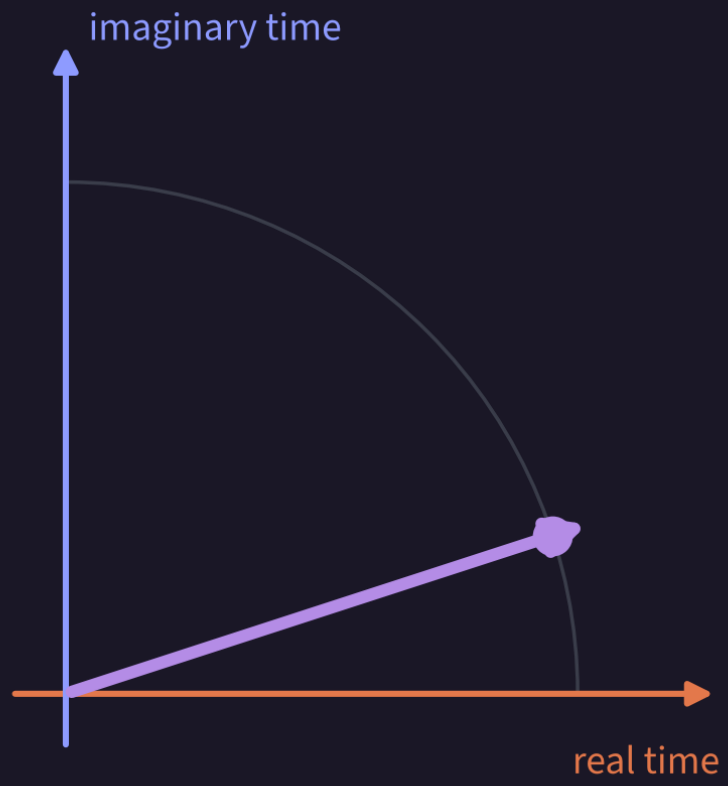
SOURCES Loubal, Sýkora, Kos 2026 · Cherman, Ünsal 2014

PART IV

From an instanton to an event

How a rate and an event generator are built — and how far to trust them.

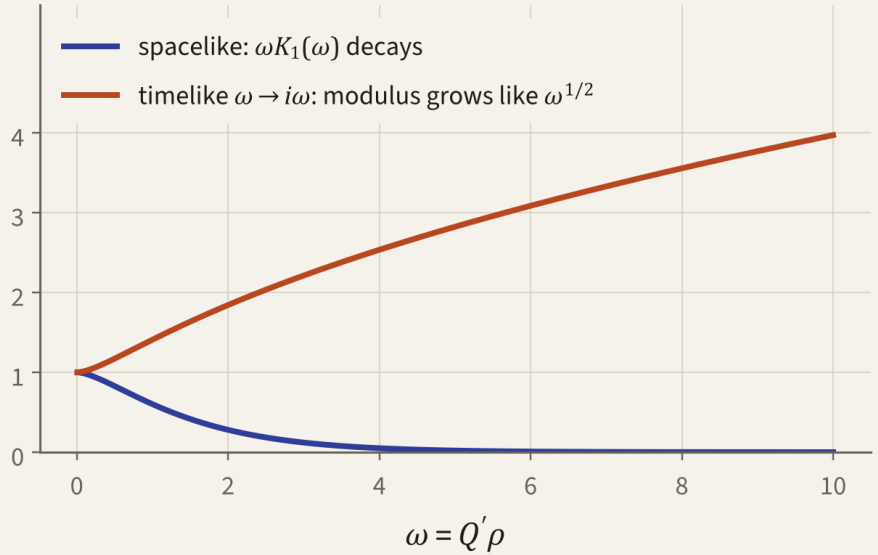
Chain: incoming partons → instanton → quarks and gluons → hadrons → detector. This part: the first arrows, and how far to trust them.

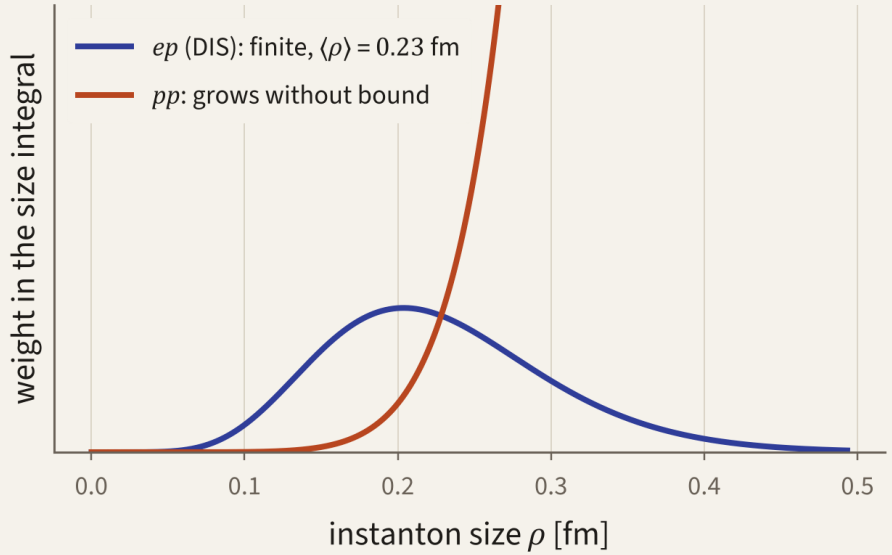


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The size problem limits reliable predictions





Why sizes are summed: every size is a solution (slide 16), so a rate integrates over all of them. Small sizes: controlled. Large sizes: the growth means the calculation fails there, not that big instantons abound.

In electron–proton collisions the photon cuts large sizes off, and the kinematics must favour small, controlled sizes. In proton–proton collisions this control is far more model-dependent.

IN DETAIL $D(\rho) \propto \rho^{\beta_0 - 5}$ (grows), $f(\omega) = \omega K_1(\omega)$, $\omega = Q'\rho$ $\beta_0 = 9$ for three light flavours

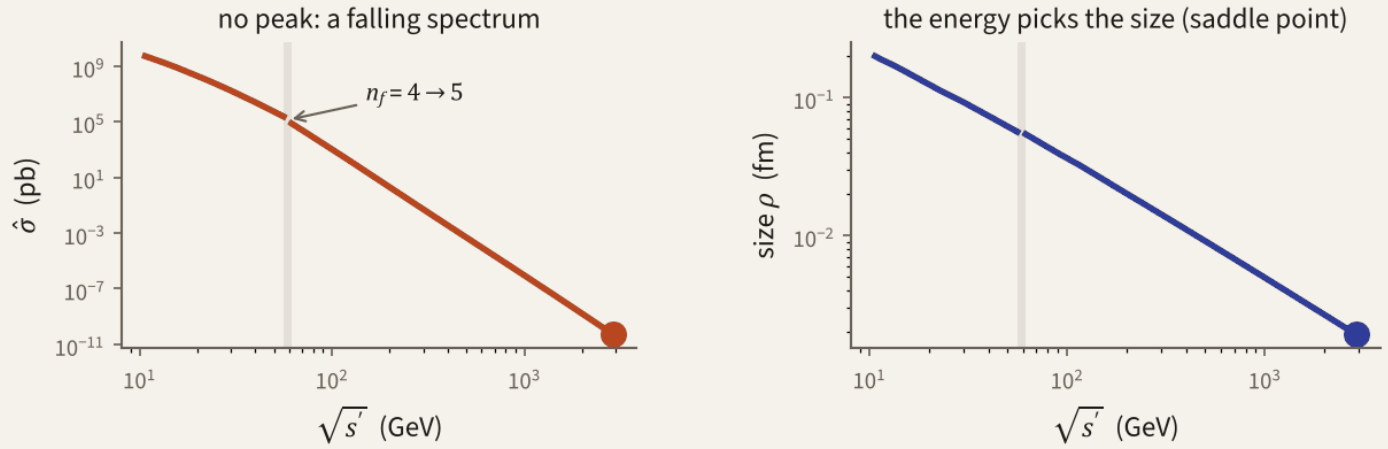
SYMBOLS ρ instanton size · $D(\rho)$ how many instantons of size ρ the vacuum contains ('t Hooft's calculation) · Q' virtuality of the quark that enters the instanton process (set by the photon kinematics) · K_1 a Bessel function · DIS electron–proton scattering · pp proton–proton

SOURCES Moch, Ringwald, Schrempp 1997 · Ringwald, Schrempp 1998 · Shuryak, Zahed 2021b

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An instanton has no mass: its size is a Euclidean number

$\sqrt{s'}$ = 2895.5 GeV → ρ = 0.002 fm, $u = \rho\sqrt{s'}$ = 28, $\alpha_s(1/\rho)$ = 0.117, $\langle n_g \rangle$ = 12.1, n_f = 5



Khoze, Krauss, Schott 2020, Table 1 (gg, leading order). Shaded: the table switches from 4 to 5 light flavours.

A particle has a mass: a pole in real time, a peak in the spectrum.

An instanton has none: a Euclidean solution, action $8\pi^2/g^2$ at every size; ρ is a collective coordinate, a Euclidean length.

What we measure: the final-state mass $M_{\text{inst}} = \sqrt{s'}$, a continuous variable.

Euclidean size → Minkowski energy is the hard step: they meet only at a saddle point, $\rho\sqrt{s'} = (4\pi/\alpha_s)\, S'(\chi)$ in pp (KKS Eq. 2.33), $\langle\rho\rangle \approx 9.8/Q'$ in DIS (MRS Eq. 76); the energy enters as $e^{R_0\sqrt{s'}}$ after a Wick rotation, and the continued solution is complex, not a real-time object (slides 16, B5).

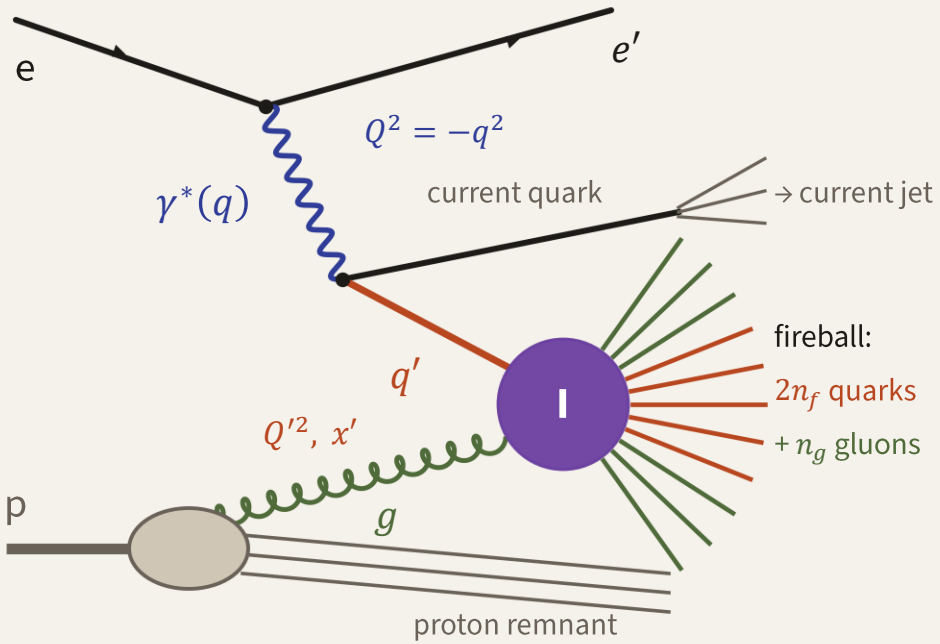
No mass peak to find: search in windows of M_{inst}, compare shapes at fixed M_{inst}.

SYMBOLS $\sqrt{s'}$ invariant mass of the instanton subsystem (= M_{inst}) · ρ instanton size · saddle point = the value of a variable that dominates an integral · χ instanton–anti-instanton separation in units of ρ · Q' virtuality of the quark entering the process (DIS) · Wick rotation = turning imaginary time into real time · R_0 separation of instanton and anti-instanton in imaginary time · KKS Khoze, Krauss, Schott (2020) · MRS Moch, Ringwald, Schrempp (1997)

SOURCES Khoze, Krauss, Schott 2020 · Moch, Ringwald, Schrempp 1997

How the ep instanton generator is built

HERA, EIC: $e\,p \to e' +$ **current jet + fireball + proton remnant**



1 · Factorise (RS98 Eq. 2): photon flux × parton density × the instanton subprocess $\sigma^{(I)}(x', Q'^2)$.

2 · Subprocess (RS98 Eq. 20): the instanton–anti-instanton valley at its saddle; the photon's form factor $\omega K_1(Q'\rho)$ makes the size integral finite (MRS Eq. 62).

3 · Event: $M_I = Q'\sqrt{1/x' - 1}$; $2n_f$ quarks + Poisson gluons from the same saddle (QCDINS Eq. 13), RAMBO, colour strings → Pythia 8.

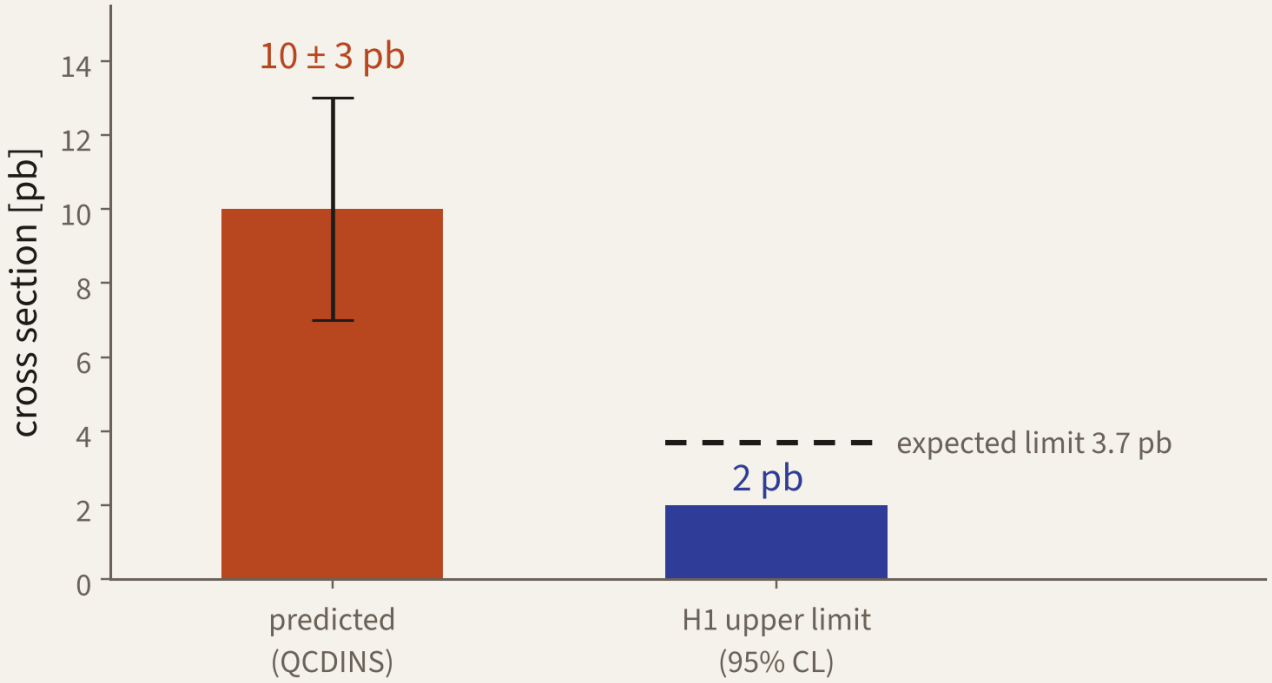
Validated: RS98 Fig. 4, median ratio 1.046 over 429 points; $\langle n_g \rangle = 3.44$ at the fiducial corner (QCDINS: ~3).

Rates from the calculation, shapes from the vertex: in ep the size is controlled — the reason the EIC is the place for a first controlled observation (slide 41).

SYMBOLS γ^* virtual photon · q' the virtual quark entering the instanton · Q'^2, x' its virtuality and Bjorken x · M_I fireball mass · RAMBO flat phase-space generator · $\langle n_g \rangle$ mean gluon number · RS98 Ringwald, Schrempp (1998) · σ cross section · EIC Electron–Ion Collider (Brookhaven) · factorisation = cross section = parton densities × a hard subprocess · fiducial region = where the calculation is trusted · form factor = how the photon's virtuality suppresses large sizes · QCDINS the HERA instanton event generator of Ringwald and Schrempp · e, e' incoming and scattered electron · $Q^2 = -q^2$ virtuality of the photon · current quark = the quark struck by the photon, seen as a jet · g gluon from the proton · p proton · I instanton · remnant = what is left of the proton · K_1 a Bessel function

SOURCES Ringwald, Schrempp 1998 · Moch, Ringwald, Schrempp 1997 · Ringwald, Schrempp 2000 (QCDINS)

HERA tested the standard prediction



What H1 looked for: a search tests a predicted distribution, not one odd-looking event. Five features of the final state (many particles in a band, spread evenly around the beam) combine into one number that selects a signal-enriched sample.

No signal: 2430 events seen, 2483–2966 expected from background. Limit 2 pb (95% CL) in this kinematic region, five times below the predicted 10 ± 3 pb.

Firm: the benchmark prediction is excluded in the tested region.

Open: the wider verdict depends on the lattice input that sets which sizes are trusted (slide 25).

SYMBOLS HERA electron–proton collider (DESY, Hamburg) · H1 one of its experiments · band = a slice of angles where the instanton's particles go · discriminant = one number combining several features · pb picobarn, unit of cross section · CL confidence level

SOURCES H1 2016 · Ringwald, Schrempp 1998 · Khoze, Krauss, Schott 2020 · Hasenfratz, Nieter 1998

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Where the calculation of instanton processes is weak

1 · A classical solution, to first order	amplitude = $e^{-8\pi^2/g^2}$ × one-loop determinant × $(1 + O(\alpha_s))$; no NLO for any collider process (MRS 1997 is the state of the art); the expansion is asymptotic, in $\alpha_s(1/\rho)$, which grows with the size
2 · Many particles	each gluon adds ρ^2 under the size integral; the gluon number is not determined (prescriptions differ by ≈ 3×); the expansion fails towards the sphaleron energy
3 · The size integral	controlled by the photon in DIS; in pp no derived regulator (slide 25)
4 · Perturbative and non-perturbative parts	separating the two contributions requires care: the perturbative series diverges, and how to combine it with an instanton term is not settled for these processes
All orders: resurgence	resurgence clarifies this interplay in quantum mechanics (Zinn-Justin–Jentschura: the two-instanton term cancels the imaginary part of the resummed series); its application to these QCD collider predictions remains open

The available rates are leading semiclassical predictions, with limited theoretical control — particularly in pp.
So we state the assumptions and test the model dependence.

SYMBOLS saddle = stationary point of the action · one-loop = Gaussian fluctuations around the saddle · trans-series = perturbative series plus terms with powers of e^{-S/g^2} · Borel resummation = a way to give a divergent series a value · renormalon = a divergence of perturbation theory itself · NLO next-to-leading order (the first correction)

SOURCES 't Hooft 1976 (PRD) · Moch, Ringwald, Schrempp 1997 · Zinn-Justin, Jentschura 2004 · Aniceto, Başar, Schiappa 2019 · InstGen v4 documentation

PART V

Can we catch one?

Two ATLAS analyses: one in the central detector, one with forward protons.

Chain: incoming partons → instanton → quarks and gluons → hadrons → detector. Part IV said how a rate is calculated and how far to trust it; this part: how to tell such an event from ordinary QCD.

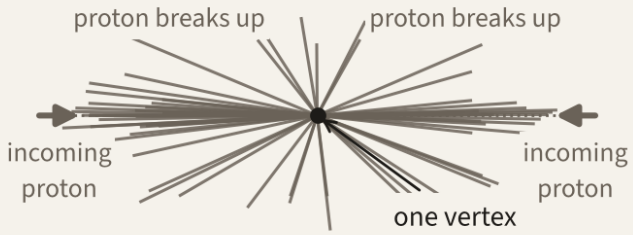


Progress marker: back in real time, where detectors live

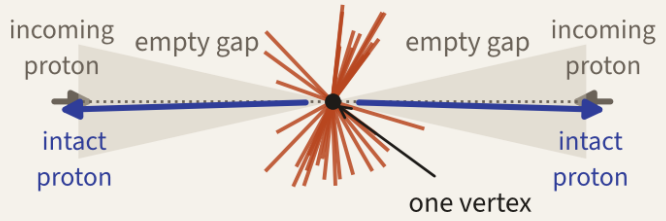
OPEN

Two ways to look at the LHC

ordinary collision: particles at all angles, most of them close to the beam



instanton with intact protons: nothing near the beam, one round burst (drawn in its rest frame)





pseudorapidity η ($|\eta|$ large = near the beam; protons at $|\eta| \approx 9.6$)



pseudorapidity η ($|\eta|$ large = near the beam; protons at $|\eta| \approx 9.6$)

Analysis 1, central: all inelastic collisions in low-pile-up runs; which of them contain a round burst?

Analysis 2, forward: demand intact protons in the Roman pots of ALFA or AFP, and empty gaps.

Neither solves the size problem (slide 25): both search for a shape, not for a precisely predicted rate.

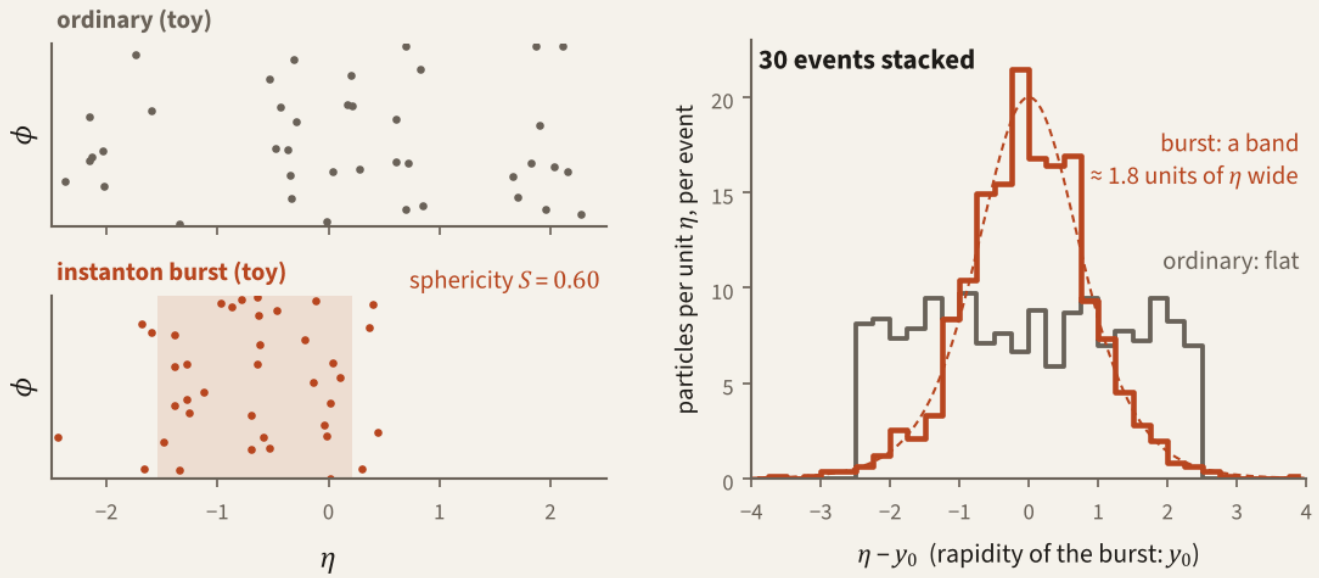
SYMBOLS LHC Large Hadron Collider · ATLAS its detector · η pseudorapidity = the angle to the beam on a log scale ($|\eta|$ large = close to the beam) · gap = no particles · intact proton = survives the collision · ALFA, AFP the two forward-proton detector systems of ATLAS · pile-up = several proton–proton collisions in one bunch crossing · Roman pot = movable vessel that brings a detector close to the beam

SOURCES Khoze, Krauss, Schott 2020 · Harris (ATLAS) 2024 · Vavřička 2026

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What an instanton event looks like in a detector

One event is noisy; stacked events show the band



Toy, not simulated: 40 massless particles, isotropic at rest at a random rapidity (S = 1: round). Dashed: $(n/2)/\cosh^2(\eta - y_0)$; shaded strip: its full width at half maximum, 1.76 units of η.

Many soft particles from one small region.

Round in its rest frame: high sphericity, low thrust; a band in η, even in φ; small $\langle|\Delta\eta|\rangle$.

Every light flavour at once (slide 20): extra strangeness, K and Λ.

Larger mass, smaller instanton: the rate falls steeply with mass (slide 26).

What both analyses measure: event shapes (sphericity, thrust, flattenicity), $\langle|\Delta\eta|\rangle$, and neural-network classifiers, some trained on the raw track pattern in η and φ.

Signal simulated with Sherpa; backgrounds with EPOS LHC and Pythia 8 (Harris 2024; Vavřička 2026). The figure is a toy, not a simulation.

SYMBOLS φ azimuth around the beam · sphericity = how round the momentum flow is (1 = round) · thrust = how much of it lines up along one axis · $\langle|\Delta\eta|\rangle$ mean η distance between two tracks · Λ lambda hyperon (u d s) · K kaon · Sherpa, EPOS LHC, Pythia 8 event generators (Sherpa with instantons; EPOS and Pythia for ordinary collisions) · classifier = a trained function scoring how signal-like an event is (here neural networks) · flattenicity = how evenly the transverse momentum spreads in η and φ

SOURCES Khoze, Krauss, Schott 2020 · Harris (ATLAS) 2024 · Vavřička 2026

OPEN

Analysis 1: the central search with the inner detector

1 · DATA

special low-pile-up 13 TeV runs, minimum-bias trigger; ≈ 1.6 nb^{-1}

2 · MODELS

signal: Sherpa; background: EPOS LHC (nominal), Pythia 8 (alternative)

3 · VARIABLES

event shapes, $\langle|\Delta\eta|\rangle$, flattenicity; three instanton-mass categories

4 · CLASSIFIERS

two classifier outputs trained to be approximately independent for background events

5 · BACKGROUND FROM DATA

ABCD: if the background approximately factorises between the two outputs, regions B, C, D predict it in A

6 · FIT

profile likelihood for μ; signal region blinded until the checks pass

PUBLIC STATUS

"The data have so far been unblinded and the paper draft is undergoing a publication process in the ATLAS collaboration." — Taševský, Ryskin, arXiv:2609.15252

Method as described in public: Y. Harris, CERN, May 2024. The result will be shown when the paper is public.

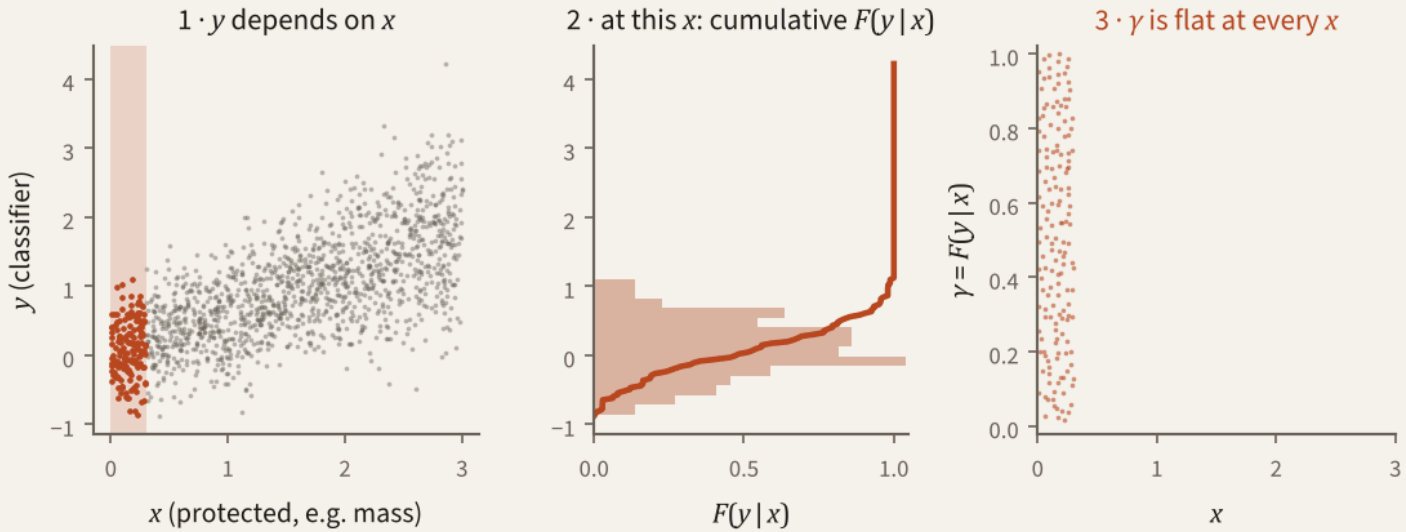
SYMBOLS nb^{-1} inverse nanobarn (amount of data) · pile-up = extra collisions in the same bunch crossing · ABCD = background in the signal region predicted from three control regions · μ signal strength (1 = the prediction) · CERN the European laboratory for particle physics

SOURCES Harris (ATLAS) 2024 · Vavřička 2026 · Taševský, Ryskin 2026

OURS

Decorrelation without retraining: the Rosenblatt transform

The Rosenblatt transform: replace y by its conditional cumulative distribution



Toy background sample ($N = 1500$); F estimated with the kernel estimator of Vavřička et al. (Eq. 7), $\sigma = 0.05\ \sigma_a$ (Eq. 8).

Definition (Eq. 4 of our paper): $\gamma(x, y) = \frac{\int_{-\infty}^{y} \rho(x, y')\, dy'}{\rho_X(x)}$

Why it works (App. A): the new density is $\rho_{X\Gamma}(x, \gamma) = \rho_X(x) \cdot 1$ on $[0, 1]$ — a product, so γ is independent of x and uniform.

From a sample (Eq. 7): $\gamma_\sigma(x, y) = \frac{\sum_i H(y - y_i)\, w_i}{\sum_i w_i}$, $w_i = e^{-(x - x_i)^2 / 2\sigma^2}$. No retraining; the ranking in y is kept at every x.

Then ABCD works: $N_A = N_B N_C / N_D$ in the plane (x, γ).

Transform afterwards instead of penalising during training: LHC dijet benchmark, correlation down by > 10^3, ABCD non-closure ~ 10^{-6}.

SYMBOLS x protected variable (here a mass) · y classifier output · $\rho(x, y)$ their joint density on background · $\rho_X(x)$ the density of x alone · γ the transformed classifier · $F(y \mid x)$ cumulative distribution of y at fixed x · H step function (0 below zero, 1 above) · w_i weight of event i, width σ · $N_A \ldots N_D$ event counts in regions A–D · DisCo distance-correlation penalty · AUC area under the ROC (receiver operating characteristic) curve · Γ the transformed classifier as a random variable (in $\rho_{X\Gamma}$) · closure test = checking that the method reproduces a known count

SOURCES Vavřička, Sýkora, Schott 2026

ESTABLISHED

The hard part: a background generators do not model reliably

CMS 2025, event shapes at about one collision per crossing: *"none of the models investigated is able to satisfactorily describe the data"; the data are "more isotropic than any of the simulations".*

Central answer: do not trust the generators for the background — estimate it from data (ABCD, as in the central search).

Forward answer: remove most of it. Every extra parton collision fills a rapidity gap with particles; demand a gap, and the multi-parton background is suppressed (Khoze, Khoze, Milne, Ryskin 2021).

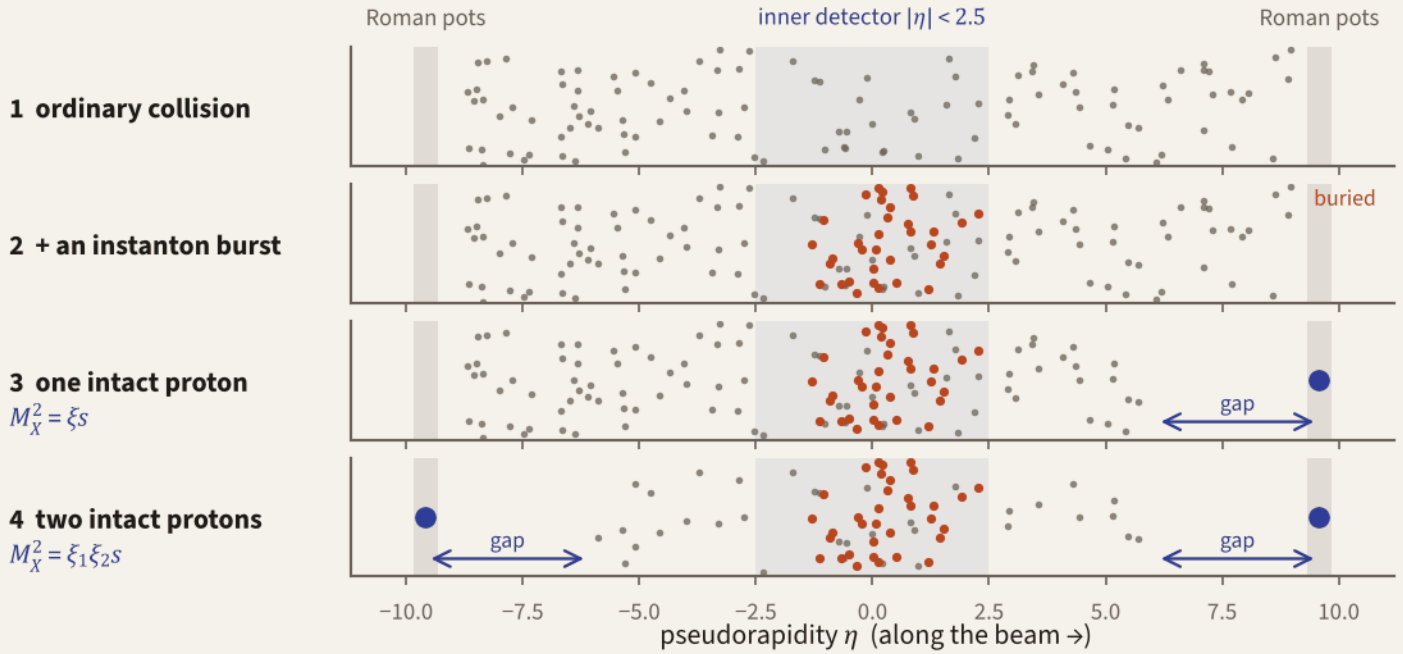
The signal resembles a background that the available generators do not model reliably. Either constrain that background from data, or suppress it with gaps and tagged protons.

SYMBOLS underlying event = everything in a collision besides the process of interest · MPI multi-parton interactions = several parton collisions in one proton–proton collision · generator = program that simulates collisions · isotropic = the same in every direction · CMS the second large general-purpose LHC detector

SOURCES CMS 2025 · Khoze, Khoze, Milne, Ryskin 2021

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Analysis 2: suppress the background with forward protons



Toy (not simulated). Copper: instanton burst; blue: intact protons (beam rapidity 9.6 at 13.6 TeV); ξ = 0.03.

Cleaner: a gap removes most of the multi-parton background. One gap, M_{inst} = 20–60 GeV: signal above background by a factor ≥ 8 in Khoze et al. (2021).

A kinematic handle: ξ from each tagged proton; two tags give $M_X^2 = \xi_1 \xi_2 s$, an upper bound on M_{inst}. One tag gives a gap, not the mass.

The price: gap survival $S^2 \approx 0.05$–0.1, not calculable from first principles.

Clean events and a bound on the mass, at the cost of rate. The gap is the response to the background problem of the previous slides.

SYMBOLS ξ fractional momentum loss of a proton · s squared collision energy · M_X mass of the central system · M_{inst} mass of the instanton system · S^2 gap survival probability · diffraction = a collision in which a proton survives intact, next to an empty gap · multi-parton interactions = several parton collisions in one proton–proton collision

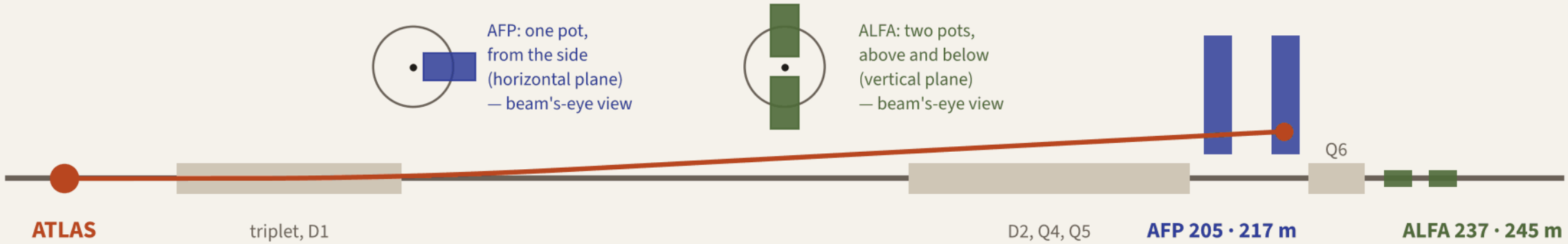
SOURCES Khoze, Khoze, Milne, Ryskin 2021 · Taševský et al. 2023 · Taševský, Ryskin 2026

ESTABLISHED

ALFA and AFP: Roman pots 205–245 m from ATLAS

proton momentum loss $\xi = 0.038$ → tagged by AFP

one tag: $M_X = \sqrt{\xi s} = 2.65$ TeV at $\sqrt{s} = 13.6$ TeV two tags ($\xi_1 = \xi_2 = \xi$): $M_X = \xi\sqrt{s} = 517$ GeV



Schematic top view of one side, not the LHC optics (offset exaggerated; magnet positions schematic). AFP Run 3 acceptance $0.02 < \xi < 0.12$ (Erland, ICHEP 2022).

AFP: luminosity, but pile-up. One pot per station, from the side, at 205 and 217 m; Run 3: 34.1, 26.3, 108 fb^{-1} (2022–24), 196 fb^{-1} in all; in 2025 also in the oxygen and neon runs. Acceptance $0.02 < \xi < 0.12$.

ALFA: clean, but little data. Further out, at 237 and 245 m, two pots per station from above and below; larger acceptance, but only in special high-β^* runs: 1.67 nb^{-1} (8 TeV, 2012); 0.65–0.73 pb^{-1} (2015), 340 μb^{-1} ($\beta^* = 2.5$ km, 2016), 0.79 + 3.9 pb^{-1} (2018).

Ours: summer-student project of M. Draskovits (2023): instantons with forward-proton tagging — feasibility, and a larger AFP acceptance.

SYMBOLS Roman pot = movable vessel that brings a detector to about a millimetre from the beam · β^* beam-optics parameter (large β^*: special runs, tiny angles, low luminosity) · fb^{-1}, μb^{-1} amounts of data · acceptance = the range of events a detector can record

SOURCES Erland (ATLAS) 2022 · Clawson (ATLAS) 2025 · Lysenko (ATLAS) 2025 · Sobień, Trzebiński 2026 · ATLAS 2022 (ALFA, 13 TeV) · ATLAS 2020 (ALFA, SD) · Draskovits 2023

OPEN

ATLAS instanton search with ALFA: signatures

TOPOLOGY & TRIGGER

Double tag: p + X + p, both protons in ALFA.

Single tag: p + X + dissociated proton.

Trigger: ALFA arm AND central activity (MBTS or a track count); an elastic trigger alone rejects the signal.

NEW VARIABLES

Protons: ξ, t, acoplanarity of the two protons.

Event: the size of the rapidity gap.

Kept: $\langle\eta\rangle$, $\langle|\Delta\eta|\rangle$, sphericity, thrust; a classifier on the raw tracks in η–φ.

BACKGROUNDS & SYSTEMATICS

Accidental tags: a proton from another collision fakes the coincidence; measure it with zero-bias data.

New systematics: pot alignment and optics, acceptance at the pot edge, luminosity, the Pomeron-flux model.

Statistics: the ABCD method of the central search (slide 33) extended to a combined central + forward plane; blinded profile-likelihood fit; a differential result in t or ξ would need unfolding.

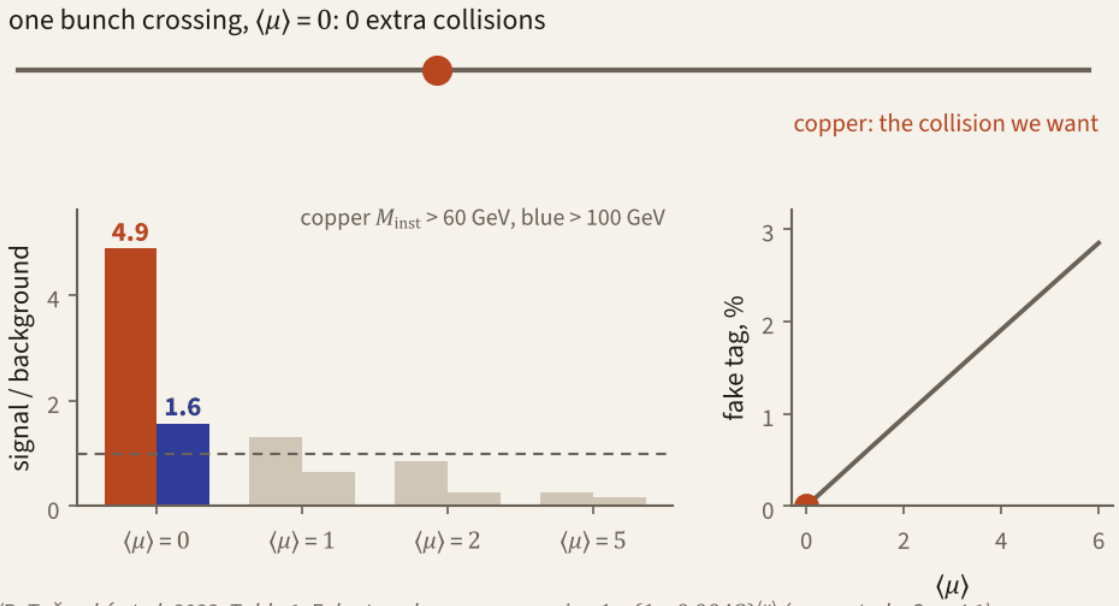
Ideal: a dedicated 13 TeV run at $\beta^* \approx 90$ m with a combined ALFA + MBTS/track trigger. ATLAS ALFA instanton search (R. Vavřička, CZ+SK HEP Workshop 2026)

SYMBOLS t squared momentum transfer to the proton · acoplanarity = how far two tagged protons are from back-to-back in azimuth · zero-bias = randomly triggered crossings · unfolding = correcting a distribution for the detector · single tag, double tag = one or both intact protons detected · CZ+SK HEP Workshop the Czech–Slovak high-energy-physics workshop · MBTS minimum-bias trigger scintillators of ATLAS · Pomeron = the colourless exchange behind diffraction

SOURCES Vavřička 2026 · Taševský et al. 2023

AFP in Run 3: the single-tag channel

Pile-up: protons from other collisions fake the tag



S/B: Taševský et al. 2023, Table 1. Fake-tag chance per crossing $1-(1-0.0048)^{\langle\mu\rangle}$ (same study, Sec. 4.1).

Proposal of Taševský, Khoze, Milne, Ryskin (2023): one proton in AFP ($0.02 < \xi < 0.05$), $M_{\text{inst}} > 60$ or 100 GeV, many central tracks, an empty forward region.

Pile-up is the enemy: a proton from another collision fakes the tag. S/B falls from 4.9 to 0.3 as $\langle\mu\rangle$ goes from 0 to 5.

Conclusion of that study: a special run with $\langle\mu\rangle \sim 1$ and ~ 0.1 fb^{-1}; double tags are swamped by fake coincidences.

Ours: V. Loubal's central-diffractive generator reproduces published instanton cross sections to ≈ 1%.

Ratios at fixed mass: within a common subprocess model, diffractive over inclusive reduces the sensitivity to the instanton normalisation; gap survival and acceptance remain essential.

SYMBOLS $\langle\mu\rangle$ mean pile-up · ξ_{calo} momentum loss estimated from the calorimeter · S/B signal over background · combinatorial background = a tag from an unrelated collision

SOURCES Taševský et al. 2023 · Clawson (ATLAS) 2025 · Khoze, Milne, Spannowsky 2021

ESTABLISHED OPEN

Numbers: what is predicted, what is excluded, what data exist

Predicted, pp 13 TeV	$\sigma(\sqrt{s'} > 20$ GeV) = 6.3 mb · > 50: 41 µb · > 100: 80 nb · > 200: 105 pb · > 500 GeV: 3.5 fb (KKS, leading order; uncertain by orders of magnitude; the partonic spectrum: slide 26)
Excluded, ep (HERA)	H1: $\sigma < 2$ pb at 95% CL in the region of slide 28 (1.5–6 pb over the kinematic regions studied)
Data, central	2015, 13 TeV, $\langle\mu\rangle \approx 0.03$: 1.6 nb^{-1} recorded (13.4 nb^{-1} delivered, prescaled trigger) — nominally $\approx 10^7$ produced events above 20 GeV at leading order, normalisation highly uncertain
Data, ALFA	8 TeV: 1.67 nb^{-1} (2012) · 13 TeV: 0.65–0.73 pb^{-1} (2015), 340 $\mu\mathrm{b}^{-1}$ (2016, $\beta^* = 2.5$ km), 0.79 + 3.9 pb^{-1} (2018)
Data, AFP	$\langle\mu\rangle \leq 0.05$: ≈ 0.15–0.25 pb^{-1} · $\langle\mu\rangle \approx 1$: 13 pb^{-1} · high pile-up: 196 fb^{-1} (36 fb^{-1} with ToF)

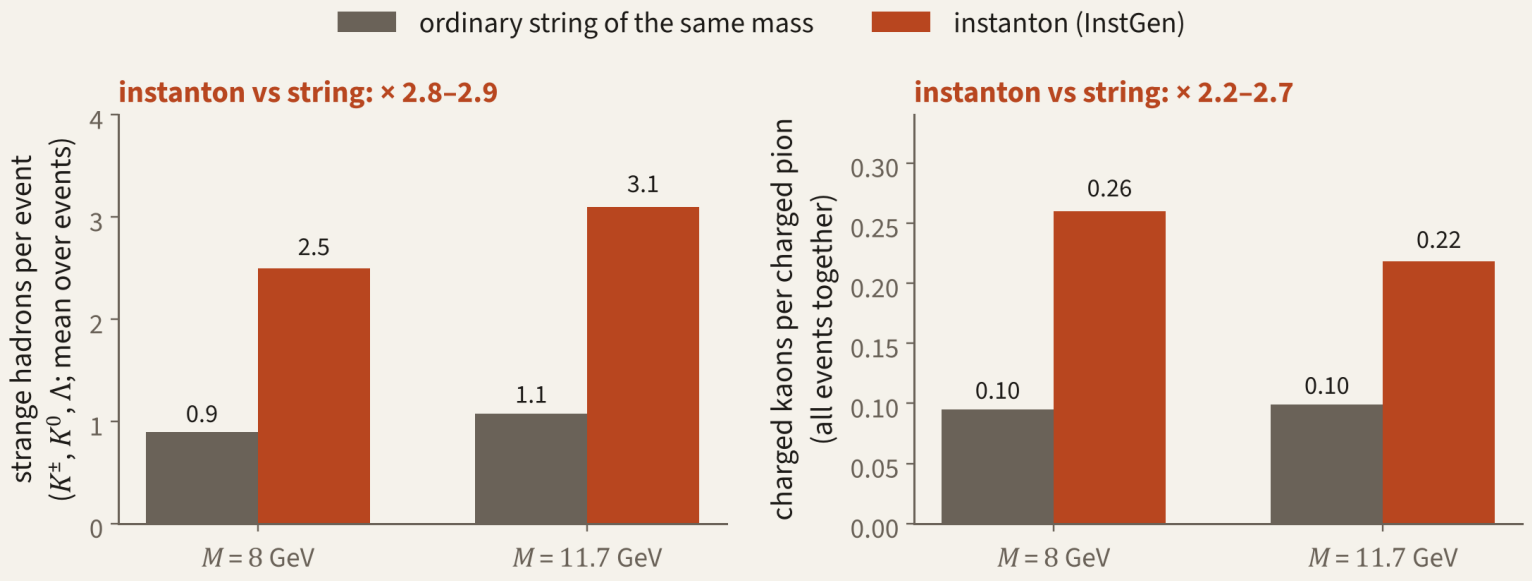
The central challenge is distinguishing a contribution from ordinary QCD. About 20 pb^{-1} with tagged protons at $\langle\mu\rangle \lesssim 1$ were recorded — at different energies and triggers; how much is usable is being worked out. The predicted spectrum falls by nine orders of magnitude and its normalisation is uncertain.

SYMBOLS σ cross section · mb, µb, nb, pb, fb units of cross section, each 1000 times smaller than the one before · nb^{-1}, pb^{-1}, fb^{-1} amounts of data · $\sqrt{s'}$ mass of the instanton system · LO leading order · $\langle\mu\rangle$ mean number of collisions per bunch crossing · β^* beam-optics parameter · ToF time-of-flight detector · CL confidence level

SOURCES Khoze, Krauss, Schott 2020 · H1 2016 · Harris (ATLAS) 2024 · ATLAS 2020 (ALFA, SD) · ATLAS 2022 (ALFA, 13 TeV) · AFP Run 3 overview 2023 · Erland (ATLAS) 2022

OURS OPEN DEEPER DIVE ›

Why the EIC: InstGen says every instanton event is strange



InstGen + Pythia 8, instanton system at rest, all final hadrons, 3000 events per bar, statistical error ≤ 0.03 (our study, 11 Sep 2026)

Why strange: the vertex (slide 20) puts an $s\bar{s}$ pair into every event before hadronisation; a string makes strangeness only at its breaks.

Instanton vs string: × 2.8–2.9 strange hadrons per event; $K^{\pm}/\pi^{\pm}$ 0.22–0.26 vs 0.10.

With a detector (ePIC study, July 2026; to be redone with the present InstGen): DIS 0.6 vs signal 1.2–3.3 strange hadrons per event; two cuts keep 54% signal, 6% background.

Why the EIC: HERA's control of the size; 10 fb^{-1} in 30 weeks at 10^{33} cm^{-2} s^{-1} (Yellow Report), against H1's 0.35 fb^{-1} in all; particle identification, which H1 lacked. Flavours: backup B15.

Ratios at fixed mass, not rates. Step 2: the real DIS background at the same kinematics, and the detector.

SYMBOLS EIC Electron–Ion Collider (Brookhaven, 2030s) · ePIC its detector · InstGen our event generator · strange hadron = $K^{\pm}$, K^0, Λ · $K^{\pm}/\pi^{\pm}$ = number of charged kaons per charged pion, all events · M mass of the instanton system · string = an ordinary quark–antiquark pair turning into hadrons · hadronisation = quarks and gluons turning into the hadrons we detect

SOURCES Moch, Ringwald, Schrempp 1997 · 't Hooft 1976 (PRD) · EIC Yellow Report 2021

OPEN DEEPER DIVE ›

Open questions, and where to start

COMPLEX TIME

Which complex-time solutions contribute in real time?

We know these solutions and their actions (slide 23; backup B7). First step: decide numerically which of them contribute, for one member of our family.

FORWARD PROTONS

Can a tagged-proton sample see an instanton?

First step: the 2012 MBTS + ALFA sample and our generator in single-tag mode — the mass spectrum in the ALFA acceptance.

EIC

Is every instanton event strange?

First step: our events and ordinary DIS events through the ePIC detector simulation, at the same kinematics.

All three are open, in theory and in experiment — come and talk to us. A fourth, speculative: are the quarks from one instanton entangled? (backup B14)

SOURCES Tanizaki, Koike 2014 · ATLAS 2020 (ALFA, SD) · Taševský et al. 2023

The team



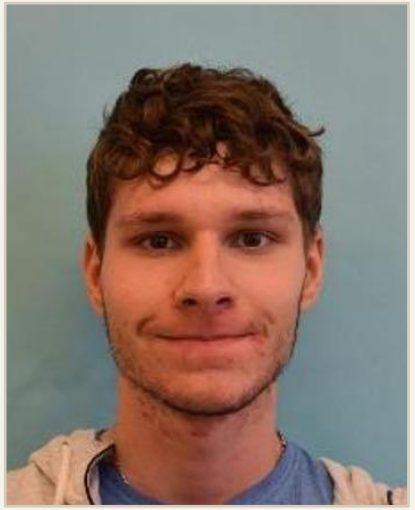
Radek Vavřička

PhD: ATLAS instanton search with ALFA; the Rosenblatt decorrelation, with M. Schott (slides 38, 34)



Šimon Kos

the complex-time solutions of Part III (slide 23; Loubal, Sýkora, Kos 2026)



Vojtěch Loubal

the double-well to triple-well solutions of Part III — also his bachelor thesis (slide 23); a central-diffractive instanton generator (SFG 2026)



Tomáš Sýkora

ATLAS forward-proton and instanton analyses; InstGen and the EIC study (slide 41)

?

anyone interested?

open problems in theory and in experiment (slide 42) — come and talk to us

Thanks also to the CERN summer students who worked on the searches — M. Draskovits (2023): instantons with forward proton tagging; T. Ketheeswaran (2024): instantons with the ALFA detector (feasibility studies).

SYMBOLS SFG = Student Faculty Grant (Charles University summer projects)

What to take home

What is an instanton?

ESTABLISHED

A finite-action solution in imaginary time that describes tunnelling; its action sets the exponential in measured quantities.

What changes when a field tunnels?

ESTABLISHED

Topology labels the vacua: $\bar{\theta}$ and the strong CP problem; the six-quark 't Hooft vertex; in the weak force, B and L change together.

How does an instanton become an event?

OPEN

No mass and no peak: the calculation relates the subprocess kinematics to a dominant size. Many soft particles, round and strange. Rates remain uncertain, especially in pp: prioritise discriminating shapes and ratios, with explicit tests of model dependence.

Can we catch one?

OPEN

Not yet. HERA set limits. Next: the ATLAS central search (unblinded, paper in the publication process); the forward search with ALFA and AFP, on data already recorded; later the EIC, with the size under control.

What would decide it?

OPEN

Support: a consistent excess of round, strangeness-rich events whose yield falls with mass as predicted, in more than one channel. Constraint: at the EIC, exclusion of a prediction fixed beforehand, where the size is controlled and the detector is sensitive. HERA began this test (slide 28).

Messengers from imaginary time: their traces are measurable; catching one directly is our programme.
Whether they tell us that spacetime is complex stays open.

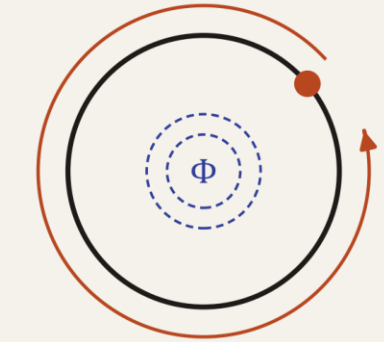
Backup: symbols and abbreviations

t real time · τ imaginary time · α angle of the time axis · $\hbar$ reduced Planck constant · k_B Boltzmann constant · T temperature · $\beta = 1/(k_B T)$ · Z partition function · H Hamiltonian · S action · S_E Euclidean action · m mass · V potential · x position · Λ, λ couplings · $\pm\eta$ minima · ω well frequency · τ_c kink centre · S_0, S_I instanton action · $E_\pm$ lowest energies · K fluctuation prefactor · g coupling · $\alpha_s = g^2/4\pi$ · a_n perturbative coefficients · θ vacuum angle · $\bar{\theta}$ effective angle · p_φ angular momentum · I moment of inertia · N_{CS} winding number · F field strength · Q topological charge · ρ instanton size · n_L, n_R zero modes · N_f flavours · Q_5 chirality · d_n neutron EDM · e elementary charge · κ surface gravity · M_{sun} solar mass

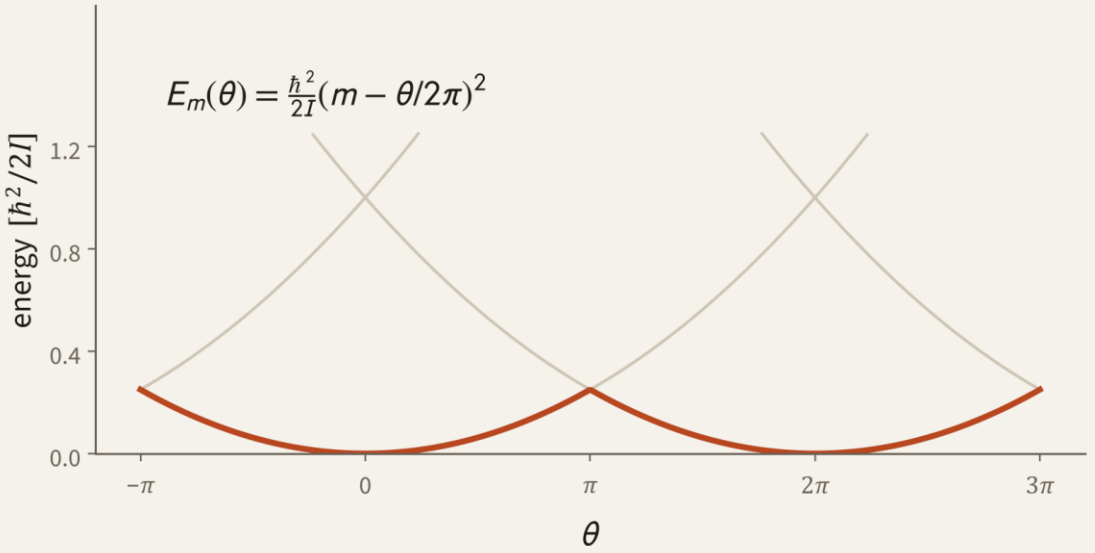
ξ proton momentum loss · M_X central mass · S^2 gap survival · $\langle\mu\rangle$ pile-up · ALFA, AFP forward proton detectors · MBTS trigger scintillators · ABCD background method · B, L baryon, lepton number · N_{gen} generations · α_W weak coupling · p family parameter · ξ length scale · b shape parameter · $u = \omega t/2$ · $T = 4\pi/\omega$ period · $N(\alpha)$ circuits picked up · S_{period} period action · $|\psi\rangle$ state · ρ_A reduced density matrix · S_A entanglement entropy · K boost generator · a acceleration · n replicas · $\mathcal{E}$ electric field · $\mathcal{E}_c$ critical field · d creation distance · $D(\rho)$ size distribution · Q' quark virtuality · K_1 Bessel function · QCD quantum chromodynamics · CP charge conjugation × parity · EDM electric dipole moment · EPR Einstein–Podolsky–Rosen · DIS deep-inelastic scattering · HERA, H1 · LHC, ATLAS, CMS · RHIC, STAR · EIC, ePIC · CL confidence level · QCDINS, InstGen event generators

ESTABLISHED

A bead on a ring, threaded by a magnetic flux



winding n = times around



$$H = \frac{\left(p_\varphi - \hbar\theta/2\pi\right)^2}{2I}, \qquad p_\varphi = -i\hbar\frac{\partial}{\partial\varphi} \qquad \Rightarrow \qquad E_m = \frac{\hbar^2\,(m - \theta/2\pi)^2}{2I}$$

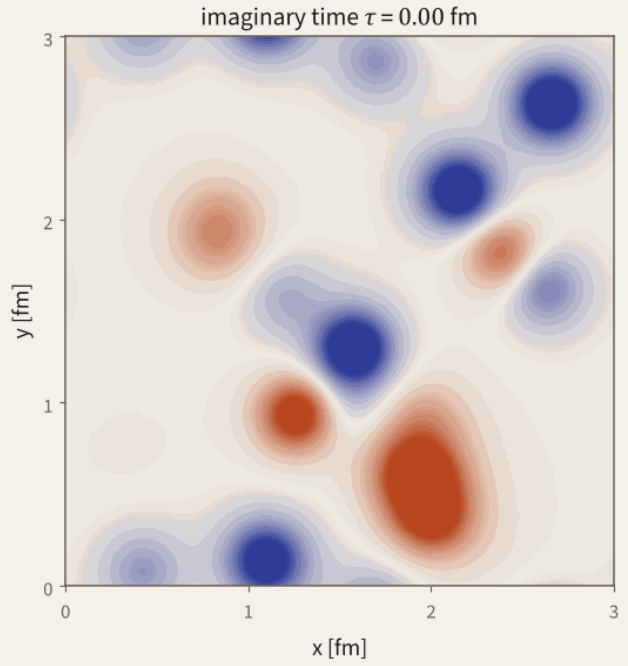
New ingredient: paths fall into classes by how often they wind; θ gives each class a phase. No classical force acts, yet the spectrum depends on the flux. The strong force has its own θ (slide 19).

IN DETAIL $Z(\theta) = \sum_{n\in\mathbb{Z}} e^{in\theta}\, Z_n$ paths sorted by winding n, each with the phase $e^{in\theta}$

SYMBOLS H Hamiltonian (the energy operator) · φ angle of the bead · p_φ its angular momentum · I moment of inertia · θ phase picked up by going once around the flux (Aharonov–Bohm) · $m = 0, \pm1, \ldots$ · n winding number of a path · for the staff: Z_n sum over paths winding n times

ESTABLISHED

A model of the vacuum: the instanton liquid



Why many? Configurations with many instantons, far apart, also contribute. The liquid model approximates that ensemble: about ⅓ fm, one per fm^4.

Imaginary time runs as it plays: slices of a Euclidean picture, not a real-time movie. Copper: instanton; blue: anti-instanton.

Illustration, not lattice data: BPST lumps with liquid-model parameters, a 2D view of a 4D box. Lattice pictures: Leinweber.

One instanton is a saddle; the model asks whether many of them capture the vacuum. What follows does not depend on this particular picture.

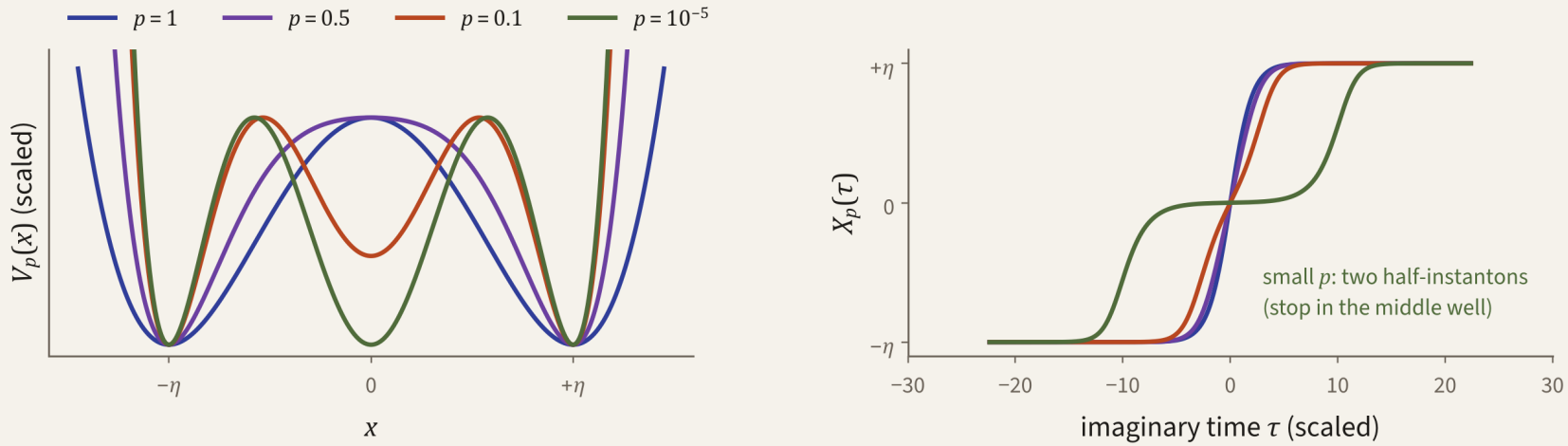
IN DETAIL $\rho \approx 1/3$ fm, $n_{I+\bar{I}} \approx 1\ \mathrm{fm}^{-4}$ instanton liquid (Shuryak–Zahed 2021, Eq. 6)

SYMBOLS fm femtometre = 10^{-15} m · copper blob: instanton (winding +1) · blue blob: anti-instanton (winding −1) · liquid model = a model of the vacuum as many instantons with fitted size and density · for the staff: ρ size, n number per fm^4 · saddle = a stationary point of the action; the instanton is one · instanton liquid = this model

SOURCES Shuryak, Zahed 2021b · Leinweber 2000 · Belavin et al. 1975

OURS OPEN

Why we went back to quantum mechanics



1 · What is imaginary time? A trick — or does complexified time, real and imaginary together, carry physics?

2 · How does the particle content change with the potential? From three wells to two the minima, the quanta and the tunnelling all change. How, exactly?

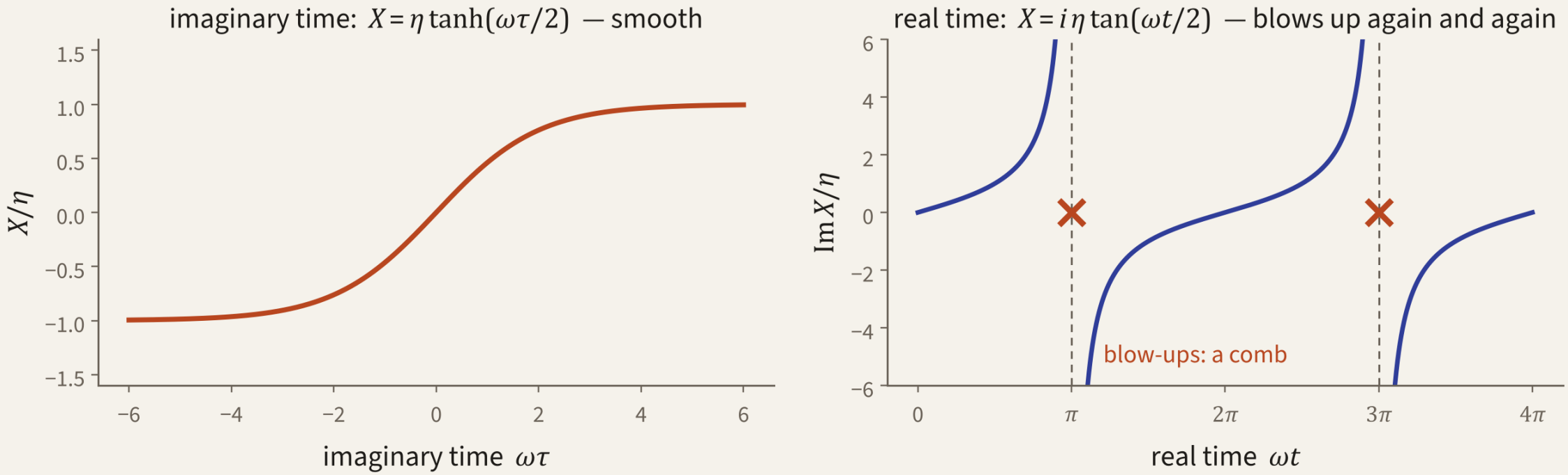
A laboratory: a family from three wells to two, instanton in closed form for every member (Loubal, Sýkora, Kos 2026). Question 1: next slides. Question 2: still open.

SYMBOLS p family parameter (p = 1: two wells; small p: three) · complexified time = time allowed to be a complex number, real and imaginary parts together

SOURCES Loubal, Sýkora, Kos 2026 · Cherman, Ünsal 2014

ESTABLISHED

Continue the instanton back to real time



Put real time into the instanton of slide 10: tanh becomes tan, which blows up again and again, a comb. Is this singular behaviour stable when we deform the potential?

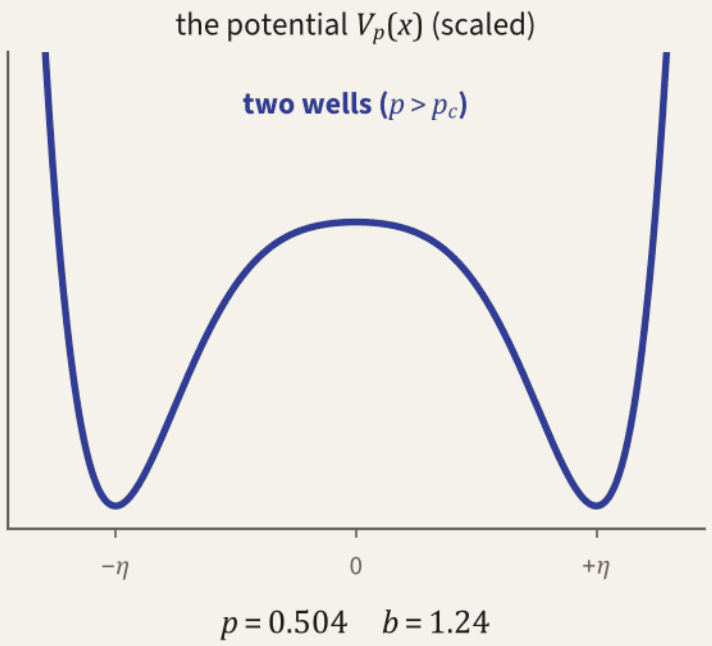
IN DETAIL $t = e^{-i\alpha}\tau$, $\alpha = \frac{\pi}{2}$: Euclidean, $\alpha = 0$: real time — rotate the time axis step by step (Cherman–Ünsal 2014)

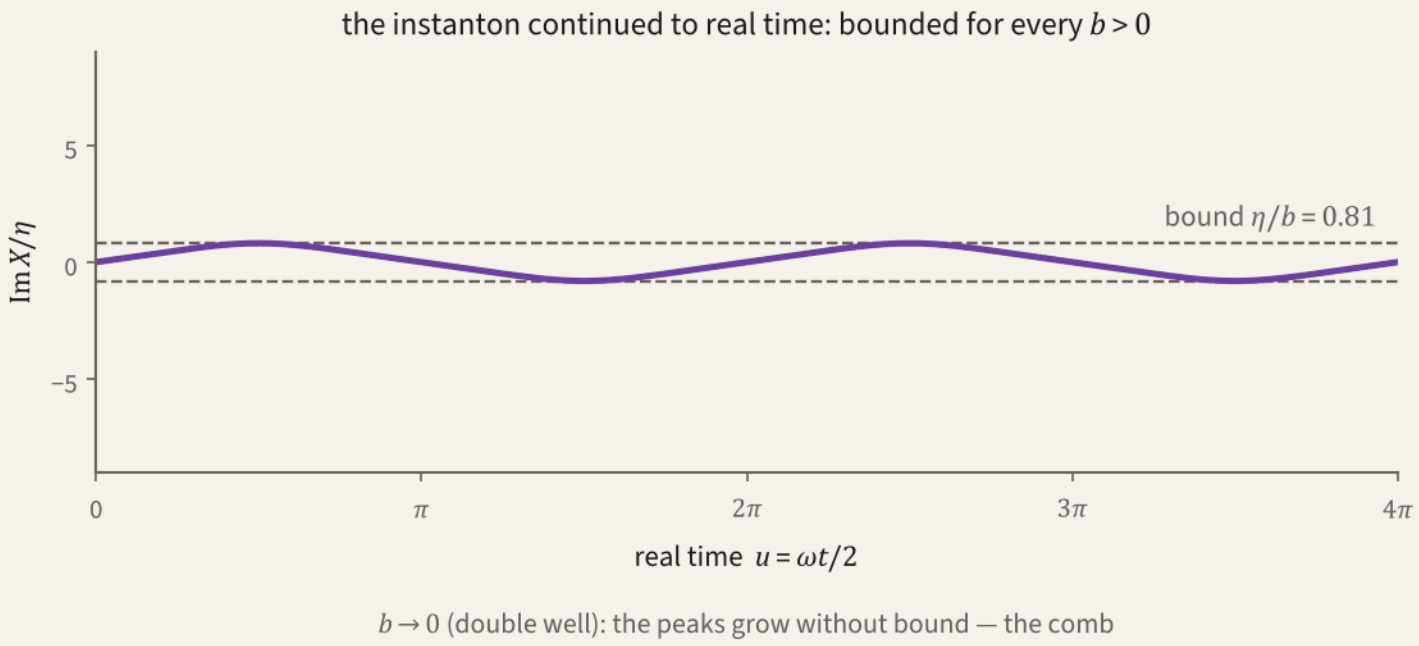
SYMBOLS τ imaginary time · t real time · X the double-well instanton of Part I, continued: a complex number, not a measured position · i imaginary unit · comb = infinitely many equally spaced blow-ups

SOURCES Cherman, Ünsal 2014 · Tanizaki, Koike 2014

OURS

In real time the instanton is regular





$b \to 0$ (double well): the peaks grow without bound — the comb

Plotted: $\mathrm{Im}\, X$ (in real time X is purely imaginary). For every $b > 0$ it stays bounded; only $b = 0$, the double well, blows up. The comb is not generic.

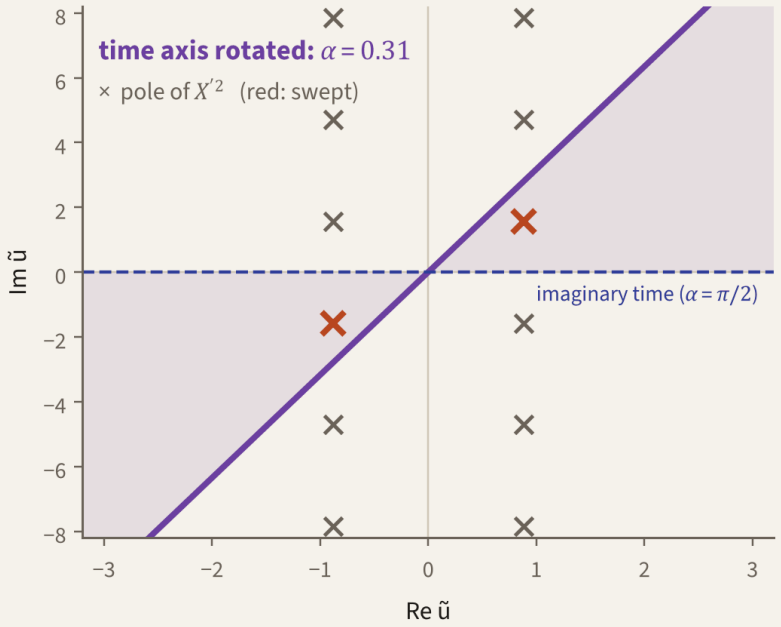
IN DETAIL $$X^{\alpha=0} = \frac{i\eta \sin u}{\sqrt{\cos^2 u + b^2}} \in \left[-\frac{i\eta}{b}, \frac{i\eta}{b}\right]$$ $|X| \le \eta/b$: the bound diverges as $b \to 0$ (non-uniform limit)

SYMBOLS $u = \omega t/2$ rescaled real time · $\mathrm{Im}\, X$ imaginary part of the continued position (in real time X is purely imaginary) · b shape parameter ($b = 0$: the double well)

SOURCES Loubal, Sýkora, Kos 2026 · Cherman, Ünsal 2014

OURS DEEPER DIVE ›

Rotate time: the cost stays, the phase clicks

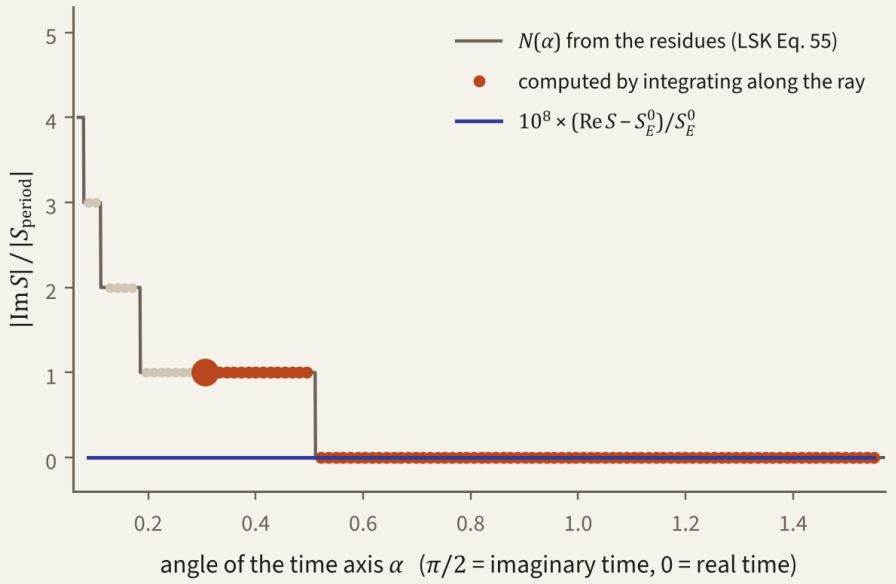


pole pairs swept

$N = 1$

$\mathrm{Im}\, S = -N \cdot S_{\mathrm{period}}$

$\mathrm{Re}\, S$ unchanged



$b = 1$ · Loubal, Sýkora, Kos, arXiv:2609.10621v2, Eqs. 50–55

Each step = one pole pair swept by the rotating ray: Im S jumps by one real-time period; Re S never changes for $\alpha > 0$. LSK v2 (2026) §5.1, Eqs. 50–55

A saddle contributes $e^{-S/\hbar}$. For this saddle we know how the suppression exponent and the phase change as the contour rotates. Which saddles contribute to the real-time amplitude, and how they combine, is open (slide 42).

IN DETAIL $S(\alpha) = S_E^0 - i\, N(\alpha)\, S_{\mathrm{period}}$ S_{period}: the action of one real-time period

SYMBOLS α angle of the time axis (90° imaginary, 0° real) · $S(\alpha)$ action along the rotated axis · Re S its real part (the suppression) · Im S its imaginary part (a phase) · S_{period} action of one real-time period · $N(\alpha)$ number of periods · $\tilde{u}$ rescaled complex time, $\theta = \mathrm{arsinh} b$ the position of the pole rows · LSK Loubal, Sýkora, Kos

SOURCES Loubal, Sýkora, Kos 2026 · Cherman, Ünsal 2014

ESTABLISHED

From the path integral to an effective vertex

Integrate the instanton out	position, size, colour orientation → an effective interaction, like Fermi's constant of the weak force
't Hooft's effective Lagrangian (schematic)	$\mathcal{L}_{\text{eff}} \propto \int d\rho\, D(\rho)\, \rho^{3n_f} \prod_f (\bar{\psi}_R \psi_L)_f$ — every light flavour flips chirality, $\Delta Q_5 = 2n_f$
Amplitude with gluons (KKS Eq. 2.6)	$\mathcal{A} \sim \int d^4x_0\, d\rho\, D(\rho)\, e^{-2\pi/\alpha_s} \prod A_{\text{LSZ}} \prod \psi_{\text{LSZ}}$
Each leg brings powers of the size	gluon $4\pi^2\rho^2/g$ (KKS 2.22), quark $2\pi\rho^{3/2}$ (MRS 52–53): $\mathcal{A} \propto \int d\rho\, D(\rho)\, \rho^{3n_f+2n_g+4}$

What a generator emits: one q q̄ pair per light flavour, all flipping chirality, plus gluons — weighted by a size integral that every extra leg makes harder.

SYMBOLS L_{eff} effective Lagrangian · $\psi_{\text{L}}, \psi_{\text{R}}$ left- and right-handed quark fields · n_f light flavours, n_g gluons · $A_{\text{LSZ}}, \psi_{\text{LSZ}}$ amputated gluon field and quark zero mode · x_0 instanton position · KKS Khoze, Krauss, Schott (2020) · MRS Moch, Ringwald, Schrempp (1997) · LSZ Lehmann–Symanzik–Zimmermann: amputating external legs to get an amplitude · effective vertex = a point-like interaction that stands in for the instanton

SOURCES 't Hooft 1976 (PRD) · Khoze, Krauss, Schott 2020 · Moch, Ringwald, Schrempp 1997

ESTABLISHED OPEN

A generator author's motivation: instantons in diffraction

WHY

"Predicted by QCD but never measured." A burst of gluons and one $q_L \bar{q}_R$ pair per light flavour, isotropic.

"Problem: UE has the same signature." Way out: diffraction (Taševský et al. 2023; Khoze et al. 2021).

But in hadron collisions diffraction does not factorise: the gap survival must be modelled.

HOW SHERPA DOES IT

Dynamical gap survival: generate the multi-parton interactions at the impact parameter given by the Pomeron flux; veto events with any — "no separate tuning".

Tested on CMS–TOTEM single-diffractive dijets (8 TeV).

Status: public with Sherpa 3.1 (planned).

The generator side reaches the same conclusion: switch the underlying event off with gaps — and model their survival dynamically.

P. Meinzinger, ATLAS SM workshop, Annecy, 16 Sep 2026 — motivation only.

SYMBOLS MPI multi-parton interactions · gap survival = the probability that nothing else fills the gap · b impact parameter = transverse distance between the two protons · UE underlying event = everything in a collision besides the hard process · SM Standard Model · TOTEM the forward experiment at the CMS interaction point · diffraction = a collision in which a proton survives intact, next to an empty gap · Pomeron = the colourless exchange behind diffraction

SOURCES Meinzinger 2026 · Taševský et al. 2023 · Khoze, Khoze, Milne, Ryskin 2021

OPEN

ATLAS instanton search with ALFA: which data exist?

YEAR	$\sqrt{s}$	β^*	DATA	TRIGGER	SUITABILITY
2012	8 TeV	90 m	500 μb^{-1}	elastic	far less data than central; trigger rejects the signal
2016	13 TeV	2.5 km	340 μb^{-1}	elastic	only extremely small angles: poor overlap
2017/18	13 TeV	90 m	dedicated fills	elastic	best energy match; luminosity to be confirmed
2012	8 TeV	high β^*	**1.67** nb^{-1}	MBTS one side + ALFA other	**single-tag topology, already recorded: new lead**

The analysis' assessment: ALFA runs are low pile-up by construction — good; but the elastic campaigns have two to three orders of magnitude less data than the central sample, and triggers built for elastic scattering.

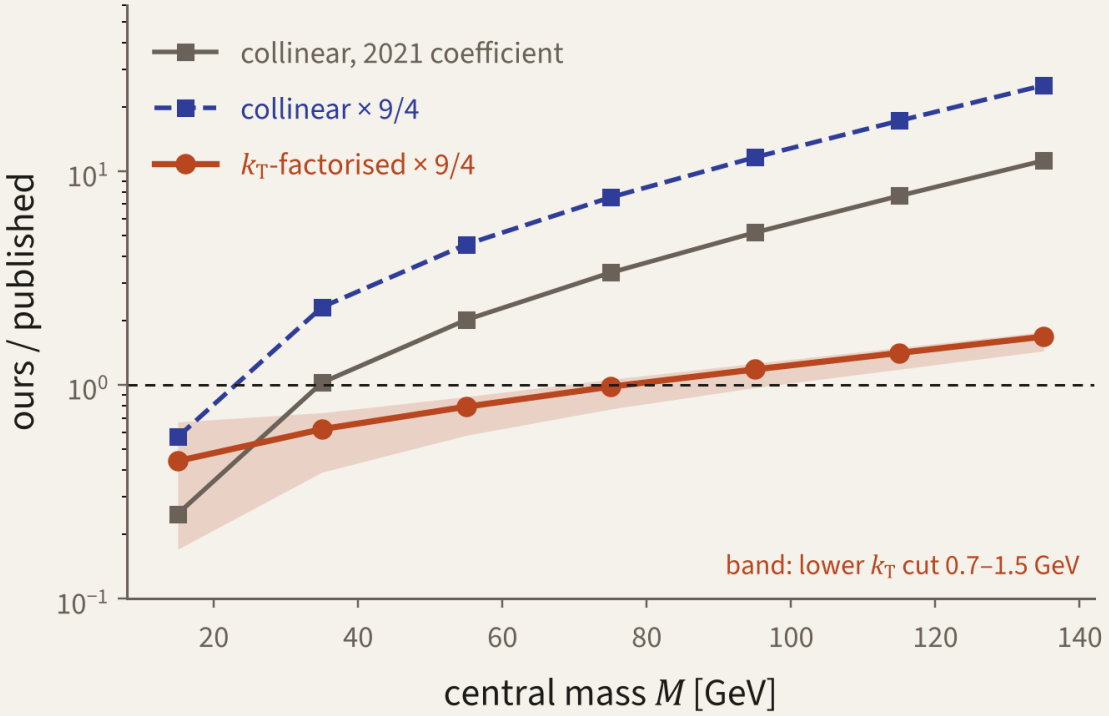
New lead, to check: the 2012 single-diffraction sample was triggered by MBTS + ALFA: 1.67 nb^{-1}, pile-up below 0.08, ALFA acceptance ≈ 30% in ξ (ATLAS, arXiv:1911.00453).

ATLAS ALFA instanton search, presented by R. Vavřička, CZ+SK HEP Workshop 2026 (public). Rows 1–3 from that talk; row 4 from the ATLAS paper.

SYMBOLS MBTS minimum-bias trigger scintillators · elastic trigger = a proton in both arms, back to back · nb^{-1} = 1000 μb^{-1} · μ pile-up · CZ+SK HEP Workshop the Czech–Slovak high-energy-physics workshop

SOURCES Vavřička 2026 · ATLAS 2020 (ALFA, SD) · ATLAS 2016 (ALFA, 8 TeV) · ATLAS 2022 (ALFA, 13 TeV)

Re-deriving the published central-diffractive rate (V. Loubal)



Goal: a minimal generator for pp → p + X_I + spectators + p, H1 Fit B Pomeron gluons, LHE → Pythia 8.

Benchmark: KMS 2021 and KKMR 2021 tables to ≈ 1%, but KKMR 2022 missed by 0.25–11× (grey).

Two causes: the gluon insertion changed by 9/4 between the papers; the published column is k_T-factorised, not collinear.

With both, nothing fitted: within a factor 2 over 15–135 GeV (red); 6.93 vs 7.06 pb at 75 GeV.

Next: k_T-factorised event weights with sampled spectator momenta, the single-tag mode for ALFA and AFP, a regenerated sample.

SYMBOLS k_T transverse momentum of the incoming gluon · collinear = gluons along the beam · H1 Fit B = measured Pomeron parton densities · SFG Student Faculty Grant · LHE Les Houches event file · X_I the instanton-induced system · KMS Khoze, Milne, Spannowsky · KKMR Khoze, Khoze, Milne, Ryskin

SOURCES Loubal (SFG) 2026 · Khoze et al. 2022 · Khoze, Milne, Spannowsky 2021

Two analyses, one question

	CENTRAL SEARCH	FORWARD: ALFA / AFP
Data	special low-pile-up 13 TeV runs, $\approx 1.6\ \mathrm{nb}^{-1}$	ALFA: special high-β^* runs; AFP: needs a low-pile-up run
Events	all inelastic collisions	intact proton(s) and rapidity gap(s)
Background	underlying event — from data (ABCD)	much smaller; accidental proton tags instead
Gains	statistics; the first full analysis	the protons measure ξ, hence the mass; ratios at fixed mass
Theory input	pp rate depends on the size treatment	the same, times gap survival $S^2 \approx 0.05$–0.1
Status (public)	unblinded; paper in the ATLAS publication process	ALFA: ATLAS search in preparation; AFP: feasibility

Why both: an excess in the central search needs an independent check with different systematics; without one, the forward channel is the cleaner way to lower masses.

SOURCES Harris (ATLAS) 2024 · Taševský, Ryskin 2026 · Vavřička 2026 · Taševský et al. 2023

OPEN

Open problem: does factorisation hold in diffraction?

ep, inclusive DIS	standard framework (RS98 Eq. 2) — but the instanton vertex, a $2n_f$-fermion non-perturbative operator, is not covered by the proofs	ASSUMED
ep, diffractive DIS	proven (Collins 1998): diffractive parton densities are universal in DIS	PROVEN
pp, diffractive	broken: soft rescattering fills the gaps; repaired by a global $S^2 \approx 0.05$–0.1, or dynamically (Sherpa: veto extra interactions)	BROKEN
pp, scheme	collinear or k_T-factorised gluons: up to 25× apart for the published rate at 135 GeV (slide 55)	OPEN

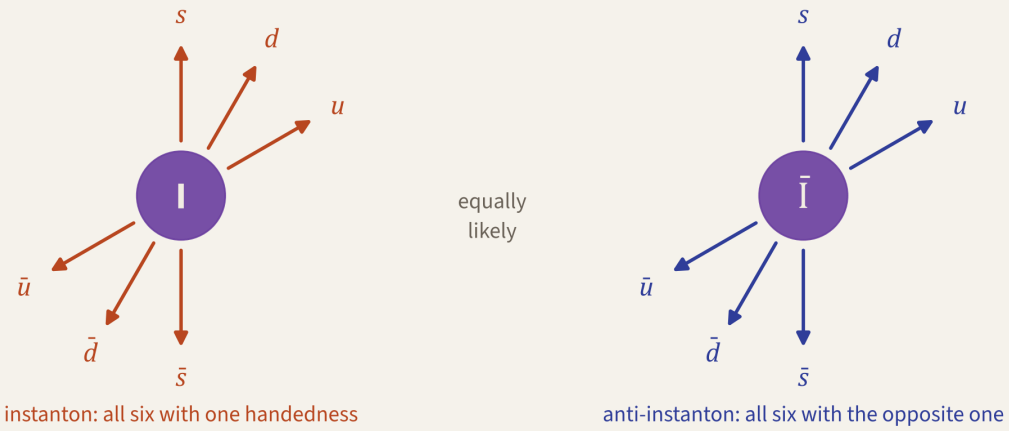
What would settle it: diffractive over inclusive at the same mass, for single tag, double tag and gap only, against the survival models — where Meinzinger's generator programme and the two ATLAS analyses meet.

SYMBOLS factorisation = cross section written as (flux or parton density) × (hard subprocess), each part universal · DPDF diffractive parton density · rescattering = soft interactions between the other partons of the two protons

SOURCES Collins 1998 · Meinzinger 2026 · Khoze, Khoze, Milne, Ryskin 2021 · Loubal (SFG) 2026

SPECULATIVE DEEPER DIVE ›

Speculation: are the quarks from one instanton entangled?



Entanglement: one joint state, not a product; each side alone random, the comparison correlated. Bell tests certify it (loophole-free since 2015).

One instanton, one quantum event: a q q̄ pair of every light flavour (slide 20); within the event all flip handedness the same way.

Correlation is not yet entanglement: a mixture of instanton and anti-instanton events gives the same pattern. The spin state is an open calculation.

Speculation: a pair born by tunnelling starts from one imaginary-time event, then flies apart in our spacetime. Does something connect the partners there? Such a link would have to be instantaneous and hidden (Bancal et al.; Salart et al.: above 10^4 c). No working theory.

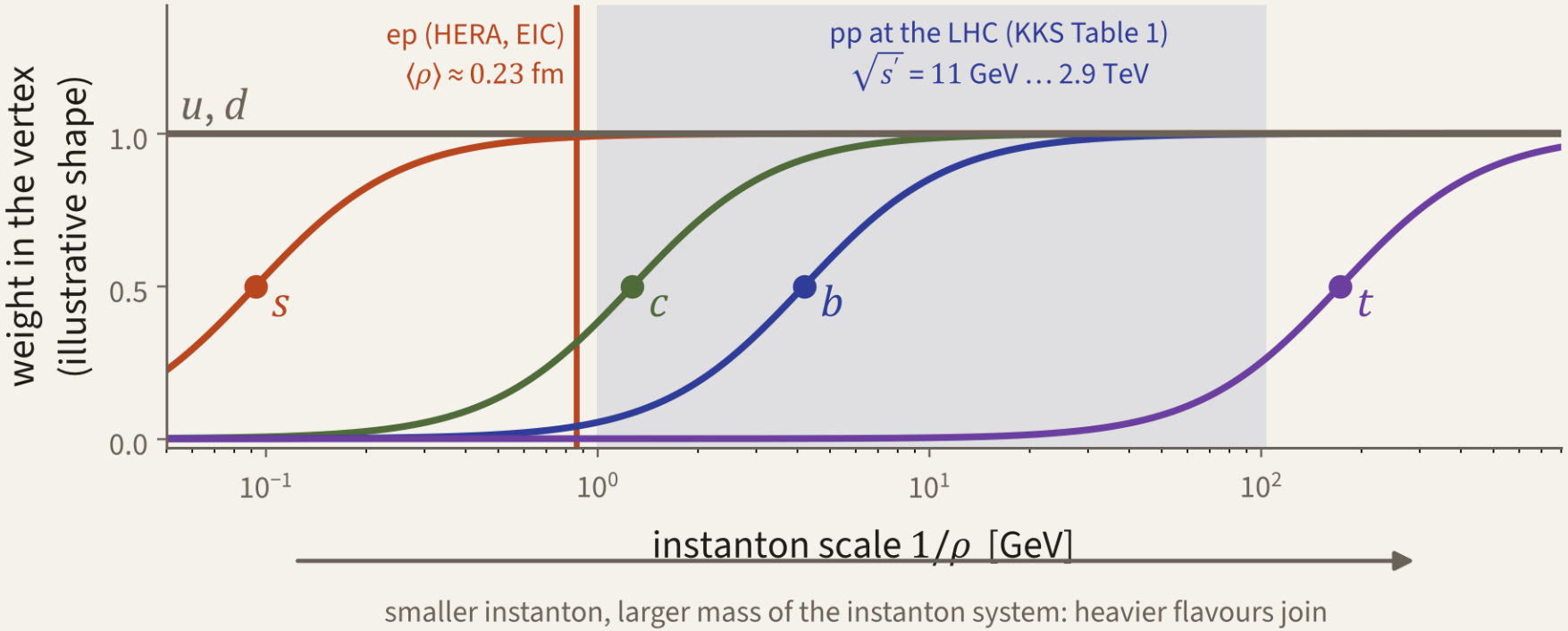
A handle: $\Lambda\,\bar{\Lambda}$ spin correlations, the Λ 's spin sits on its strange quark. STAR's (18 ± 4)% motivates the question, it does not answer it.

SYMBOLS entanglement = two particles in one joint quantum state that is not a product of one state for each · Bell test = a measurement that certifies it · q quark, q̄ antiquark · Λ, $\bar{\Lambda}$ lambda hyperon (u d s) and its antiparticle · c speed of light · EIC Electron–Ion Collider · STAR a detector at the RHIC collider (Brookhaven)

SOURCES Einstein, Podolsky, Rosen 1935 · Hensen et al. 2015 · STAR 2026 · Bhattacharyya, Hung, Melby-Thompson 2017 · Bancal et al. 2012 · Salart et al. 2008

ESTABLISHED

Which flavours take part: light on the instanton's scale



smaller instanton, larger mass of the instanton system: heavier flavours join

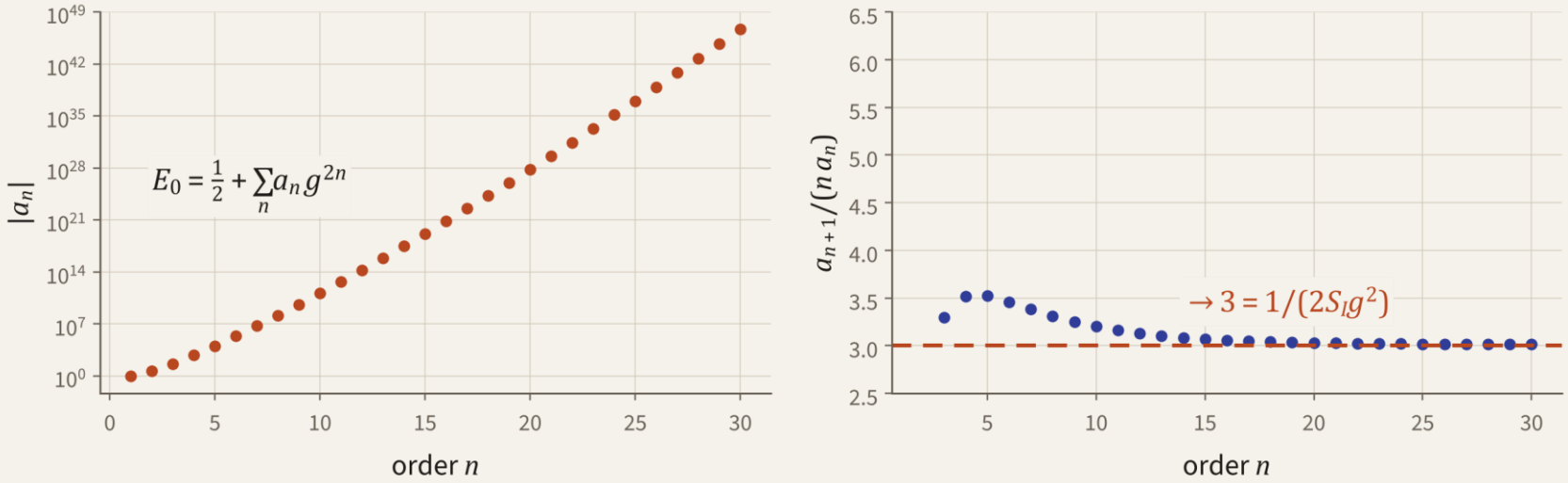
A flavour takes part when $m_q \ll 1/\rho$. In ep: u, d, s, with charm at the border. In pp the count grows with the mass: KKS use four flavours, then five above $\sqrt{s'} \approx 60$ GeV.

SYMBOLS ρ instanton size · $1/\rho$ its scale · m_q quark mass · u, d, s, c, b, t the six quark flavours · n_f number of flavours taking part · ep electron–proton · pp proton–proton · KKS Khoze, Krauss, Schott (2020)

SOURCES Khoze, Krauss, Schott 2020 · Moch, Ringwald, Schrempp 1997 · 't Hooft 1976 (PRD)

ESTABLISHED DEEPER DIVE ›

Perturbation theory knows anyway



The model: the double well of slide 10, shifted and rescaled to $V = x^2(1-gx)^2/2$. Right panel: the ratios $a_{n+1}/(n\,a_n)$ approach $3 = 1/(2S_I g^2)$, a value set by the action of an instanton–anti-instanton pair.

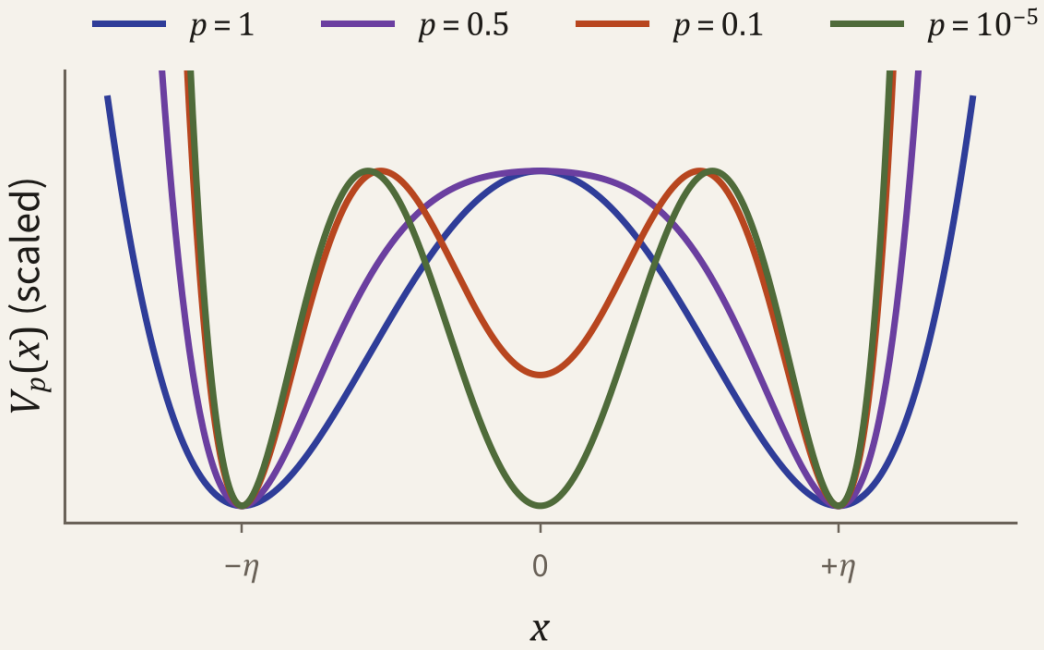
Finite orders miss tunnelling, but their large-order pattern carries information about the non-perturbative saddles: perturbation theory 'knows'.

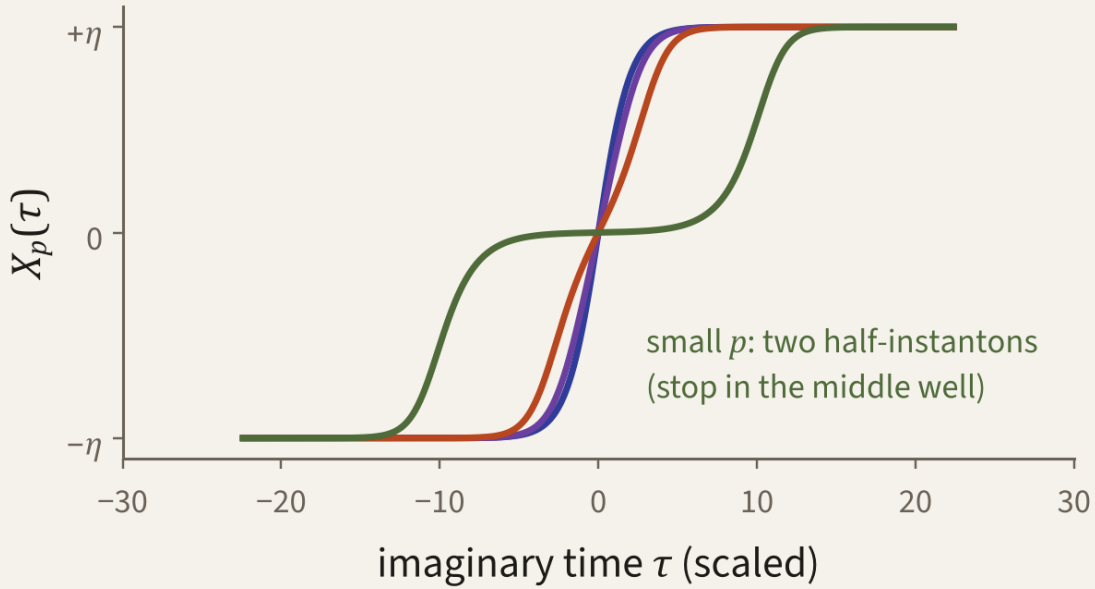
IN DETAIL $$E_0 = \frac{1}{2} - g^2 - \frac{9}{2}g^4 - \frac{89}{2}g^6 - \cdots, \qquad \frac{a_{n+1}}{n\,a_n} \to 3 = \frac{1}{2S_I\,g^2}$$ pair action $2S_I$ sets the growth

SYMBOLS anharmonic oscillator = a spring that does not quite obey Hooke's law · a_n coefficient of g^{2n} in the ground-state energy E_0 · $n!$ factorial · for the staff: $S_I = 1/(6g^2)$ the action of one instanton of this double well, $2S_I$ that of an instanton–anti-instanton pair

SOURCES Bender, Wu 1969 · Dyson 1952

A family in closed form: from three wells to two





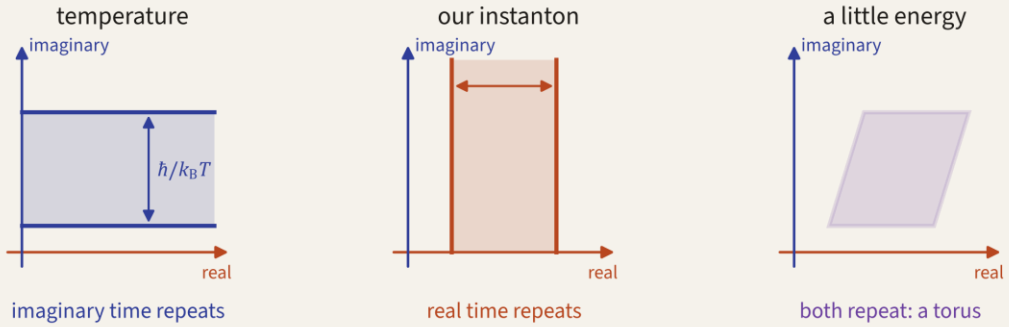
Our test bed: from triple to double well, the instanton exact for every member, always joining the outer minima $\pm\eta$; below p_c it is one smooth solution with two steps and a stay near the middle well (right). Exact means the classical solution, for $0 < p \leq 1$.

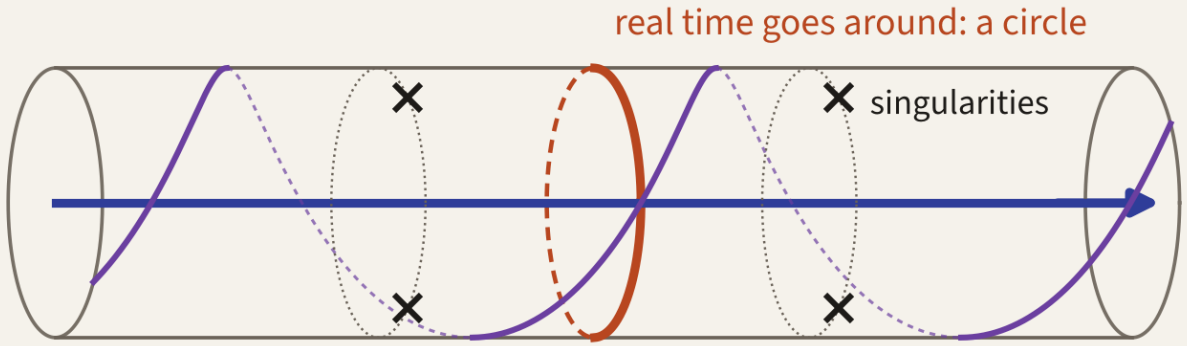
IN DETAIL $V_p(x) = \lambda\,(x^2 - \eta^2)^2\,[(1-p)\,x^2 + \xi^2 p], \qquad 0 < p \leq 1$ $b^2 = (\eta^2/\xi^2)(1-p)/p$: $b = 0$ is the double well

SYMBOLS p family parameter ($p = 1$ double well, $p \to 0$ triple well) · b shape parameter used next: $b = 0$ is the double well, b grows as p decreases · $\pm\eta$ the outer minima · for the staff: λ coupling, ξ second length scale, p_c value at which the middle well appears

SOURCES Loubal, Sýkora, Kos 2026

The solution repeats: complex time as a cylinder





Left: temperature; imaginary time closes because of the trace (slide 7).

Middle: our solution repeats in real time, so we identify points one period apart. Time itself is not a circle.

Right, an aside: a little energy, both repeat.

Drawn on a cylinder: imaginary time along it, real time around it. Crosses: singularities of the solution; the violet spiral, a rotated time axis, passes them as it rotates (next slide).

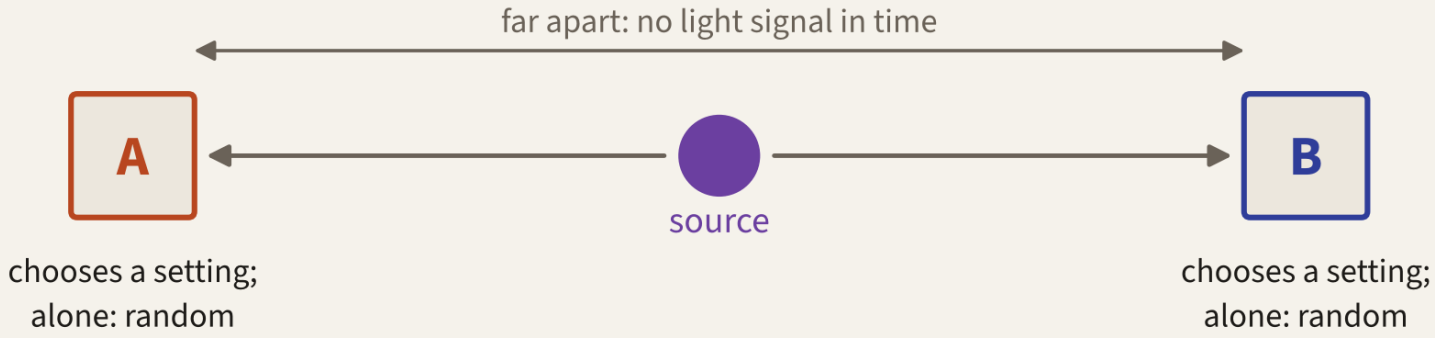
IN DETAIL $X(t+T) = X(t), \qquad T = \frac{4\pi}{\omega}$ the continued solution repeats

SYMBOLS complex time = time with a real and an imaginary part, drawn as a plane · cylinder: a plane rolled up in one direction · spiral = a time axis rotated by an angle · for the staff: T period of the continued solution

SOURCES Loubal, Sýkora, Kos 2026

ESTABLISHED

Entanglement: two particles, one state



compared later: the results are correlated — more strongly than any instructions from the source allow

Entanglement: the joint (pure) state cannot be written as one state for each particle times the other. Each side alone is then described by a mixed reduced state: its outcomes alone are probabilistic.

EPR 1935, "Can quantum-mechanical description of physical reality be considered complete?": then each must carry its answers from the start. **Bell:** no local answers carried from the source reproduce the correlations; experiments agree (loophole-free, 2015).

Entanglement (the joint state) · Bell violation (a stronger test that can certify it) · no signalling (one side alone never sees what the other does). The puzzle: how can two far-apart particles behave as one?

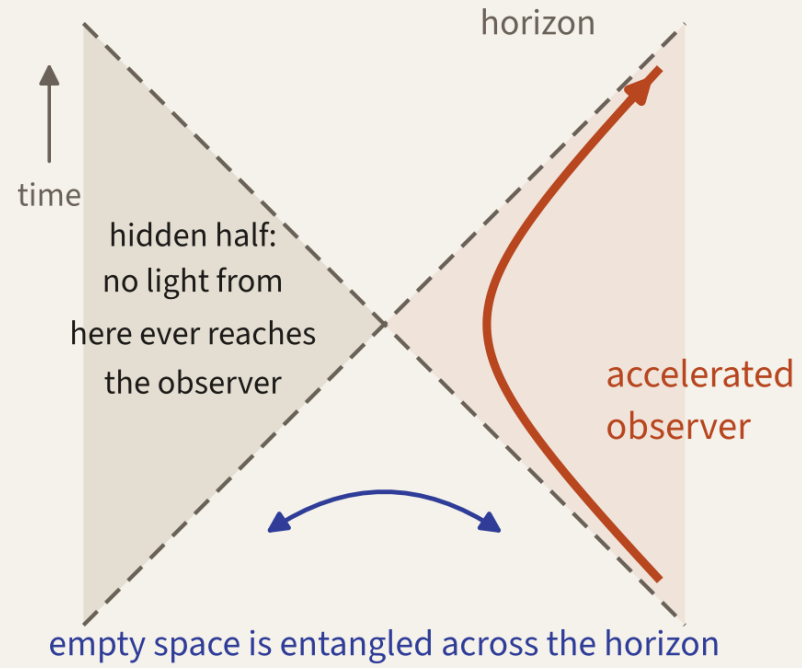
IN DETAIL $|S| \le 2$ (any instructions carried from the source), $|S| \le 2\sqrt{2}$ (quantum mechanics)

SYMBOLS joint state = one quantum state for both · EPR Einstein, Podolsky, Rosen · Bell violation = correlations no local hidden variables can give · reduced state = one particle alone · no signalling = one side alone never sees what the other does · for the staff: S the CHSH combination

SOURCES Einstein, Podolsky, Rosen 1935 · Hensen et al. 2015 · Witten 2018

ESTABLISHED DEEPER DIVE ›

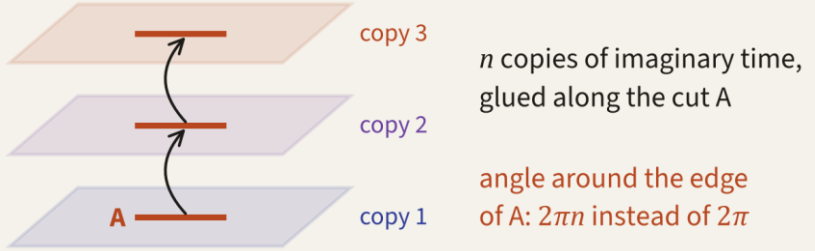
Entangled across a horizon, warm on one side



The accelerated observer never sees the other half. To predict what it sees, we ignore the hidden half: what remains is a reduced state.

Why thermal, not just mixed? In imaginary time the boost around the horizon is a rotation that must close after a full turn, as on slide 8: a fixed period, hence a temperature.

Aside (backup B25): the same reduced state also gives the entanglement entropy, by the replica construction.



Thermal for the accelerated observer's own time evolution, generated by boosts: not a gas at one temperature for everyone. Special to this vacuum and this horizon.

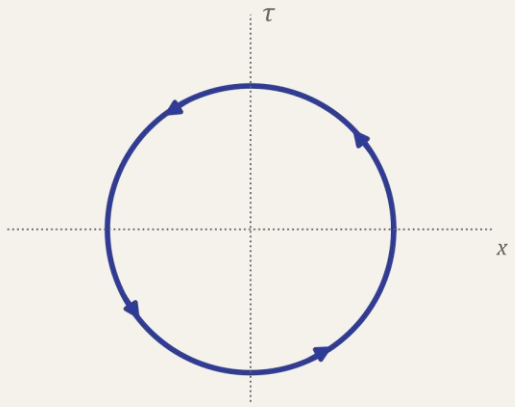
IN DETAIL $\rho_A \propto e^{-2\pi K} \Rightarrow T_{\text{Unruh}} = \dfrac{\hbar a}{2\pi c\, k_B}$ the visible half is a Boltzmann state of boosts (Bisognano–Wichmann)

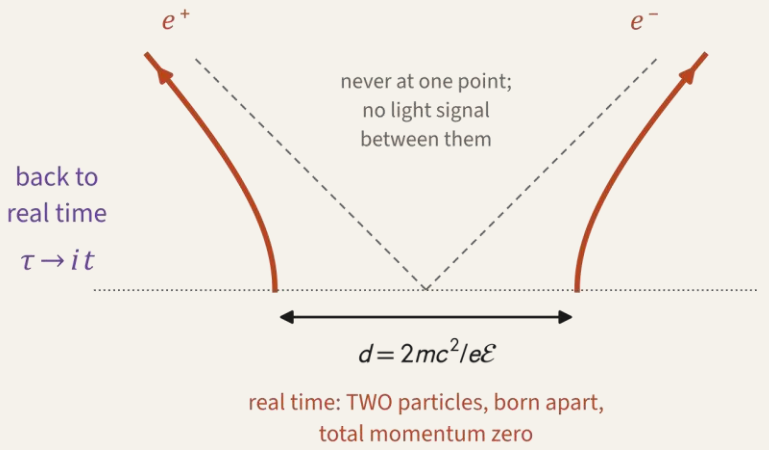
SYMBOLS horizon = the boundary beyond which no light can ever reach the observer · reduced state = the description of one side after ignoring the other · replica trick = glue copies of imaginary time to count entanglement · for the staff: ρ_A state of the visible half, K boost generator, T_{Unruh} Unruh temperature

SOURCES Bisognano, Wichmann 1975 · Unruh 1976 · Calabrese, Cardy 2004

ESTABLISHED

The Schwinger effect: one loop, two particles





$$H = \sqrt{p^2c^2 + m^2c^4} - e\mathcal{E}x: \qquad (x - x_0)^2 - c^2t^2 = R^2 \quad \overset{t=-i\tau}{\Rightarrow} \quad (x - x_0)^2 + c^2\tau^2 = R^2, \qquad R = \frac{mc^2}{e\mathcal{E}}$$

Same Hamiltonian, two times. Real time: a hyperbola, two particles flying apart. Imaginary time: a circle, one loop, joining the two branches at their turning points, $2mc^2/e\mathcal{E}$ apart.

IN DETAIL $\Gamma \propto \exp\left(-\frac{\pi m^2 c^3}{e\,\mathcal{E}\,\hbar}\right)$ the action of the loop (Affleck–Alvarez–Manton 1982)

SYMBOLS H Hamiltonian · p momentum · x position along the field · $\mathcal{E}$ electric field · e charge · m electron mass · c speed of light · $R = mc^2/e\mathcal{E}$ · $\mathcal{E}_c \approx 1.3 \times 10^{18}$ V/m the critical field · turning point = where the imaginary-time loop meets real time

SOURCES Schwinger 1951 · Affleck, Alvarez, Manton 1982

ESTABLISHED

Schwinger production and an EPR-type pair state

EPR 1935 (an idealised state)

two particles at a fixed distance, total momentum zero

Schwinger pair (an analogy, not the same state)

born about $2mc^2/e\mathcal{E}$ apart, total momentum zero

State: to illustrate, take the one-pair part, with the electron and the positron as the two subsystems: $\sum_p c_p \ |p\rangle_{\text{electron}} \, | -p\rangle_{\text{positron}}$, a coherent superposition (standard form, not derived here). Zero total momentum alone would not guarantee coherence.

Geometry: the imaginary-time loop of the previous slide represents the production process; the joint state was prepared by that one process. The boxes above are an analogy, not evidence that the states match.

Limit: the production geometry and the correlated quantum state describe different aspects of one process. The loop is not a communication channel and does not derive the Bell correlations.

SYMBOLS d distance between the particles · p momentum · $p_1 + p_2$ total momentum · c speed of light

SOURCES Einstein, Podolsky, Rosen 1935 · Schwinger 1951

Speculation: a signal through imaginary directions?

SPECULATIVE

A separate conjecture, not an implication of the imaginary-time picture. The idea: the two particles are far apart in our spacetime but close in complexified spacetime; perhaps something connects them there.

Known constraints: finite-speed hidden-influence models of quantum correlations would allow signalling (Bancal et al.). Experiments give frame-dependent lower bounds, e.g. above $10^4 c$ if the Earth moves slower than $10^{-3} c$ in the preferred frame (Salart et al.).

And: it would have to stay hidden and uncontrollable, or it would carry messages. Such a channel has no defined distance or propagation rule.

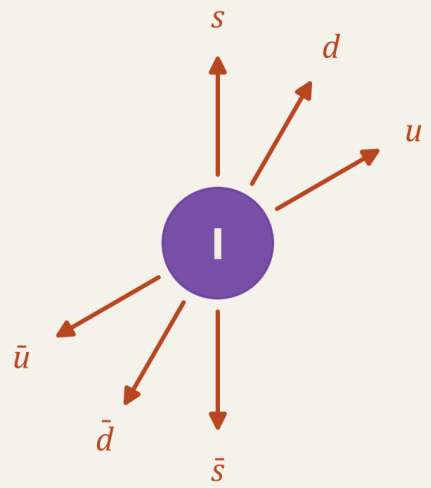
Status: nobody has a working theory of such a channel.

The established picture ends here. The next question, about quarks from QCD instantons, does not need this conjecture.

SYMBOLS c speed of light · signalling = sending a message that the other side can read

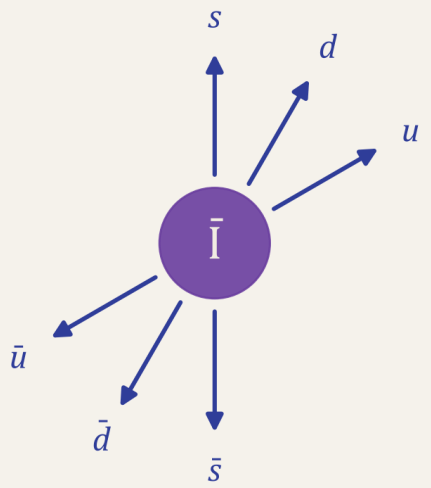
SOURCES Bancal et al. 2012 · Salart et al. 2008

From the instanton vertex to observable quark correlations



instanton: all six with one handedness

equally likely



anti-instanton: all six with the opposite one

Recall slide 20: the instanton vertex ties together flavour and handedness. In the production channel considered here: a quark–antiquark pair of every light flavour in one event.

The open calculation: what spin state follows for an observable pair, such as s s̄ → $\Lambda\,\bar{\Lambda}$? Only then can the observable spin correlations and entanglement be quantified.

Why Λ and $\bar{\Lambda}$? in the quark model a Λ 's spin is carried by its strange quark, and its decay shows the spin's direction. How much s s̄ spin survives into the $\Lambda\,\bar{\Lambda}$ (spin transfer) is itself uncertain. **STAR's (18 ± 4)% motivates the question; it does not test this not-yet-calculated prediction.**

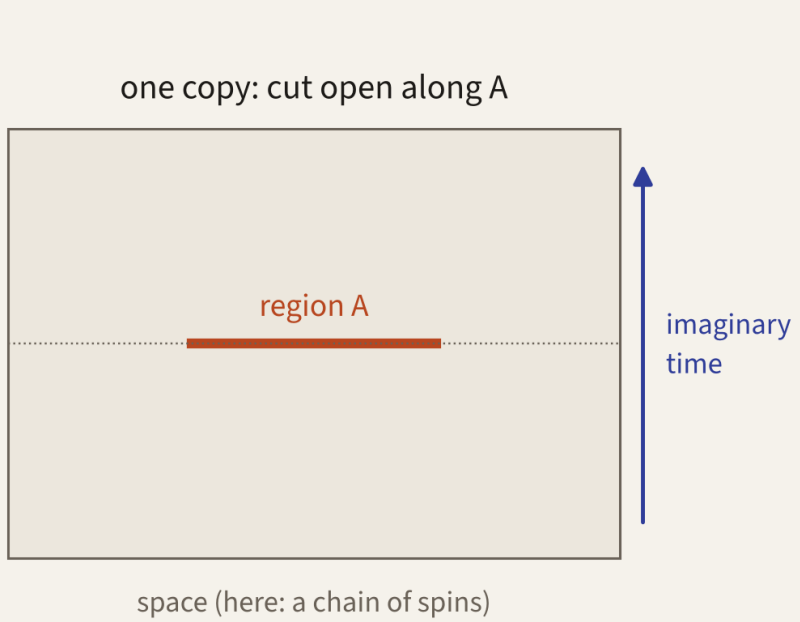
SYMBOLS I instanton, Ī anti-instanton · u, d, s up, down, strange quarks · handedness = chirality (slide 20) · Λ, $\bar{\Lambda}$ lambda hyperon (u d s) and its antiparticle · hadronisation = quarks turning into the hadrons we detect

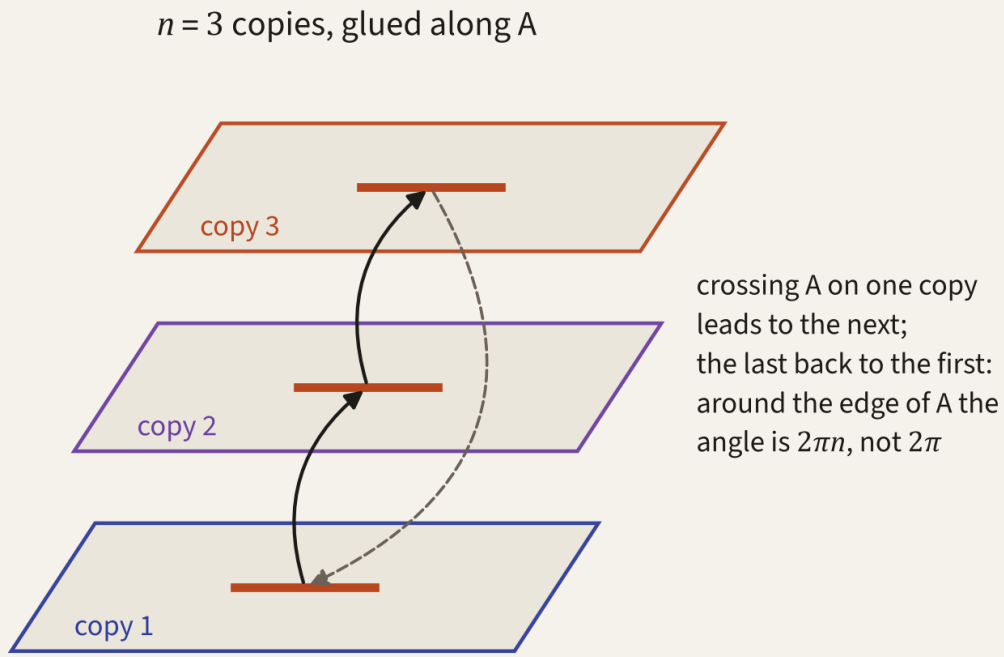
SOURCES 't Hooft 1976 (PRD) · STAR 2026 · Bhattacharyya, Hung, Melby-Thompson 2017

ESTABLISHED

← back to slide 64

Entropy from opening an angle





Glue n copies of imaginary time along A: the angle around its edge becomes $2\pi n$. Opening it measures entanglement; for a critical spin chain $S \approx (c/3)\ln(\ell/a)$. The same geometry as the black hole of slide 8.

IN DETAIL

$$S_A = -\,\partial_n \ln \mathrm{Tr}\,\rho_A^n\Big|_{n=1}, \qquad \mathrm{Tr}\,\rho_A^n = \frac{Z_n}{Z_1^n}$$

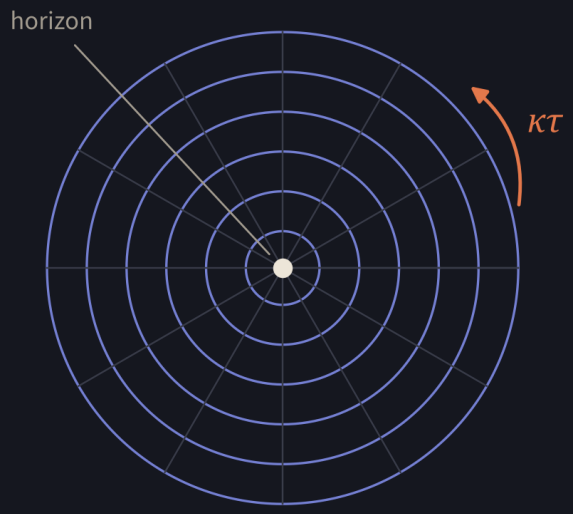
SYMBOLS region A = a piece of the system (here a stretch of a chain of spins) · n number of copies · ℓ length of A · a spacing of the chain · c a number characterising the chain ($c = 1/2$ for the critical Ising chain) · S_A entanglement entropy

SOURCES Calabrese, Cardy 2004 · Callan, Wilczek 1994

Hawking temperature is a smoothness condition

$$ds^2 = f(r)\,c^2 d\tau^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2, \qquad f(r) = 1 - \frac{2GM}{c^2 r}$$

$$ds^2 \simeq \rho^2\, d\left(\frac{\kappa\tau}{c}\right)^2 + d\rho^2 \quad \Rightarrow \quad \tau \sim \tau + \frac{2\pi c}{\kappa}$$



period $= 2\pi/\kappa$: smooth disk



any other period : cone, singular tip

$$T_H = \frac{\hbar\kappa}{2\pi c\, k_B} = \frac{\hbar c^3}{8\pi G M k_B} \approx 6 \times 10^{-8}\,\mathrm{K} \times \frac{M_\odot}{M}$$

No conical tip ⇒ fixed period ⇒ temperature.

Pure differential geometry: the same argument as for temperature, now forced by the geometry.

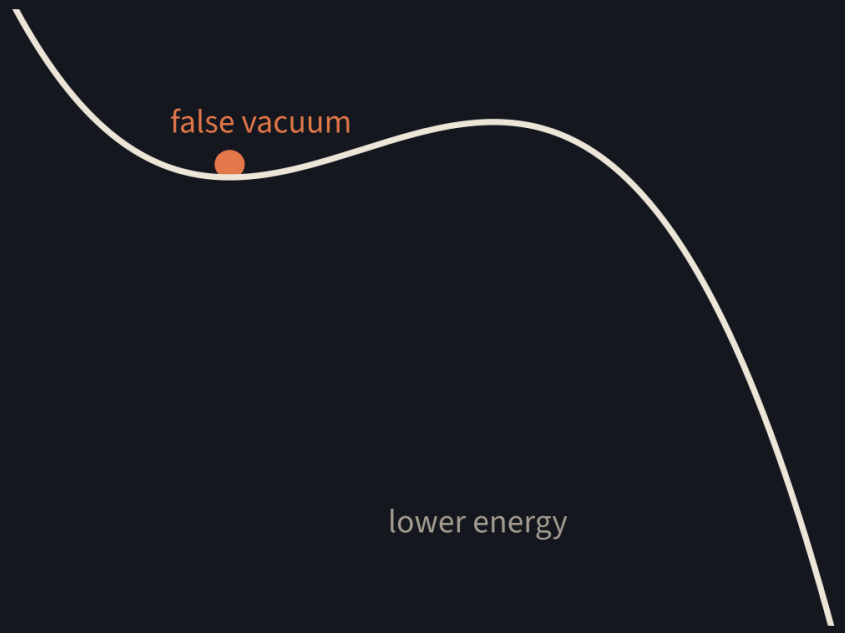
Checked by script: $\kappa = 1/4M$ $(G = c = 1)$, $T = 1/8\pi M$; 6.2×10^{-8} K for one solar mass.

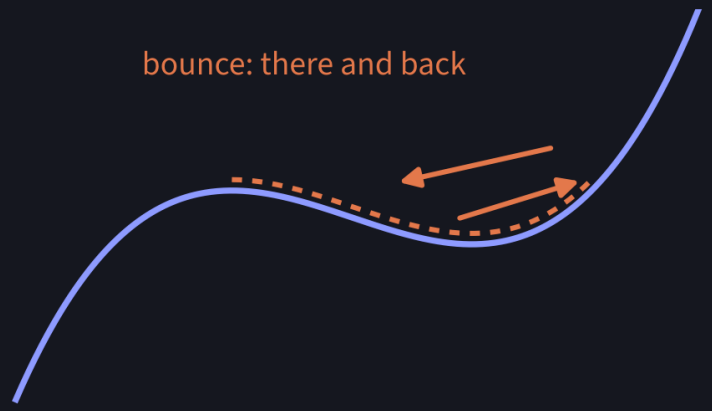
← back to slide 8

SYMBOLS ds^2 line element · $f(r)$ metric function · $d\Omega^2$ sphere angles · $\kappa = c^4/4GM$ surface gravity · ρ proper distance from the horizon · M_{sun} solar mass

SOURCES Hawking 1975 · Gibbons, Hawking 1977

One formula for boiling, vacuum decay and our Universe





$$\Gamma = -\frac{2}{\hbar}\,\mathrm{Im}\,E_0 \quad \propto \quad e^{-S_B/\hbar}$$

The bounce has one unstable direction → an imaginary part of the energy = decay rate (Callan–Coleman).

Same mathematics: droplets in a supercooled liquid; bubbles of a new vacuum (Coleman–De Luccia, with gravity); the metastability of our Higgs vacuum.

Recovered directly in real time on Lefschetz thimbles: Ai, Garbrecht, Tamarit (2019).

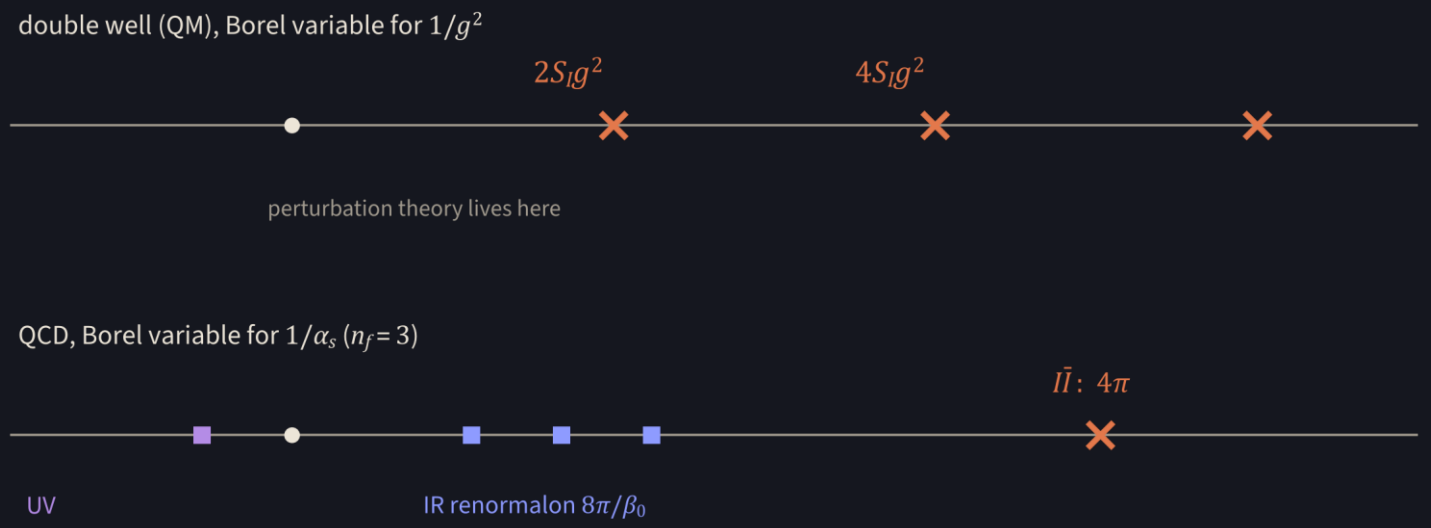
← back to slide 9

SYMBOLS Γ decay rate · E_0 energy of the metastable state · Im imaginary part · S_B action of the bounce

SOURCES Coleman 1977 · Callan, Coleman 1977 · Coleman, De Luccia 1980 · Ai, Garbrecht, Tamarit 2019

The Borel plane: where the two worlds meet

$$B(t) = \sum_n \frac{a_n}{n!}\, t^n, \qquad E_0 \sim \int_0^\infty d\,t\; e^{-t/g^2}\, B(t)$$



Quantum mechanics: the ambiguity at $t = 2S_I g^2 = 1/3$ cancels against the instanton–anti-instanton contribution (resurgence).

QCD: renormalons sit 4.5 times closer and dominate; their semiclassical meaning is open.

Physicists' 'streamlines' = Lefschetz thimbles (Shuryak–Zahed 2021, footnote 1).

$$t_{\mathrm{IR}} = \frac{8\pi}{\beta_0} \approx 2.8 \;<\; t_{I\bar{I}} = 4\pi \approx 12.6 \qquad \left(\beta_0 = 11 - \frac{2}{3} n_f = 9\right)$$

← back to slide 60

SYMBOLS $B(t)$ Borel transform · t Borel variable (conjugate to $1/g^2$ for the double well, to $1/\alpha_s$ for QCD) · $\beta_0 = 11 - 2n_f/3$ first coefficient of the QCD β-function · n_f number of light quark flavours · IR, UV infrared, ultraviolet

SOURCES Dunne, Ünsal 2012 · Shuryak, Zahed 2021

Self-duality: one equation, three signatures

τ

$$\star\star = +1 \ \text{(signatures (4,0) and (2,2))}, \qquad \star\star = -1 \ \text{(signature (3,1))} \ \Rightarrow \ \mathbf{E} = \pm i\,\mathbf{B}$$

Euclidean (4,0)

real self-dual solutions: the instantons

Lorentzian (3,1)

$\mathbf{E} = \pm i\mathbf{B}$: no real instanton in our spacetime

Kleinian (2,2)

real again: the natural home of twistors

Gravity has its own instantons: Riemannian metrics with self-dual curvature — Eguchi–Hanson, Taub–NUT.

All Yang–Mills instantons on $\mathbb{R}^4$ are known: the ADHM construction (1978); size and position are coordinates on their moduli space.

← back to slide 16

SYMBOLS * Hodge dual · signature (p,q): number of + and − signs in the metric · E, B electric and magnetic fields · ADHM Atiyah–Drinfeld–Hitchin–Manin construction

SOURCES Eguchi, Hanson 1978 · Atiyah et al. 1978

τ

Topology weighs the η' meson

$$m_{\eta'}^2 + m_\eta^2 - 2m_K^2 = \frac{2N_f\,\chi_{\mathrm{YM}}}{f_\pi^2}, \qquad \chi_{\mathrm{YM}} = \frac{\langle Q^2 \rangle}{V_4}$$

$$\chi_{\mathrm{YM}}^{1/4} \approx 179\ \mathrm{MeV} \qquad (\eta' \text{ alone would give } 190\ \mathrm{MeV})$$

9

light pseudoscalars expected

8

light ones found

958 MeV

the ninth: η'

Large N_c: instantons are suppressed like $\exp(-N_c)$, yet the η' mass survives — it is topology, not specifically a dilute instanton gas (Witten).

← back to slide 20

SYMBOLS $m_{\eta'}, m_\eta, m_K$ meson masses · $f_\pi \approx 92$ MeV pion decay constant · χ_{YM} topological susceptibility of gluons alone · $\langle Q^2 \rangle$ mean square topological charge · V_4 Euclidean four-volume · N_c number of colours

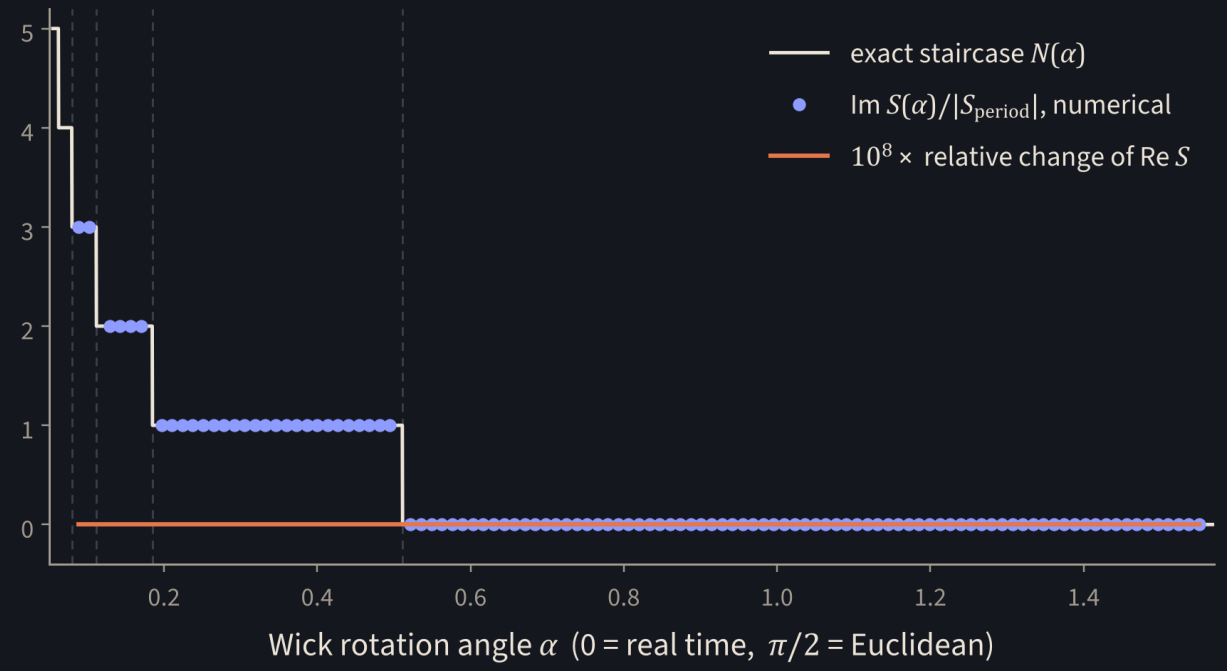
SOURCES Witten 1979 · Veneziano 1979

S9 *"… the real part of the action, which determines the tunnelling suppression, stays fixed, while its imaginary part grows in steps, each step the action accumulated over one circuit of that circle."*

The phase counts windings: a staircase

$$S(\alpha) = S_E^0 - i\, N(\alpha)\, S_{\text{period}}$$

$$\Delta S = 2\pi i\, c_{-1} = -i\, S_{\text{period}}, \qquad \sqrt{2V_p(X)} = \cdots + \frac{c_{-1}}{X} + \cdots \quad (X \to \infty)$$



$$S_{\text{period}} = -\frac{\pi\eta^2\omega\,(1+4b^2)}{8b^3\sqrt{1+b^2}}$$

Re S **is fixed** for every angle short of real time — exact by contour deformation, checked to 10 digits. At exactly $\alpha = 0$ the ray runs along the periodic direction: the steps accumulate and the saddle is a pure phase.

Im S **grows** by $|S_{\text{period}}|$ each time the contour crosses a pole pair: one more circuit of the real-time orbit.

Double well: $\sqrt{2V}$ is a polynomial, $c_{-1} = 0$, hence no α dependence (Cherman–Ünsal).

Loubal, Sýkora, Kos, arXiv v2 (2026), §5.1, Fig. 5.2

← back to slide 51

SYMBOLS $S(\alpha)$ action of the continued instanton · S_E^0 Euclidean action · $N(\alpha)$ number of pole pairs (singular angles) crossed · S_{period} action of one real-time period (negative) · c_{-1} coefficient of $1/X$

SOURCES Loubal, Sýkora, Kos 2026

τ

Entanglement and instantons: known, measured, open

KNOWN

area law

Instantons do not break the area law in the models studied.

Bhattacharyya, Hung, Melby-Thompson 2017

COMPUTED

$\theta = \pi$

Entanglement of the θ-vacuum peaks near $\theta = \pi$ (Schwinger model).

Grieninger, Kharzeev, Marroquin 2026

MEASURED

18 ± 4 %

Λ–$\bar{\Lambda}$ spin correlation, 4.4σ, read as spin-aligned s s̄ pairs from the vacuum.

STAR, Nature 650 (2026) 65

OPEN

The 't Hooft vertex fixes the chirality of $2N_f$ quarks — **their entanglement has never been quantified or connected to these data.**

Our family at finite energy has a finite imaginary-time period — **the first ingredient of a reduced density matrix; which bipartition, open.**

← back to slide 68

SYMBOLS area law: entanglement entropy grows with the boundary, not the volume · Λ lambda hyperon (u d s) · σ standard deviation · STAR experiment at RHIC

SOURCES Bhattacharyya, Hung, Melby-Thompson 2017 · Grieninger, Kharzeev, Marroquin 2026 · STAR 2026

From the size integral to events

← back to slide 41

Size integral

$\omega K_1(Q'\rho)$ form factor (MRS 1997) × density $D(\rho)$ (RS98)

↓

InstGen: one shared core

density · valley saddle · 't Hooft vertex · gluon number · colour, helicities

↓

Hadronisation

LHE events → Pythia 8

↓

Observable at fixed M

K/π , strange yield, event shape — ratios, not rates

line	size regulator	status
ep: HERA, EIC	photon form factor (MRS), spacelike — derived	implementation reproduces the RS98 formula (their Fig. 4): median ratio 1.05 over 6.7 decades
$\gamma^*\gamma$ **: FCC-ee**	spacelike — derived, partial	shares the $\gamma^* g$ core; no published anchor yet
pp: LHC	none derived	size integral diverges with the perturbative density — open

Forward protons (pp) do not regulate the size, but they clean the event and measure the mass: $M^2 = \xi_1 \xi_2 s$. Shapes at fixed M are what we trust.

One core for every machine; the size regulator is the machine's job — and only the virtual photon provides one.

SYMBOLS InstGen our instanton event generator · LHE Les Houches event file · $\langle n_g \rangle$ mean number of gluons · M mass of the instanton system · ξ_1, ξ_2 fractional momentum losses of the two forward protons · s squared collision energy

SOURCES Moch, Ringwald, Schrempp 1997 · Ringwald, Schrempp 1998 · Ringwald, Schrempp 2000 (QCDINS) · Taševský et al. 2023

Three open questions, one per field

τ

QUANTUM MECHANICS

Which complex saddles really contribute to real-time tunnelling?

Thimble intersection numbers: open, also in our family (Tanizaki–Koike 2014).

STATISTICAL PHYSICS

What is the entanglement of the θ-vacuum in four dimensions?

At $\theta \neq 0$ the Boltzmann weight is complex: the sign problem.

GRAVITY

Which complex geometries belong in the path integral?

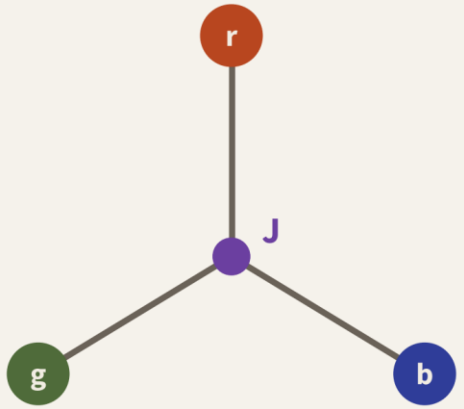
A criterion proposed by Witten (2021), building on Kontsevich–Segal.

← back to slide 42

SYMBOLS Lefschetz thimble = steepest-descent contour through a complex saddle · sign problem = a complex Boltzmann weight that defeats Monte Carlo sampling

Backup: what carries baryon number? The junction proposal

$$B(J) = \epsilon^{abc} \left[\mathcal{P} e^{ig \int_{x_1}^{J} A} q(x_1)\right]_a \left[\mathcal{P} e^{ig \int_{x_2}^{J} A} q(x_2)\right]_b \left[\mathcal{P} e^{ig \int_{x_3}^{J} A} q(x_3)\right]_c$$



three colour lines meet at the junction J

$$\varepsilon_{abc}\, q_a\, q_b\, q_c$$

the colour singlet is the fully antisymmetric state of three colour 'qutrits' — a genuinely three-body entangled state

Proposal, not established: tested by where the baryon number stops in collisions — RHIC now, the EIC next.

Not an instanton effect: the junction is required by gauge invariance. **Link to entanglement:** each quark's colour alone is maximally mixed — entropy ln 3.

SYMBOLS J junction point · ϵ^{abc} totally antisymmetric symbol · a, b, c colour indices · $\mathcal{P}$ path ordering · A gluon field · q quark field · RHIC Relativistic Heavy Ion Collider · EIC Electron–Ion Collider

SOURCES Rossi, Veneziano 1977 · Kharzeev 1996 · Pihan et al. 2024

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IV · Two ATLAS analyses

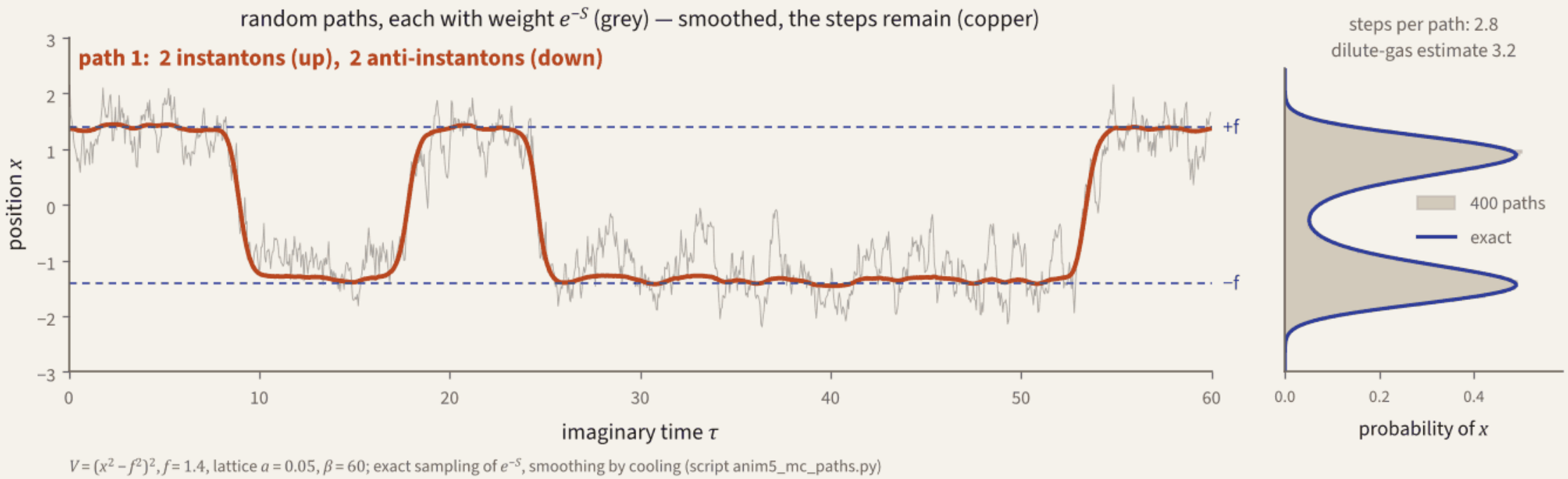
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Backup: the instanton gas, seen in random paths

← back to slide 11



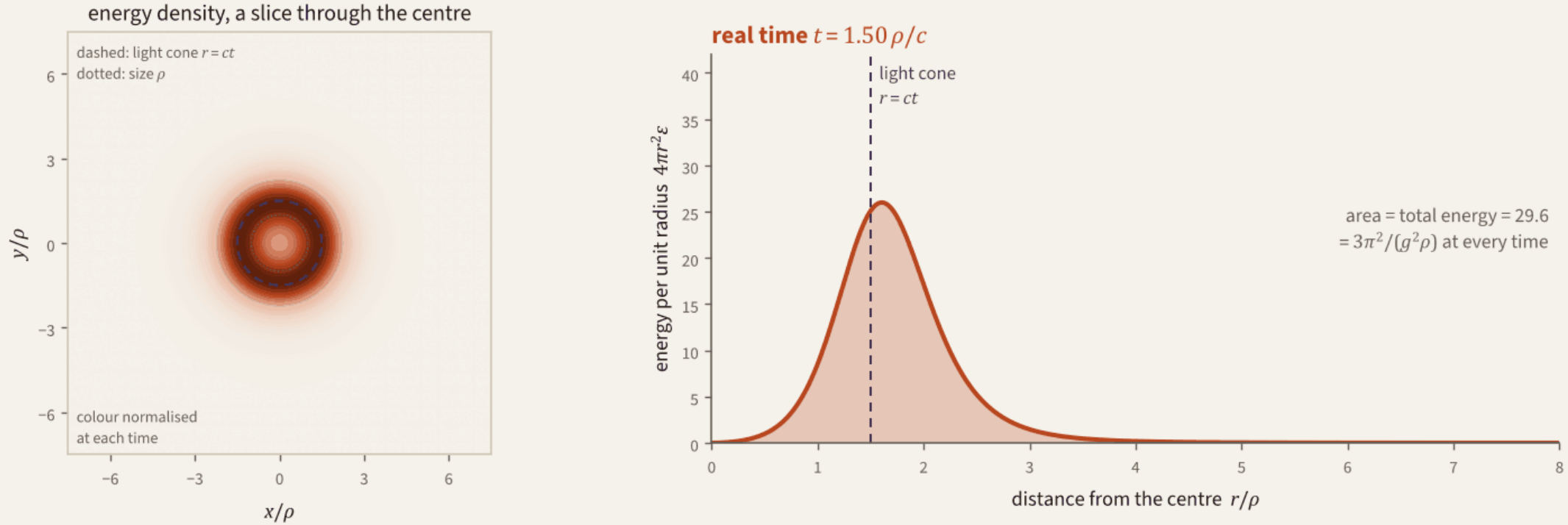
$V = (x^2 - f^2)^2$, $f = 1.4$, lattice $a = 0.05$, $\beta = 60$; exact sampling of e^{-S}, smoothing by cooling (script anim5_mc_paths.py)

Typical paths are noise; smoothed, they are steps between the wells: a dilute gas of instantons and anti-instantons. The sampled paths reproduce the exact probability of x (right).

SYMBOLS τ imaginary time · x position · S the (Euclidean) action of the path · $\pm f$ the two minima · β total imaginary time · cooling = repeatedly lowering the action locally, which removes the small wiggles

Backup: the exploding shell, animated

← back to slide 16



Shuryak–Zahed real-time solution (arXiv:2102.00256, Sec. 4, Eq. 16) at fixed size ρ; computed by sphaleron_explosion.py — not the instanton continued to real time

Not the instanton continued to real time: the barrier-top member at fixed size, energy $3\pi^2/(g^2\rho)$, evolved by the real-time equations. The energy leaves as a shell on the light cone, and the total stays constant.

SYMBOLS t real time · r distance from the centre · ρ the fixed size · c speed of light · energy density = energy per unit volume · $4\pi r^2\varepsilon$ = energy per unit radius (its area is the total energy) · light cone = $r = ct$

Backup: how the numbers were checked

Ammonia inversion line 23.87 GHz	Paris Observatory dictionary, 'ammonia maser' (opened 26 Oct)
Double-well series $-1,\ -9/2,\ -89/2$; ratio $\to 3 = 1/(2S_I g^2)$	figs60.py f09 (exact recursion, 30 orders in g^2)
Splitting: exact ΔE vs instanton formula, ratio $\to 1$	cheat_numbers2.py (diagonalisation)
$T_H = 6.2 \times 10^{-8}\,\mathrm{K}$ for one solar mass; $e^{-4\pi/\alpha_W} = 10^{-164}$	checks_talk60.py (sympy); cheat_numbers.py
$e^{-2\pi/\alpha_s} = 2.3 \times 10^{-14}$ at $\alpha_s = 0.2$	checks_talk60.py
Staircase: $\mathrm{Re}\,S$ constant to 10 digits	LSK v2 §5.1, Eq. (55), Fig. 5.2
Schwinger pair: $d = 2mc^2/e\mathcal{E} = 772\,\mathrm{fm}$ at $\mathcal{E}_c = 1.32 \times 10^{18}\,\mathrm{V/m}$	complex_signature_and_schwinger.py
The two branches are one connected complex curve	one_complex_worldline.py (sympy + numeric path)
EPR 1935: fixed distance d, total momentum zero	Stanford Encyclopedia, 'The EPR argument' (opened 26 Oct)
Influence speed $> 10^4 c$ if Earth $< 10^{-3} c$ in the frame; finite speed $\Rightarrow$ signalling	Salart et al. 0808.3316; Bancal et al. 1110.3795 (abstracts, re-read 26 Oct)
$\langle\rho\rangle = 0.23\,\mathrm{fm}$ at $Q' = 8\,\mathrm{GeV}$ ($b = 7.8 \leftrightarrow \alpha_s \approx 0.34$)	cheat_numbers.py (MRS Eqs. 62, 76)
K/π 0.22–0.26 vs 0.10; strange yield ×2.8–2.9	strangeness_content.py (STRANGENESS_CHECK, 11 Sep 2026)
H1: 2430 vs 2483/2966; limit 2 pb (3.7 expected); 10 ± 3 pb	H1, EPJC 76 (2016) 381, §2, §5, Tab. 1 (re-read 26 Oct)



Backup: the new numbers of Part IV

AFP single tag S/B: 4.9, 1.3, 0.9, 0.3 (M_{inst} > 60 GeV) at $\langle\mu\rangle$ = 0, 1, 2, 5	TKMR Table 1 (event counts), ratios: fzu_numbers.py
Fake-tag probability $1-(1-0.0048)^{\langle\mu\rangle}$	TKMR Eq. 7, Sec. 4.1
AFP acceptance $0.02 < \xi < 0.15$; baseline 0.02–0.05	TKMR Sec. 1, Sec. 4
Double tag with AFP: $M_X \geq 272$ GeV at 13.6 TeV	$M_X = \xi\sqrt{s}$, symmetric; fzu_numbers.py
S^2 = 0.1 (one proton), 0.05 (two); ±50%	TKMR Sec. 3; KKMR Sec. 7.2
One gap, M_{inst} = 20–60 GeV, S/B ≥ 8	KKMR abstract
Central sample ≈ 1.7 nb^{-1} (1.693)	Y. Harris, CERN talk, May 2024, slide 9
2012 SD sample: MBTS + ALFA, 1.67 nb^{-1}, $\mu \leq 0.08$, acceptance ≈ 30%	ATLAS arXiv:1911.00453, Secs. 2, 4, 5
ALFA: 237 / 245 m; 500 μb^{-1} (8 TeV), 340 μb^{-1} (13 TeV)	ATLAS arXiv:2207.12246 Sec. 2; arXiv:1607.06605
AFP: NEAR ≈ 205 m, FAR 217 m; Run 3 34.1, 26.3, 108 fb^{-1}	arXiv:2310.11783 Sec. 1; Clawson PoS DIS2025 121
Public ATLAS status: unblinded, paper in the publication process	Taševský–Ryskin arXiv:2609.15252, Sec. 1

Backup: numbers added in revision 2

k-instanton moduli space of SU(N): dimension $4kN$	Dorey et al., hep-th/0206063, Sec. II.3
one SU(3) instanton: 4 + 1 + 7 = 12 collective coordinates	D: $4N-5$ = dim SU(N)/S(U(N−2)×U(1))
$$E(\theta) = \hbar\omega/2 - 2\hbar K\, e^{-S_0/\hbar}\cos\theta$$	D: tight-binding sum of single hops
RS98 Fig. 4 reproduced: median ratio 1.046 (429 points)	Kalamata deck 2026 (project), validation slide
$\langle n_g \rangle$ = 3.44 at x' = 0.35, Q' = 8 GeV	Kalamata deck 2026; QCDINS Eq. 13
KKMR 2022 Table 1: collinear 0.25–11×; ×9/4: 0.57–25×	Loubal SFG report, Tables 1–2
kT-factorised ×9/4: 0.44–1.68; 75 GeV: 6.93 vs 7.06 pb	kt_resolution (project), kt_recompute.py
9/4 check by Sobol integration: 2.2499, 2.2499, 2.2501	Loubal SFG report, Sec. 4.1
Rosenblatt: distance correlation down > 10^3; ABCD closure ~ 10^{-6}	Vavřička, Sýkora, Schott, arXiv:2509.25521, abstract